

NAVIGATION

Math Un 170-2

8^u

~~Mathem~~

~~p 121~~

R

Robertson

THE

ELEMENTS

OF

NAVIGATION.

BY JOHN ROBERTSON, F.R.S.

Lately published by J. Nourse,

In One Volume in Octavo,

(Price 6 s. bound)

The ELEMENTS of ASTRONOMY ;

With practical rules, whereby the principal phænomena are
determined.

To which is annexed,

A TREATISE of PROJECTION in general, and the computations thence arising, and in particular, the calculation from projections peculiar to eclipses of the sun, and to occultations of the fixed stars by the moon.

Designed for Students in Universities.

The whole translated from the French of M. DE LA CAILLE,
Member of the Royal Academy of Sciences at Paris,

By JOHN ROBERTSON, F. R. S.

With Additions and Corrections communicated by the Author.

THE
ELEMENTS
OF
NAVIGATION;

CONTAINING THE
THEORY and PRACTICE.

With all the necessary TABLES.

To which is added,

A TREATISE of
Marine Fortification.

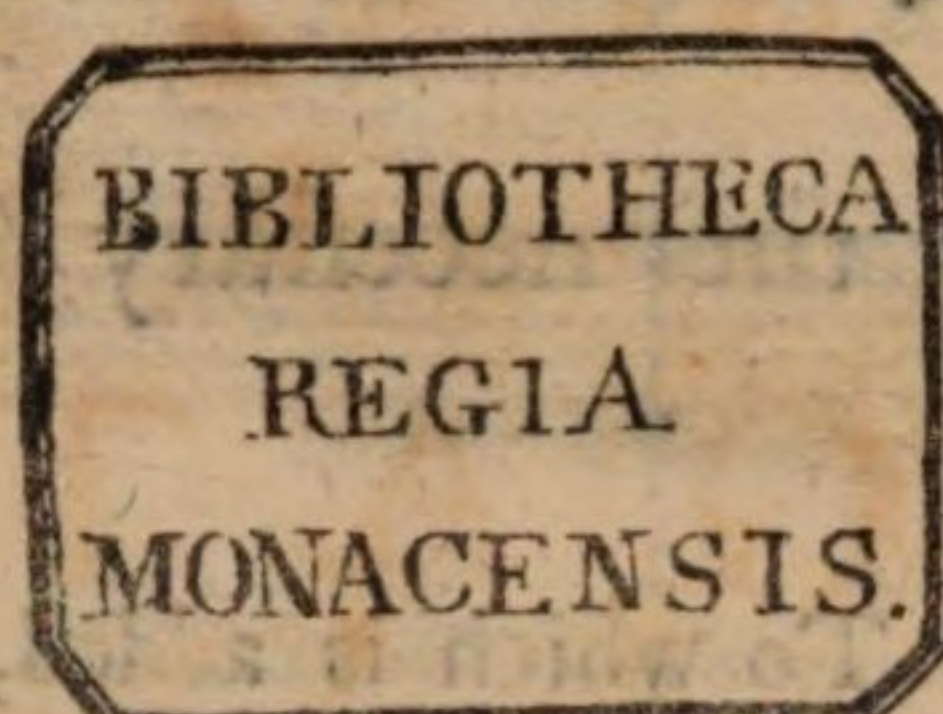
For the USE of the ROYAL MATHEMATICAL SCHOOL at
CHRIST'S HOSPITAL, and the GENTLEMEN of the NAVY.

VOL. II.

By J. ROBERTSON, F.R.S.
Mathematical Master at CHRIST'S HOSPITAL.

LONDON,
Printed for J. Nourse, at the *Lamb*, opposite *Katherine-*
Street, in the *Strand*. MDCCLIV.

THE
ELEMENTS
OF
NAVIGATION;



Marine Fortification.

For the Use of the Royal Mathematical School at
Christ's Hospital, and the Gentlemen of the Navy.

VOL. II.

By J. ROBERTSON, F.R.S.

Mathematical Master at Christ's Hospital.

L O N D O N,

Printed for J. No. 1, at the Lane, opposite Kew
Gardens, in the Strand. 1793.



THE ELEMENTS OF NAVIGATION.

BOOK VII. OF GLOBULAR SAILING.

SECTION I.

Definitions and Principles.

713. **B**Y Globular sailing, is meant the manner of estimating either the difference of latitude, difference of longitude, departure, course or distance, from any two of them being known, upon principles deduced from the spherical figure of the earth.

In the preceding parts of sailing, the operations were performed as if the courses and distances of ships had been run on a plane, or flat surface: But that this cannot be true, appears from what was said in art. 553. To which, as a farther proof, there may be added, that mariners, as they sail from northern or southern latitudes towards the equator, observe the stars about the poles constantly to diminish their altitude or height above the apparent horizon: It is also well known, that in eclipses of the moon, the darkened part is bounded by a circular curve; and consequently, the body which casts the shade, or obstructs the light, must be bounded by a like curve: But as this obscuration is caused by the earth, consequently its surface must be limited by a circular figure, that is, it must be globular.

714. As the meridians are circles on this globe, meeting in the poles, it is obvious that any two of these circles must recede more from one another at greater distances from the poles; and at equal distances from both poles, or at the equator, the distance between the meridians is the greatest.

The true place of a ship at sea, depends both upon its distance from the equator, and from some noted meridian; and because the meridional distance, that is, the distance between two meridians, varies in every latitude; therefore it is convenient this distance should be reckoned in a fixed latitude, and where the degrees are of the same magnitude with those of the meridian, which can be no where but at the equator, where 60 nautical miles make a degree.

715. Now a ship sailing from any latitude upon an oblique rhumb, until she has made a known departure, will, by the rules of plane sailing, reckon herself under the same meridian, whether she sails to the northward or southward: But from what has been said, this cannot be true; for the same meridional distance, which in this case is taken as departure, will not serve two different latitudes on the same side of the equator: Therefore the common methods of plane sailing alone, are not sufficient to shew the ship's true place, a necessary correction being wanted with respect to her longitude.

This incorrect method of plane sailing was founded on the manner in which the sea charts were at first constructed: For it being the most convenient way to represent the meridians, parallels of latitude, and rhumb lines in such charts by right lines, where the meridians should be parallel, and the east and west lines cut them at right angles: Consequently the distances of the meridians in every parallel were equal to their distances at the equator; and a degree on any parallel of latitude, as large as a degree on the equator; whereby each degree of longitude in those parallels was increased beyond its just size, in proportion as the equator, or its radius, is greater than the parallel, or its radius.

716. Now to remove these material objections against the sea charts, a different method of construction should be used; and it had been hinted, some hundred years since, by several persons, that the degrees of latitude in such charts should be lengthened as they approach the poles; and accordingly, about the year 1550, the first chart of this kind was published by Mr. *Gerrard Mercator* a Fleming; wherein the degrees of latitude were gradually lengthened from the equator towards the poles; but in what manner, or how the chart was constructed, he did not shew.

717. About the year 1590 Mr. *Edward Wright* an Englishman discovered the true principles upon which such a chart should be constructed; which was to increase the degrees of latitude as they approached the poles, in the same proportion as the degrees of longitude between any two meridians decreased on the globes: But it was not till the year 1599 that

that he published his errors in Navigation; wherein he shews the principles of the true sea chart, with the method of constructing it: Nevertheless, the charts made on this construction are called Mercator charts.

A true method of computing a ship's place at sea does not appear to have been known till Mr. Wright published the said book; therefore altho' the chart may properly enough be called *Mercator's*, yet the method of sailing called Mercator's, should, in justice, be named from Mr. *Wright*.

718. The difficulty in constructing a true sea chart, seems to have consisted in the finding a proper manner how to apply the surface of a globe to a plane, which Mr. *Wright* happily accomplished by a most ingenious conception; the substance whereof is as follows.

1st. Suppose a rectangular plane was rolled about the globe until the edges of the plane met, and formed a kind of concave cylinder inclosing the globe, and touching its equator.

2d. Conceive the surface of this globe to swell (like a bladder while it is blowing up), from the equator towards the poles, proportionally in latitude as it does in longitude, until every part of its surface meets that of the concave cylinder, and thereon impress the lines that were drawn on the globular surface.

3d. Then the cylinder, or rectangular plane, being unrolled, it will represent a sea chart, whose parts bear the same proportion to one another, as do the corresponding parts on the globe: But herein all the lines will be right lines.

For, in this formation of the nautical chart, every parallel of latitude on the globe will be increased till it is equal to the equator; and so the distance of the meridians in those parallels will become equal to their distance at the equator; consequently the meridians on the chart are expressed by parallel right lines.

Also, the meridians being lengthened as the parallels increased, every degree of latitude is lengthened in the same proportion as the degrees of longitude are increased; therefore the distances of the parallels of latitude become wider and wider as they approach the poles: But these parallels are also right lines.

Again. As the rhumb lines on the globe cut the meridians at equal angles, they will also on the chart cut the meridians at equal angles, and consequently be expressed by right lines; since none but right lines can cut several parallel right lines, at equal angles.

The construction of such a chart, Mr. Wright deduced from the following principles.

SECTION II.

Of the reckoning of longitude.

719.

PROPOSITION I.

*The distance between any two meridians at the equator,
Is in proportion to their distance in any parallel of latitude;
As the Radius,
To the cosine of that latitude.*

DEMONSTRATION. See Fig. 1. Pl. VIII.

Let PDFE represent the fourth part of a sphere; E being the centre, P the pole, ED the radius of the equator, AB the radius of a parallel of latitude: Then the arcs PBD, PCd, will each represent a quadrant of a meridian; Dd is an arc of the equator, and BC an arc of a parallel of latitude; also the arc DB expresses the latitude, and PB the complement of the latitude, whose right sine is BA. (298)

Now (262) the circumference of a circle whose radius is ED, is to the circumference of a circle whose radius is AB, as ED is to AB; that is, as the radius of the equator, to the radius of the parallel; or, as radius to the cosine of the latitude.

But like arcs are in the same ratio of their circumferences. (233, 262)

And like arcs Dd, BC, are intercepted between the same two meridians; therefore

An arc of the equator, is to a like arc in any parallel of latitude;
As the Radius, to the cosine of that latitude.

720. COROL. Hence it is easy to construct a table shewing, from the equator to the poles, the approach of two meridians in any latitude; or in what proportion the degrees of longitude diminish in every latitude.

721.

PROPOSITION II.

Any part of a parallel of latitude, is to a like part of a meridian, as Radius to the secant of the lat. to that parallel.

DEMONSTRATION. See Fig. 2. Pl. VIII.

Let PDE represent a quadrant of a meridian, where P is the pole, and DE the radius of the equator; AB is the radius of a parallel of latitude, or the cosine of the latitude whose sine is BF, and secant EC.

Then $EF : EB :: ED : EC$. (329)

Or $\cos. lat. : Rad :: Rad : \secant \text{ of the lat. to that parallel.}$

But $\cos. lat. : Rad :: \text{part of a parallel} : \text{like part of the equator.}$ (719)

Therefore, part of a parallel of latitude, is to a like part of the equator;
As the Radius, is to the secant of the lat. to that parallel.

Now like parts of the meridian and equator are equal.

Therefore any part of a paral. of lat. is to a like part of a meridian;
As Radius, to the secant of the lat. to that parallel.

722.

PROPOSITION III.

The distance of any parallel of latitude (A) from the equator, is expressed by the sum of the secants of all the arcs between the equator and that parallel.

DEMONSTRATION.

Now (721) As Radius, to the secant of the latitude A ;

So is a diminish'd deg. long. in lat. A , or a degree of that parallel, to a degree of the meridian.

But the degrees of latitude, or of the meridian, are to be lengthened in proportion as the degrees of longitude diminish. (717)

Therefore, As Radius, to the secant of the latitude A ;

So is a natural deg. of the meridian, to a lengthened deg. in lat. A .

Here radius being as unity, and one natural deg. as unity also;

Therefore the length of a degree in any latitude, is as the secant of that latitude, or may be expressed by that secant.

Now the distance of any parallel from the equator, is the sum of all the successive arcs between the equator and that parallel.

Consequently the distance of that parallel, is expressed by the sum of the secants of all these arcs between the equator and that parallel of latitude.

723. COR. Hence it appears, that by the addition of the secants of small arcs, the distances of the parallels of latitude from the equator are obtained.

And these several distances, which are called *meridional parts*, being disposed in a table corresponding to the degrees and minutes in a quadrant, form a table of meridional parts.

And this was the excellent discovery of the celebrated Mr. Wright, by which the art of Navigation received greater improvements than by whatever had been done before, the discovery of the mariner's compass excepted; and indeed, in this and other things, he so far perfected the art, that little else was wanted beside an accurate method to find the longitude at sea. And altho' there have been since his time many eminent men who have done excellent things towards the rendering of Navigation more perfect, yet if the real use of such things be strictly considered, it will, perhaps, be found that they amount to no more than other methods of performing the useful operations; except what has been done by Mr. Richard Norwood and Dr. Halley.

724. Mr. Wright made his table for the division of the nautical meridian, or the table of meridional parts, as follows.

The meridional parts for 1 min. he made equal to the secant of 1 min.

The mer. parts to 2 m. equal to the sum of the sec. of 1 and 2 min.

The mer. parts to 3 m. equal to the sum of the sec. of 1, 2 and 3 min.

The mer. parts to 4 m. equal to the sum of the mer. parts of 3 min. and the sec. of 4 min.

And so on by a constant addition of the secants,

Now

Now although a table thus made is abundantly sufficient for all nautical uses, yet had the secants of smaller parts than minutes been taken, the table would have been more correct: And therefore Mr. Oughtred, Sir Jonas Moore, Dr. Wallis, Dr. Halley, and others, have been induced to find methods of constructing these tables with more accuracy than by the addition of the secants to every minute.

But a table of meridional parts constructed by the most accurate method, only shews that Mr. Wright's table does no where exceed the true meridian parts by half a minute, and this only near the pole; for in latitudes as far as navigation is practicable, the difference is scarce sensible.

A table of meridional parts being obtained (see art. 827), the meridians and parallels of latitude for a sea chart, wherein the situation of places will bear the same proportion to one another as on the globe, may be constructed by the following

725.

PROPOSITION IV.

To construct a sea chart which shall contain a given number of degrees of latitude and longitude.

Suppose from the latitude of 23 degrees N. to the lat. of 59 degrees N., and for to contain 80 degrees of west longitude from the meridian of London.

SOLUTION.

1st. Write the several included degrees of latitude orderly under one another in a column, the greatest being uppermost.

2d. Then beginning at the lower one, against each degree write, in another column, the differences between the meridional parts of the lesser latitude, and those of the other given degrees of latitude.

3d. Divide each of these meridional parts by 60, setting the quotients against their respective dividends, in another column; and these will be the meridional parts reduced to degrees of longitude, and express the measures of the divisions for the meridional degrees.

If the meridional parts to divisions between the degrees were wanted, they are to be taken from the table of meridional parts in the same manner.

These three articles are exemplified in the following table; which, altho' made to serve from the equator to 72 degrees of latitude, yet it will serve for any intermediate limits, by subtracting the meridional degrees belonging to the latitude where the chart is to begin at, from those of all the latitudes the chart is to contain; of the several remainders make a table, and these will be the numbers to be transferred to the chart, as shewn in what follows.

A TABLE

A TABLE for constructing a Mercator chart.

Lat.	Mer. Parts	Mer. Deg.	Lat.	Mer. Parts	Mer. Deg.	Lat.	Mer. Parts	Mer. Deg.	Lat.	Mer. Parts	Mer. Deg.
72	6335	105,6	54	3865	64,4	36	2318	38,6	18	1098	18,8
71	6146	102,4	53	3764	62,7	35	2244	37,4	17	1035	17,2
70	5966	99,4	52	3665	61,1	34	2171	36,2	16	0973	16,2
69	6795	96,6	51	3569	59,5	33	2100	35,0	15	0910	15,2
68	5631	93,8	50	3475	57,9	32	2028	33,8	14	0848	14,1
67	5474	91,2	49	3382	56,4	31	1958	32,6	13	0787	13,1
66	5324	88,7	48	3292	54,8	30	1888	31,5	12	0725	12,1
65	5179	86,3	47	3203	53,4	29	1819	30,3	11	0664	11,1
64	5040	84,0	46	3116	51,9	28	1751	29,2	10	0603	10,1
63	4905	81,7	45	3030	50,5	27	1684	28,1	9	0542	9,0
62	4775	79,6	44	2946	49,1	26	1616	26,9	8	0482	8,0
61	4649	77,5	43	2863	47,7	25	1550	25,8	7	0421	7,0
60	4527	75,5	42	2782	46,4	24	1484	24,7	6	0361	6,0
59	4409	73,5	41	2702	45,0	23	1419	23,6	5	0300	5,0
58	4294	71,6	40	2623	43,7	22	1354	22,6	4	0240	4,0
57	4183	69,7	39	2545	42,4	21	1289	21,5	3	0180	3,0
56	4074	67,9	38	2468	41,1	20	1225	20,4	2	0120	2,0
55	3968	66,1	37	2393	39,9	19	1161	19,3	1	0060	1,0

4th. Draw a line AB (see Plate VII.) to represent a part of the parallel of latitude where the chart is to begin (which in this example is the parallel of 23°); and on this line, by the help of a scale of equal parts, lay a number of successive divisions to express the proposed number of degrees of longitude.

Thus 80 degrees of longitude being proposed; from some scale take one of the primary divisions, and this distance being repeated 8 successive times in the line AB, will give divisions to every 10th degree of longitude; and then let the intermediate degrees be inserted, whereby AB will be divided into 80 equal parts of that scale; which in this example, is the scale where 13 subdivisions or degrees make an inch: And in like manner the parts of degrees may be inserted.

5th. From the ends A, B, draw lines AD, BC, perpendicular to AB; and on these lines lay the divisions expressed by the meridional degrees in the foregoing table, beginning at the line AB.

Thus did the chart begin at the equator, and go to 70 degrees of latitude; for 70 deg. take 99,4; for 69 deg. take 96,6; for 68 deg. take 93,8, &c.; and these divisions taken from the same scale, and laid severally on AD, BC, will give the divisions for the degrees of latitude increased in their true proportion; and the like for the intermediate parts of degrees.

But

But the chart beginning at 23 deg. of lat. the meridional degrees 23,6 answering to 23 deg. of lat. are to be subtracted from those belonging to all the latitudes introduced in the chart, and the respective remainders are to be successively laid off from A and B towards D and C. Thus

Latit.	59°	58°	57°	56°	55°	54°	53°	52°	51°	&c.
From	73,5	71,6	69,7	67,9	66,1	64,4	62,7	61,1	59,5	&c.
Take	23,6	23,6	23,6	23,6	23,6	23,6	23,6	23,6	23,6	&c.
Leaves	49,9	48,0	46,1	44,3	42,5	40,8	39,1	37,5	35,9	&c.

For the several meridional degrees of latitude to be taken from the same scale used for the degrees of longitude.

6th. Draw lines thro' the several corresponding divisions of latitude, and they will represent the parallels of latitude: Also lines drawn thro' the divisions of longitude, will express the meridians or parallels of longitude: But in this example, the meridians and parallels are drawn only to every 10 degrees.

7th. From some point of the chart as a centre, let the rhumb lines be drawn.

726. Note, the difference between the meridional parts to two given latitudes, is called the meridional difference of latitude; to distinguish it from the difference of latitude of those two places, reckoned in miles of the equator, and which is called the proper difference of latitude.

In a chart thus constructed, every place may, by its latitude and longitude, be laid down in its true position, with regard to other places; and with as much accuracy as on the globe itself; the distances of the meridians and parallels of latitude having the same proportion to one another in both: Thus on the globe, the distance of two meridians in the parallel of 60 degrees is half their distance at the equator, because the cosine of 60 degrees is half the radius (319); so in the chart, at the latitude of 60 degrees, the meridional difference of latitude to 1 minute is double to the meridional difference of latitude of 1 minute from the equator, the secant of 60 degrees being double to the radius (329); and the like analogy holds in other corresponding latitudes.

Now as the meridians are parallel right lines, the parallels of latitude cutting the meridians at right angles, and the rhumbs also being right lines; therefore such a chart is of more service to the mariner for most purposes, than the globe is.

The properties of the several lines and angles concerned in sailing by this chart, will be shewn in the following propositions.

PROPOSITION

PROPOSITION V.

727. *If a ship sails on any oblique rhumb, the proper difference of latitude to the departure, the meridional difference of latitude to the difference of longitude, and the radius to the tangent of the course, are all in the same ratio. See Fig. I. Plate VII.*

DEMONSTRATION.

Suppose a ship from P in the latitude of 30 deg. N. should sail on the rhumb Ps, to s in the latitude of 40 degrees north.

Now because she has made 10 deg. of diff. lat.; make PO, in the meridian, equal to 10 degrees of the equator; and draw OR parallel to QS, meeting the rhumb Ps in R.

Then PO expresses the proper difference of latitude.

OR the departure.

PR the distance sailed, in equatorial measure.

$\angle OPR$ is the angle of the course.

And R is the apparent place of the ship.

But she having made 10 deg. diff. lat. should be in the parallel of 40 degrees; therefore her place, when corrected by longitude, will be in s.

Then QS will be the difference of longitude.

And PQ the meridional difference of latitude.

Now the triangles POR, PQS, are similar.

Therefore $PO : OR :: PQ : QS.$ (250)

Also $PO : OR :: \text{Rad} : \text{t. } \angle OPR.$ (341)

Consequently. As the proper diff. lat., to the departure;

So the merid. diff. lat., to the diff. longitude.

And So is the Radius, to the tang. of the course.

PROPOSITION VI.

728. *The tangents of the angles which different rhumbs make with the meridian, are directly proportional to the differences of longitudes made on those rhumbs to equal meridional differences of latitude: Or, are reciprocally proportional to unequal meridional differences of latitudes on those rhumbs, when the differences of longitude are equal. See Fig. I. Plate VII.*

DEMONSTRATION.

Let QS, QV, be the differences of longitudes made on the rhumbs Ps, PV, to the common merid. diff. lat. PQ; also PW, PQ, be the meridional differences of latitudes, on the same rhumbs, to the equal differences of longitudes WX, QV.

Now $PQ : QV :: R : \tan. \angle QPV$ (341). Therefore $QV \times R = PQ \times \tan. \angle QPV.$ (245)

Also $PQ : QS :: R : \tan. \angle QPS.$

Consequently $QV : QS :: \tan. \angle QPV : \tan. \angle QPS.$ (238)

Again. $PW : (WX =) QV :: R : \tan. \angle WPX.$

Therefore $QV \times R = PW \times \tan. \angle WPX.$ (245)

But $QV \times R = PQ \times \tan. \angle QPV,$ found above.

Therefore $PW \times \tan. \angle WPX = PQ \times \tan. \angle QPV.$ (137)

Consequently $PQ : PW :: \tan. \angle WPX : \tan. \angle QPV.$ (246)

A a a

PROPOSI-

PROPOSITION VII.

729. *If a ship sails on several different rhumbs, the whole meridional diff. lat. is equal to the sum, or differences of all the successive meridional differences of lat.: And the whole diff. long. is also equal to the sum or differences of all the successive differences of longitudes: Observing to take the sums, when of the same name; and the differences when of contrary names, See Fig. 2. Plate VII.*

DEMONSTRATION.

1st. Suppose a ship sails from E in the lat. of 23° N. to G in the parallel of 26° deg. N. Then $Eg = \text{mer. diff. lat.}$ and $Gg = \text{diff. long.}$

2d. From G to H in the parallel of 28° deg. Then $Gh = \text{diff. lat.}$ and $hH = \text{diff. long.}$

Now $Em = Eg + Gh$, is the whole diff. latitude.

And $mH = mh - hH$, is the whole diff. longitude.

3d. She sails west from H to I: Here she only alters her longitude by HI; so that her present diff. long. is mI .

4th. In sailing from I to K, she increases her diff. long. by kK , from mI to gK ; and decreases her diff. lat. by Ik , from Em to Eg .

5th. In sailing north from K to P, the diff. long. pP continues the same: But the diff. lat. is increased by KP , from Eg to Ep .

6th. In running east from P to M, the diff. lat. Ep does not alter: But the diff. long. is diminished from pP to pM .

7th. By going from M to N; the diff. lat. is increased by Ml , from Ep to Eq : And the diff. long. is changed from pM , or ql to qN .

Consequently, the whole diff. lat. Eq is equal to the sums, or differences of all the differences of lat.; and the same of qN , the whole difference of longitude.

PROPOSITION VIII.

730. *If the course between any two places, outwards and homewards, be on the opposite rhumbs; then in each case, the differences of latitude, departures, and differences of longitude, will always be the same. See Fig. 3. Plate VII.*

DEMONSTRATION.

Suppose a ship at a in lat. 25° deg. N. should sail on one rhumb to the latitude 40° deg. N. so as at the end of every third day to alter her diff. lat. 5° deg. or 300 miles; and to return on the opposite rhumb in the same manner.

Thro' the intersections a of this rhumb with the parallel of every 5° deg. draw meridians; make ad , af , each equal to 5 common deg. of the equator; and draw db , fe , parallel to the parallels of latitude.

Then are the lines ad , the proper differences of latitudes outwards to each third day's account, which must be every where equal; and also, the equal lines af , are the differences of latitudes homewards, to the like accounts.

The

The lines db outwards, and fe homewards, are the departures to each third day's account.

The lines ab outwards, and ae homewards, are the distances run to each third day's account.

The meridian lines ac outwards, and an homewards, are the meridional differences of latitude; and the west and east lines ca outwards and na homewards, taken on the parallels of latitude, are the differences of longitude.

Now it is evident from the construction, that the triangles adb , afe , in each account are equal to one another.

Also, that the meridional differences of latitude, and the differences of longitude, to the accounts between the same parallels of latitude outwards and homewards, are respectively equal to one another.

Consequently the truth of the proposition is manifest.

PROPOSITION IX.

731. *The divisions of the meridian line on the Mercator's chart, is a table of the differences of longitude answering to each minute of latitude on the rhumb line making an angle of 45 degrees with the meridian. See Fig. 1. Plate VII.*

DEMONSTRATION.

Let PV be a rhumb of 45 degrees.

Then since PQ is at right angles to QV .

Therefore $\angle QPV = \angle QVP$, each being half a right angle:

Consequently $QV = QP$. (194)

Now PQ is the sum of all the meridional differences of latitude between the parallels thro' P and Q . (723)

And QV is the sum of all the differences of longitude between the meridians thro' Q and v .

Therefore on a rhumb of 45 degrees, the sum of all the differences of longitude, is equal to the sum of all the meridional differences of latitude.

But the sum of all the meridional differences of latitude, is the same as the nautical meridian. (724)

Consequently. On a rhumb of 45 deg. the difference of longitude, or the sum of all the intermediate differences of longitude to every minute of latitude, is equal to the corresponding parts of the nautical meridian line.

732. In the Philosophical Transactions, N^o 219, Dr. Halley has given a very curious paper relating to the division of the nautical meridian, by a quite different method from Mr. Wright's; and because it contains a method of performing the problems of sailing according to the true chart, by the help of the common table of logarithmic tangents, without a table of meridional parts; therefore the substance of that paper is here annexed.

The Doctor says: "It was first discovered by chance, and, as far as I can learn, first published by Mr. *Henry Bond*, as an addition to Norwood's epitomy of Navigation, about the year 1645, that the meridian line was analogous to a scale of logarithmic tangents of half the complements of the latitude. The difficulty to prove the truth of this proposition, seemed such to Mr. Mercator," (an ingenious mathematician then living, and) "the author of the *Logarithmotechnia*, that he proposed to wager a good sum of money, against whoso would fairly undertake it, that he should not demonstrate either, that it was true or false: And about that time Mr. John Collins holding a correspondence with most of the eminent mathematicians of the age, did excite them to this enquiry.

"The first that demonstrated the said analogy, was the excellent Mr. James Gregory in his *Exercitationes Geometricæ*, published anno 1668, which he did, not without a long train of consequences and complication of proportions, whereby the evidence of the demonstration is in a great measure lost, and the reader wearied before he attains it:" The Doctor proceeds and says, "nor hath any one, that I know of, yet discovered the rule for computing independently the interval of the meridional parts answering to any two given latitudes."

The Doctor then in a very elegant manner gives the demonstration of the said analogy; and draws several useful conclusions: But as himself confesses, he is too concise for learners; and the writers on this subject have been rather more, or at least as concise as the Doctor; therefore in order to set this affair in a clear light, and keeping to the Doctor's principles, it was thought convenient to dispose of the subject in the following manner: But the said proposition may be proved from very different principles than those used by the Doctor.

PROPOSITION X.

733. *On the Globe, the rhumb lines oblique to the meridians are spirals, which continually approach the poles but never meet them.*

DEMONSTRATION.

In any place on the surface of the globe, the rhumb running north and south, coincides with the meridian of that place; the east and west rhumbs are perpendicular to the meridian, and the other rhumbs are oblique to it: But this obliquity is the same under every meridian; therefore all the rhumbs, except the north and south, cut the meridians at equal angles.

When right lines are parallel to each other, a right line will cut them at equal angles (185): But not so when the lines are inclined to one another; therefore several inclining lines cannot be cut at equal angles but by a curve line bending towards the place where those lines would meet.

Now the meridians on the globe meeting in the poles, are inclined to one another; therefore the oblique rhumbs cutting them at equal angles, are curve lines continually approaching the poles.

But

But in every latitude, an oblique rhumb runs between the present parallel and the pole.

And a line cannot cut several other lines at equal angles in one point.

Consequently the rhumb lines are spirals, which continually wind round the poles without ever falling into them.

PROPOSITION XI.

734. *The spiral rhumbs on the globe are of the same kind with the proportional spiral. See Fig. 3. Plate VIII.*

DEMONSTRATION.

Let CAEB represent part of the stereographic projection of the sphere on the plane of the equator; where AEB is part of the equator, c the pole, CA, CD, CE, &c. are meridians; and the spiral Adef one of the rhumbs.

Now in every stereographic projection, the lines therein, intersecting each other, form angles equal to the angles on the sphere which they represent. (432)

Therefore the projection of the rhumb line Adef, cuts the radii or meridians AD, AE, &c. at equal angles.

But this is a property also of the proportional spiral. (291)

Therefore the spiral rhumbs are analogous to the proportional spiral.

735. COR. Hence the differences of longitude AD, AE, AF, &c. are the logarithms of the intercepted parts cd, ce, cf, &c. of the meridians. (292)

PROPOSITION XII.

736. *The logarithmic tangents of the half complements of the latitudes, are analogous to the lengthened degrees in the nautical meridian line.*

DEMONSTRATION. See Fig. 3. Plate VIII.

Let the figure CADEB be part of the stereographic projection of the sphere on the plane of the equator as in the last proposition: And on any rhumb, let the points A, d, e, f, &c. represent certain places on the globe, whereof A is a point in the equator; then will Dd, Ee, Ff, &c. express their latitudes; and the arcs AD, AE, AF, &c. their differences of longitude from A; also c being the pole, the lines cd, ce, cf, &c. are the complements of latitude.

Now in this projection, the lines cd, ce, cf, &c. are equal to the tangents of half the co-latitudes. (428)

And the differences of longitude AD, AE, AF, &c. are the logarithms of those tangents, or of the lines cd, ce, cf, &c. (292)

Therefore the differences of longitude are as the logarithmic tangents of the half co-latitudes.

But the lengthened degrees on the nautical meridian, are as the differences of longitude on the rhumb of 45 deg. (731)

Conf.

Consequently, the log. tangents of the half co-latitudes, are as the lengthened degrees on the nautical meridian.

737. COR. I. When the angle between the rhumb line and the meridian is equal to 45 deg.; then the longitudes of places on that rhumb are expressed by logarithms of Nepier's kind (295); whose corresponding numbers are natural tangents of the half co-latitudes (not to degrees and minutes, but) to arcs expressed in parts of the radius.

738. COR. II. Hence to any two places on a rhumb of 45 deg. the difference of longitude, or the meridional difference of latitude (731), is equal to the difference of the Neperian logarithm tangents of the half co-latitudes of those places, estimated in parts of the radius.

739. COR. III. As there may be an indefinite variety of rhumbs, and therefore as many different kinds of logarithms (294); consequently every species of logarithms has its peculiar rhumb, distinguishable by the angle it makes with the meridian: And among these must be two, whereto the differences of longitudes are the differences of the logarithm tangents of the half co-latitudes, estimated in minutes of a degree, one of which belongs to Nepier's form of logarithmic tangents, and the other to Briggs's, or the common logarithmic tangents.

740. COR. IV. Hence in every point of these rhumbs, the difference of the logarithms answering to the radii of those points (or of the logarithmic tangents of the half co-latitudes of those points) are the true differences of longitude made by a ship in sailing on that rhumb thro' the latitudes of those successive points.

PROPOSITION XIII.

741. *The common logarithmic tangents, are the same as a table of the differences of longitude to every minute of latitude, on the rhumb line making an angle with the meridian of $51^{\circ} 38' 09''$.*

DEMONSTRATION.

Suppose z to represent the meridional difference of latitude between two places on the rhumb of 45 degrees; or (736) the difference between the logarithmic tangents of the half co-latitudes of those places; estimated either in parts of the radius, or in minutes.

Then, As the circumf. in parts of the radius 62831,853, (270)

To the circumf. in min. of a degree 21600;

So is a mer. diff. lat. in parts of the radius z ,

To a mer. diff. lat. in min. of a degree $0,34377468 \times z$,

whose corresponding rhumb is different from that which z belonged to; and the angle which this rhumb makes with the meridian will be found by art. 728.

For, As the mer. diff. lat. in min. on one rhumb $0,34377468 \times z$,

To a merid. diff. lat. in min. on a rhumb of 45° , z ;

So is the natural tangent of the rhumb of 45° 10000,

To the natural tangent of the other rhumb 29088,821,

which answers to $71^{\circ} 01' 42''$; and under this angle is the meridian intersected by that rhumb line, on which the differences of Nepier's logarithmic

garithmic tangents of the half co-latitudes, are the true differences of longitude estimated in minutes and parts.

But Nepier's logarithms are to Briggs's as 2,30258, &c. to 1 *.

Therefore 2,30258, &c. : 1 :: 29088,821 : 12633,114, &c. which is the tangent of $51^{\circ} 38' 09''$, wherein the meridian is cut by the rhumb, on which the differences of Briggs's, or the common log. tangents of the half co-latitudes, are the true differences of longitude corresponding to those latitudes.

PROPOSITION XIV.

742. *The difference of the common logarithmic tangents of the half co-latitudes of any two places, to the meridional difference of latitude in minutes, is in the constant ratio of 1263,3 to 1 †.*

DEMONSTRATION.

The com. log. tangents are as the differences of longitude on the rhumb (A) of $51^{\circ} 38' 09''$, whose natural tangent is 12633114. (741)

The nautical meridian is a scale of longitudes on the rhumb (B) of 45° (731), whose tangent being equal to the radius, may be expressed by unity.

And the differences of longitude to equal differences of latitude on different rhumbs, being to each other, as the tangents of the angles those rhumbs make with the meridian. (728)

Therefore, As the tangent of A ($51^{\circ} 38' 09''$) = 1263,3,

To the tangent of B (45°) = 1000;

So is the diff. longitude on A, or the diff. log. tan. of the half co-latitudes of two places,

To the diff. longitude on B, or the meridional diff. latitudes of those places.

In this manner may a table of meridional parts be readily constructed anew, or the former ones be examined, if it was thought necessary: But the tables in common use having gone through many careful hands, they may be received as sufficiently correct.

From the preceding propositions may all the problems relating to the true method of sailing be accurately resolved; and altho' they are here all contained under the general title of Globular sailing, yet as the common custom has been to distinguish different kinds, wherein the longitude is considered; so here, in compliance with such custom, it was thought proper to retain the usual forms, viz. *Parallel sailing*, *Middle latitude sailing* and *Mercator's sailing*, or more properly *Wright's sailing*; which are distinctly treated of in what follows.

* Nepier's log. of 10 is 2,30258 &c. (78)

And Briggs's, or the common log. of 10, is 1. (75)

† When the tables of logarithms have 7 decimal places, then 1263,3 to 1 may be used for the constant ratio: But if they consist of fewer places, as of 6, then $\frac{1}{10}$ th of this number 1263,3 is to be used; if of 5 places then take $\frac{1}{100}$ th &c.; which is readily done by removing the comma as many places to the left hand as the tables consist of places less than seven.

SECTION

SECTION III.

Of Parallel sailing.

743. PARALLEL SAILING is the art of finding what distance a ship should run due east or west, in sailing from the meridian of one place to that of another given place, in any parallel of latitude.

This kind of piloting is generally used in conducting a ship to an island detached from the main land, or other islands; and also on other occasions.

The method of performing which, is by failing to the parallel of latitude the place is in, keeping a good account, so as to be certain whether the place is then to the eastward, or westward; and also, if possible, to know the longitude arrived at; and then to run due west, or east, until the ship comes near the longitude of the given place; where she is then sure to make the port required.

The computations in parallel failing depend on the following

RULE.

744. *As Radius,* = R.
To the cosine of the latitude of any parallel; = S.
So is the miles of longitude between any two meridians, = L.
To the distance of those meridians in that parallel. = D.

For the demonstration see art. 719.

From the same proposition is easily deduced the following

RULE.

745. *As the cosine of one latitude,* = S.
Is to the cosine of another latitude; = s.
So is a given meridional distance in the first parallel, = D.
To a corresponding meridional distance in the second parallel. = d.

746. And hence arise the following proportions.

$$\begin{array}{l} R : S :: L : D. \\ S : R :: D : L. \\ L : D :: R : S. \\ S : s :: D : d. \\ D : d :: S : s. \end{array}$$

Whereby all the cases that can happen in this kind of failing are readily resolved by the logarithms, and also by the following Table, calculated for a difference of longitude of 1 degree, or 60 nautical miles of the equator, by the first rule. Thus.

As Radius, to the cosine of any latitude; suppose 30°,
So 60 miles of longitude, to the meridional distance 51,96.

And the like to the other latitudes.

A TABLE

747. A TABLE shewing how many miles answer to a degree of longitude, at every degree of latitude.

Lat.	Miles	Lat.	Miles	Lat.	Miles	Lat.	Miles	Lat.	Miles	Lat.	Miles
1	59,99	16	57,67	31	51,43	46	41,68	61	29,09	76	14,51
2	59,96	17	57,38	32	50,88	47	40,92	62	28,17	77	13,50
3	59,92	18	57,06	33	50,32	48	40,15	63	27,24	78	12,48
4	59,85	19	56,73	34	49,74	49	39,36	64	26,30	79	11,45
5	59,77	20	56,38	35	49,15	50	38,57	65	25,36	80	10,42
6	59,67	21	56,01	36	48,54	51	37,76	66	24,41	81	9,38
7	59,56	22	55,63	37	47,92	52	36,94	67	23,44	82	8,35
8	59,42	23	55,23	38	47,28	53	36,11	68	22,48	83	7,32
9	59,26	24	54,81	39	46,63	54	35,27	69	21,50	84	6,28
10	59,09	25	54,38	40	45,96	55	34,41	70	20,52	85	5,23
11	58,89	26	53,93	41	45,28	56	33,55	71	19,53	86	4,18
12	58,69	27	53,46	42	44,59	57	32,68	72	18,54	87	3,14
13	58,46	28	52,97	43	43,88	58	31,79	73	17,54	88	2,09
14	58,22	29	52,47	44	43,16	59	30,90	74	16,54	89	1,05
15	57,95	30	51,96	45	42,43	60	30,00	75	15,53	90	0,00

The use of this table will be evident from what follows.

748. If the meridional distance is wanted to degrees and minutes, proceed thus.

Take the difference of the meridional distances to the given degrees, and those which are next greater; multiply this difference by the given minutes, and divide the product by 60; then the quotient subtracted from the miles against the given degrees in the table, will leave the meridional distance required.

749. But if a meridional distance, intermediate to those in the table, was given to find the corresponding degrees and minutes.

Subtract the given meridional distance from the next greater, found in the table; multiply the remainder by 60; divide the product by the difference of the meridional distances next greater and less than the given one; then the quotient annexed as minutes to the degrees of the next greater meridional distance, will give the degrees and minutes sought.

EXAM. to first rule.

How many miles answer to a degree of long. in the lat. of $48^{\circ} 26'$?

Lat. 48° 40,15 miles.

Lat. 49° 39,36 miles.

The diff. is 0,79, which multip. by 26, gives 20,55; this divided by 60 gives 0,38; which taken from 40,15 leaves 39,77, the miles required.

EXAM. to second rule.

In what lat. does 46,08 miles answer to one deg. of longitude?

From 46,63, the miles to 39° .

Take 46,08, the given miles.

Leaves 0,55; which multip. by 60, gives 33,00; this divided by 0,67, the diff. between 39° and 40° gives $49'$: So $39^{\circ} 49'$ is the lat. sought.

75C, CASE I. Given the latitude and difference of longitude: Required the distance.

EXAM. *A ship in the latitude of 32° N. sails thence due east till her difference of longitude is 384 miles: Required the distance she has run?*

BY CONSTRUCTION *. Fig. 4. Plate VIII.

From any point P, with the radius of 90° , taken from the scale of natural sines, describe the arc EQ; draw the chord EQ equal to 384 miles; and draw the radii PE, PQ.

From the same point P, with a radius equal to the sine of 58° (the comp. lat.), taken from the nat. sines, describe the arc AB; and draw the chord AB, whose measure is the distance sought.

Here P represents the north pole; the arc EQ the equator; the arc AB a parallel of latitude; the chord EQ is the difference of longitude; the chord AB is the meridional distance sought; A is the place sailed from, B the place arrived at; and the radii PE, PQ, express the meridians of the place so sailed from, and come to.

BY COMPUTATION.

As Radius	=	$90^{\circ} 00'$	10,00000
To cos. lat.	=	$32^{\circ} 00'$	9,92842
So miles long.	=	384	2,58433
To miles dist.	=	325,6	2,51275

So that she has run 325,6 miles.

BY THE TABLE.

Seek the miles to the given latitude; multiply them by the given diff. longitude; the quotient divided by 60 will give the meridional distance sought.

Thus. To lat. 32° answers 50,88 miles.

Multiply by 384 the given diff. longitude.

20352
40704
15264

60)19537,92(325,632 = merid. dist. sought.

* This and the construction of the following cases depend on the Orthographic projection † of the circles of the sphere on the plane of the equator, that circle being the primitive, and its centre or pole representing the point into which the pole of the earth is projected; consequently the distance from the pole, of any parallel of latitude, must be expressed by the complement of that latitude.

† This projection is formed by supposing that any point on the surface of a sphere is projected on a plane passing thro' its centre, by a line drawn thro' that point perpendicular to the plane; which amounts to the same, as if the visual ray came from an eye at an infinite distance.

Thus in Fig. 1. Pl. VIII. perpend. lines thro' B, b, project those points in t, z, on the plane Fdpe: Therefore small circles on that plane are described with the radii Et, Ez, = the sines of the arcs PB, Pb.

751. CASE II. Given the latitude and meridional distance: Required the difference of longitude.

EXAM. *A ship from the latitude $53^{\circ} 36' N.$ longitude $10^{\circ} 18' E.$ sails due west 236 miles: Required her present longitude?*

BY CONSTRUCTION. Fig. 4. Pl. VIII.

From the point P, with the sine of 90° , and the cosine of $53^{\circ} 36'$, describe the arcs EQ, CD. Draw the chord $CD = 326$; thro' c and D draw the radii PE, PQ; then the chord EQ being drawn, will be the difference of longitude.

BY COMPUTATION.

As PC, the cos. lat.	= $53^{\circ} 36'$	0,22664	6,0)39,8(6° 38'.
To PE, Radius	= 90 00	10,00000	Long. from 10 18 E.
So CD, merid. dist.	= 236 m.	2,37291	Diff. long. 6 38 W.
To EQ, diff. long.	= 397,7	2,59955	Long. in 3 40 E.

BY THE TABLE.

Find the meridional distance to a deg. of long. in lat. $53^{\circ} 36'$ (748), and make it a divisor to the product of the given meridional distance by 60; then will the quotient be the diff. long. sought.

Thus. The diff. merid. dist. to 53° and 54° , is 0,85; which multiplied by 36' and divided by 60', gives 0,51; this taken from 36,11, the merid. dist. to 53° , leaves 35,6 for the merid. dist. to $53^{\circ} 36'$: Then 236 multiplied by 60, and the quotient divided by 35,6, gives 397,7 for the diff. long. sought.

752. CASE III. Given the difference of longitude, and its corresponding meridional distance: Required the parallel of latitude.

EXAM. *In what latitude does 384 miles of meridional distance answer to 500 miles of diff. longitude?*

BY CONSTRUCTION. Fig. 5. Pl. VIII.

From the point P, with the sine of 90° , describe the arc EQ; draw the chord $EQ = 500$, and draw the radii PE, PQ; make $EC = 384$, draw CB parallel to EP; then the arc AB being described with the radius PB, is the parallel of lat. sought.

BY COMPUTATION.

As EQ, the diff. long.	= 500	7,30103	So in the lat. $39^{\circ} 49'$ 384 miles of merid. dist. answers to 500 miles of longitude.
To AB, merid. dist.	= 384	2,58433	
So PE, Radius	= $90^{\circ} 00'$	10,00000	
To PA, the cos. lat.	= 39 49	9,88536	

BY THE TABLE.

The given merid. dist. 384 being multiplied by 60, and the product divided by the given diff. long. 500, gives 46,08, the merid. dist. to one deg. of long. in the lat. sought: And by article 749, the latitude will be found to be $39^{\circ} 49'$.

753. CASE IV. Given the distance of two places in a given parallel of latitude: Required the distance of two other places, under the same meridians with the former, in another given parallel of latitude.

EXAM. *From two ports in the lat. $32^{\circ} 20'$ N. distant 256 miles, two ships sail directly north: How far are they distant from one another, when they come to the lat. $44^{\circ} 30'$ N.*

By CONSTRUCTION. Fig. 6. Pl. VIII.

From the point P, with radii equal to the cosines of $32^{\circ} 20'$, and $44^{\circ} 30'$, describe the arcs AB, DE; draw the chord AB = 256, and draw PA, PB; then will the chord DE be the dist. sought.

By COMPUTATION.

As PA, the cos. of lat. from	=	$32^{\circ} 20'$	0,07317
To PD, the cos. of lat. in	=	$44^{\circ} 30'$	9,85324
So AB, the first dist.	=	256	2,40824
To DE, the present dist.	=	216,1	2,33465

By THE TABLE.

The merid. distances to $32^{\circ} 20'$, and $44^{\circ} 30'$, (found by 748) are 50,7 and 42,8: Then the given merid. distance 256 multiplied by 42,8, and the product 1095,68 divided by 50,7; the quotient 216,1 is the meridian distance sought.

754. CASE V. Given the distances between the same two meridians in different parallels, and also the latitude of one parallel, to find the latitude of the other.

EXAM. *If two ships in the lat. $44^{\circ} 30'$ N., distant from one another 216 miles, should both sail thence directly south until their distance is 256 miles: What latitude are they come to?*

By CONSTRUCTION. Fig. 7. Pl. VIII.

From P, with a radius of the cosine of $44^{\circ} 30'$, the lat. sailed from, describe the arc DE; draw the chord DE = 216; and thro' D and E, draw PA, PB; continue DE till DF = 256; thro' F draw FB parallel to PA, meeting PB in B; then an arc described from P, thro' B and A, will be the parallel of latitude come to.

By COMPUTATION.

As DE, the first dist.	=	216	7,66554
To AB, the second dist.	=	256	2,40824
So PD, the cos. lat. from	=	$44^{\circ} 30'$	9,85324
To PA, the cos. lat. in	=	$32^{\circ} 17'$	9,92702

By THE TABLE.

To lat. $44^{\circ} 30'$, find (748) the mer. dist. 42,8; this multiplied by 256, and the product divided by 216, gives 50,72; which (by 749) will be found to be the merid. dist. to $32^{\circ} 17'$, the latitude sought.

755. *Other Examples to exercise the foregoing cases.*

Ex. I. *What is the distance between St. Maries in lat. $37^{\circ} 00' N.$ long. $22^{\circ} 56' W.$, and Cape Henry in lat. $37^{\circ} 00' N.$ long. $76^{\circ} 23' W.$?*

As Rad : cos. lat. $37^{\circ} 00'$:: diff. long. 3207 : distance 2561.

Ex. II. *A ship from Cape Clear sails west 265 miles : What long. is she come to?*

As cos. lat. $51^{\circ} 18'$: Radius :: merid. dist. 265 : diff. long. 423,8.

Answer. The present longitude is $16^{\circ} 54' W.$

Ex. III. *A ship having run due east for 3 days, at the rate of 5 knots an hour, finds she has altered her longitude $8^{\circ} 16'$: What parallel of latitude did she sail in?*

As diff. long. 496 : merid. dist. 360 :: Rad : cos. lat. $43^{\circ} 28'$.

Ex. IV. *Suppose two ships in the parallel of $47^{\circ} 54' N.$, their difference of long. being $9^{\circ} 35'$, should both sail thence directly south 836 miles : How far were they distant, at the beginning, and end of their courses?*

As Rad : cos. lat. $47^{\circ} 54'$:: diff. long. 575 : first merid. dist. 385,5 m.

Now 836 miles = $13^{\circ} 56'$; therefore the lat. come to is $33^{\circ} 58'$.

As Rad : cos. lat. $33^{\circ} 58'$:: diff. long. 575 : last merid. dist. 476,9 m.

Ex. V. *What is the diff. lat. between two parallels on different sides of the equator, when, between the same two meridians, the meridional distance in the northern parallel is $AB = 238$ miles, in the southern parallel is $CD = 195$ miles, and at the equator is $EQ = 352$ miles?*

As diff. long. 352 : north. mer. dist. 238 :: Rad : cos. lat. $47^{\circ} 27' N.$

As diff. long. 352 : south. mer. dist. 195 :: Rad : cos. lat. $56^{\circ} 22' S.$

Then the sum of these two latitudes, viz. $103^{\circ} 49'$, is the diff. lat. sought.

In this figure 8, Pl. VIII, P is the north pole, and s is the south pole ; but the construction on one side of the equator is sufficient, as is evident by drawing Ed parallel to PE , and dc parallel to EQ .

The difference of longitude and meridional distances may be considered as tangents to their respective arcs, as well as chords. (259)

Among the natural scales on the Gunter, are two, which used jointly answer the end of the foregoing table ; these are a scale of chords marked CH , and a scale of longitude marked ML , standing close to one another ; and are thus used.

Find the given latitude on the scale of chords, and right against it, in the scale of longitude, is the division expressing the miles of merid. distance in that lat. to one degree of longitude.

Thus against the chord 60° , stands 30 miles ; and the chord 50° , stands between 38 and 39 miles : And so of the rest.

SECTION

SECTION IV.

*Of Middle latitude, and Mercator's sailings.**Of Middle latitude sailing.*

756. MIDDLE LATITUDE SAILING, is a manner of solving the several cases of globular sailing, by the principles of plane and parallel sailing jointly.

This method, which is not quite accurate, is founded on the following supposition.

757. The departure is reckoned as a meridional distance in that latitude, which is a middle parallel between the latitude sailed from, and the latitude come to.

Now altho' the arithmetical mean of the cosines of two distant latitudes is not the cosine of the arithmetical mean of those latitudes; neither is the departure between two places on an oblique rhumb equal to the distance between their meridians in a mean latitude; yet when the parallels of those places are near the equator, or not far distant from one another in any latitude, the error becomes so small, as not to affect the nautical conclusions drawn from the foregoing supposition.

This artifice seems to have been invented on account of the easy manner in which the several cases may be resolved by the Traverse Table: And where a table of meridional parts is wanting. The computations depend on the following rules.

RULE I.

758. Add the latitude sailed from to the latitude arrived at, and take half the sum for the middle latitude.

RULE II.

759. As the cosine of the middle latitude, $= C.$
Is to the Radius; $= R.$
So is the departure, $= D.$
To the difference of longitude. $= L.$

RULE III.

760. As the cosine of the middle latitude, $= C.$
Is to the tangent of the course; $= T.$
So is the difference of latitude, $= P.$
To the difference of longitude. $= L.$

The second rule is the same as the first rule in parallel sailing, which is demonstrated at art. 719.

Demonstration

Demonstration of the third rule.

Now $\cos. \text{ mid. lat.} : \text{Rad} :: \text{departure} : \text{diff. long.}$ (719)
 Therefore $\text{Rad} \times \text{dep.} = \cos. \text{ mid. lat.} \times \text{diff. long.}$ (245)
 Also $\text{diff. lat.} : \text{depart.} :: \text{Rad} : \text{tang. course.}$ (684)
 Therefore $\text{Rad} \times \text{dep.} = \text{diff. lat.} \times \text{tang. course.}$ (245)
 Consequently $\cos. \text{ mid. lat.} \times \text{diff. long.} = \text{diff. lat.} \times \text{t. course.}$ (137)
 Therefore $\cos. \text{ mid. lat.} : \text{tan. course} :: \text{diff. lat.} : \text{diff. long.}$ (246)

From rules II and III arise the following proportions.

$C : R :: D : L.$	$C : T :: P : L.$
$L : D :: R : C.$	$P : L :: C : T.$
$R : C :: L : D.$	

Where the letters signify the names they stand against in the rules.

Of Mercator's sailing.

761. MERCATOR'S SAILING is the art of resolving the several cases of Globular sailing by Plane Trigonometry and a table of meridional parts, or logarithmic tangents.

The computations herein are performed by the following rules.

762. RULE I. Therefore
 As merid. diff. lat. $= M.$ $M : L :: R : T.$
 To diff. longitude; $= L.$ $R : T :: M : L.$
 So is the Radius, $= R.$ $T : R :: L : M.$
 To tang. course. $= T.$

763. RULE II. Therefore
 As the prop. diff. lat. $= P.$
 To the departure; $= D.$ $P : D :: M : L.$
 So mer. diff. lat. $= M.$ $M : L :: P : D.$
 To diff. longitude. $= L.$

These rules are demonstrated at Prop. IV. (727)

764. RULE III. Therefore
 As diff. log. tan. $\frac{1}{2}$ co-lats. $= G.$ $G : N :: L : T.$
 To tang. of $51^{\circ} 38' 09''$; $= N.*$ $N : G :: T : L.$
 So is a given diff. long. $= L.$ $T : L :: N : G.$
 To tang. course. $= T.$

For the demonstration of this analogy, see Prop. XIII. (742)

* The log. tangent of $51^{\circ} 38' 09''$ is 10,10151.

PROBLEM

PROBLEM I.

765. *Given the latitudes of two places.**Required the meridional difference of lat. between those places.*

CASE I. When both places are on one side of the equator.

RULE I. The difference of the meridional parts answering to each latitude, will give the meridional difference of latitude required.

RULE II. The difference of the log. tangents of the half co-latitudes divided by 12,63, will give the meridional difference of latitude required.

EXAM. I. *What is the meridional difference of latitude between the island of St. Thomas and Gibraltar?*

By the meridional parts.

Gibraltar lat.	36° 13' N.	
St. Thomas's lat.	00 00	
Merid. parts to 36 13		2334
Merid. parts to 00 00		0000
Merid. diff. lat. =		<u>2334</u>

By the log. tangents.

Co-lat. = 53° 47'	$\frac{1}{2}$ co-lat. = 26° 53'
Co-lat. = 90 00	$\frac{1}{2}$ co-lat. = 45 00
Log. tang. 10,00000	to 45 00
Log. tang. 9,70498	to 26 53

12,63) 29502 (2335

4242453°7410EXAM. II. *What is the meridian difference of latitude between the Lizard and Cape Verd?*

By the meridional parts.

C. Verd's lat.	14° 43' N.	
Lizard's lat.	49 57 N.	
Merid. parts to 49 57		3470
Merid. parts to 14 43		893
Merid. diff. lat. =		<u>2577</u>

By the log. tangents.

Co-lat. = 75° 17'	$\frac{1}{2}$ co-lat. = 37° 38'
Co-lat. = 40 03	$\frac{1}{2}$ co-lat. = 20 01
Log. tang. 9,88707	to 37 38
Log. tang. 9,56146	to 20 01

12,63) 32561 (2578 = m. p.

766. CASE II. When the given places are on different sides of the equator.

RULE I. The sum of the meridional parts answering to each latitude, will be the meridional difference of latitude required.

RULE II. The sum of the log. co-tangents, abating the index, of the half co-latitudes, divided by 12,63, will be the meridional difference of latitude required.

EXAM. *What is the meridional difference of latitude between the Cape Verd, and the island of St. Helena?*

By the meridional parts.

C. Verd's lat.	14° 43' N.	
St. Helena's lat.	16 00 S.	
Merid. parts to 16 00		973
Merid. parts to 14 43		893
Merid. diff. lat. =		<u>1866</u>

By the log. tangents.

Co-lat. = 75° 17'	$\frac{1}{2}$ co-lat. = 37° 38'
Co-lat. = 74 00	$\frac{1}{2}$ co-lat. = 37 00
Log. cot. 0,11293	to 37 38
Log. cot. 0,12288	to 37 00

12,63) 23581 (1867 = m. p.

PROBLEM

PROBLEM II.

767. Given the lat. of one place, and the merid. diff. lat. between that and another place.

Required the latitude of the other place.

CASE I. When the given lat. and merid. diff. lat. have like names.

RULE I. To the merid. parts of the given lat. add the given meridional diff. lat.; seek the sum in the tab. of merid. parts, and the corresponding deg. and min. gives the lat. sought.

RULE II. Multiply the given merid. diff. lat. by 1263; subtract the product, rejecting all to the right of the 5th decimal place *, from the log. tang. of the given half co-lat.; seek the degrees to the remainder among the log. tang. these deg. doubled, gives the co-lat. required.

EXAM. A ship from the lat. $14^{\circ} 43' N.$ sails northward till she finds her merid. diff. lat. is 2578: What lat. is she come to?

By meridional parts.

To the lat. $14^{\circ} 43' N.$
 The merid. parts = 893 N.
 Add merid. diff. lat. = 2578 N.
 The sum is = 3471

which answers to $49^{\circ} 57' N.$
 The latitude sought.

By log. tangents.

The co-lat. is $75^{\circ} 17'$; $\frac{1}{2}$ co-lat. $37^{\circ} 38'$.
 And 2578 multiplied by 1263, gives 32560,14.
 From 9,88707 = log. tan. $37^{\circ} 38'$.
 Take 0,32560
 Leaves 9,56147 = log. tan. $20^{\circ} 01'$.
 which doubled is $40^{\circ} 02'$.
 Whose comp. is $49^{\circ} 58' =$ lat. sought.

768. CASE II. When the given lat. and merid. diff. lat. have unlike names.

RULE I. Take the difference between the merid. parts of the given lat. and the merid. diff. lat. the remainder found in the table of merid. parts, will give the latitude sought.

RULE II. Multiply the given merid. diff. lat. by 1263; add the product to the log. tang. of the given half co-lat. the deg. to the tang. of the sum being doubled, gives the co-latitude required.

EXAM. A ship from the lat. $49^{\circ} 57' N.$ sails southward till her merid. diff. lat. is 2578: What latitude is she come to?

By meridional parts.

To the lat. $49^{\circ} 57' N.$
 The merid. parts = 3470 N.
 Subt. merid. diff. lat. = 2578 S.
 Remains merid. parts 892

which answers to $14^{\circ} 43' N.$
 The latitude come to.

By log. tangents.

Co-lat. $40^{\circ} 03'$; $\frac{1}{2}$ co-lat. $20^{\circ} 01'$.
 And $2578 \times 1263 = 32560,14$.
 To 9,56146 = log. tang. $20^{\circ} 01'$.
 Add 0,32560
 Sum 9,88706 = log. tang. $37^{\circ} 38'$
 which doubled is $75^{\circ} 16'$.
 Whose comp. is $14^{\circ} 44' =$ lat.

* That is, rejecting all below the number of places in the tables used.

PROBLEM III.

769. Given the latitudes and longitudes of two places.
Required their bearing and distance.

EXAM. *What is the course and distance from Cape Clear, in Ireland, to the island of St. Mary, one of the Azores?*

BY MIDDLE LATITUDE SAILING.

C. Clear's lat.	51° 18' N.	Long.	9° 50' W.
St. Mary's lat.	37 00 N.	Long.	22 56 W.
Diff. lat.	14 18 = 858 m.	Diff. long.	13 06 = 786 m.
Sum latitudes	88 18		
Mid. lat.	44 09		
Co. Mid. lat.	45 51		

BY CONSTRUCTION. Fig. 9. Plate VIII.

Draw the meridian AD; describe the quadrant A m p with the sine of 90°, and the arc EF with the cosine of the middle latitude. Make the chord mB equal to 786, the diff. longitude; draw AB cutting the arc EF in F, and draw the chord EF. Make AD equal to 858, the diff. lat. draw DC parallel to A p and equal to EF; and draw AC.

Then AC is the distance, and the angle DAC is the course.

BY COMPUTATION.

For the Course.				For the Distance.			
P	:	L	:: C : T. (760)	fin. ∠C	:	AD	:: R : AC.
As diff. lat.	=	858	7,06651	As cos. Cou.	=	33° 19'	0,07798
To diff. lon.	=	786	2,89542	To diff. lat.	=	858	2,93349
So cos. m. lat.	=	45° 51'	9,85583	So Rad	=	90° 00'	10,00000
To tan. cour.	=	33 19	9,81776	To dist.	=	1027	3,01147

BY THE TRAVERSE TABLE.

1st. With the co-mid. lat. found among the degrees, and the diff. longitude, or some part thereof, (if too large) taken among the distances, find the departure, or some part thereof, in its column.

Observing to multiply this Depart. by the same number the diff. longitude was divided by.

2d. With the diff. lat. and dep. together, or their like parts, find the course among the degrees, and the distance in its column.

Let the diff. long. 786 be broke into parts; viz. into 100 taken seven times, and into 86: And as the co-mid. lat. is 45° 51', or 45½°; proceed thus.

The departures to 100, in the columns of 45° and 46°, are 70,7 and 71,9; the diff. is 1,2; ½ parts thereof is 1,0; which added to 70,7 makes 71,7 for the dep. to 100; and for 700 it will be 501,9: Also, the departures to 86 in the columns of 45° and 46°, are 60,8 and 61,9; ½ of their diff. is 0,9, and this with 60,8 makes 61,7, the dep. to 86; and the whole dep. = 563,6.

Now

Now take $\frac{1}{20}$ of the diff. lat. 858, and of the dep. 564, viz. 42,9 and 28,2; these will be found together under $33^{\circ} 15'$ for the course; and between 51 and 52 dist.; suppose $51\frac{1}{2}$; which taken 20 times gives 1030 for the distance.

770. BY MERCATOR'S SAILING.

C. Clear long.	9° 50' W.	Lat.	51° 18'	Mer. parts	3597
St. Mary's long.	22 56	Lat.	37 00	Mer. parts	2393
Diff. long.	<u>13 06</u>	Diff. lat.	<u>14 18</u>	Mer. diff. lat.	<u>1204</u>
Or 786 miles.		Or 858 miles.			

BY CONSTRUCTION. Fig. 17. Plate VIII.

Draw the meridian AB, and describe the quadrant A m p; make AD = 858, the proper diff. lat. and AB = 1204, the merid. diff. lat.; draw DC, BE, parallel to A p; make BE equal to 786, the diff. long. and draw AE.

Then A is the place of Cape Clear, c that of St. Mary's; AC is the distance, CD the departure, and the angle DAC is the course.

COMPUTATION by Meridional parts.

<i>For the Course.</i>	<i>For the Distance.</i>
M : L :: R : T. (762)	fin. $\angle C$: AD :: R : AC.
As mer. d. lat. = 1204 6,91937	As cos. Cour. = $33^{\circ} 08'$ 0,07707
To diff. long. = 786 2,89542	To prop. d. lat. = 858 2,93349
So Rad = $45^{\circ} 00'$ 10,00000	So Rad = $90^{\circ} 00'$ 10,00000
To tan. Cour. = 33 08 <u>9,81479</u>	To Dist. = 1024 <u>3,01056</u>

COMPUTATION by Logarithmic tangents.

C. Clear's lat.	51° 18'.	Co-lat.	38° 42'.	$\frac{1}{2}$ co-lat.	19° 21'.
St. Mary's lat.	37 00.	Co-lat.	53 00.	$\frac{1}{2}$ co-lat.	26 30.
To $\frac{1}{2}$ co-lat. 20° 30', its log. tang.			=	9,09774	
To $\frac{1}{2}$ co-lat. 19 21, its log. tang.			=	9,54552	
Difference			=	<u>1522,2</u>	= C.

For the Course.

G : N :: L : T. (764)
As G 1522 6,81759
To N 10,10151
So diff. long. 786 2,89542
To tang. Cour. 33° 07' <u>9,81452</u>

For the Distance.

The distance will be the same as in the former computation; for the diff. of near a min. in the course would not happen in working by tables containing more places.

BY THE TRAVERSE TABLE.

Seek the mer. diff. lat. and diff. long. together, like diff. lat. and dep., and the course is given among the degrees.

Then with the course and proper diff. lat. find the dist.

Thus taking $\frac{1}{20}$ th of 1204 and 786, viz. 60,2 and 39,3; they will be found together under $33^{\circ} 15'$ deg. for the course.

And taking $\frac{1}{20}$ th of the diff. lat. 858, viz. 42,9, and the course $33^{\circ} 15'$, gives about $51\frac{1}{2}$ for the dist., which taken 20 times gives 1022 dist.

PROBLEM IV.

771. Given the bearing and latitudes of two places;
Required their distance and difference of longitude.

EXAM. *A ship who had taken her departure from a place in lat. 37° 00' N. long. 22° 56' W., steers N. 33° 19' E.; and cloudy weather coming on, she got no observation till eight days after, and then found herself in lat. 51° 18' N.: Required her true distance and present longitude?*

BY MIDDLE LATITUDE SAILING.

Departed lat.	37° 00' N.	Sum latitudes	88° 18'
Present lat.	51 18 N.		
		Mid. lat.	44 09
Diff. lat.	14 18 = 858 m.		
		Co. Mid. lat.	45 51

BY CONSTRUCTION. Fig. 10. Plate VIII.

Draw the meridian $AD = 858$, the diff. lat. describe the quadrant Amp ; make mc equal to $33^\circ 19'$, the course: Draw DC parallel to $A p$ meeting in c , the rhumb AC drawn thro' c : With the sine of the co-mid. lat. $45^\circ 51'$ describe the arc EF , in which apply the dep. DC , from E to F ; thro' F draw AB , and draw the chord mB .

Then is AC the distance run, and mB the diff. longitude.

By COMPUTATION.

For the distance.			For the diff. longitude.		
fin. $\angle C$	$AD :: R : AC.$		$C : T :: P : L. (760)$		
As cos. Cou.	$= 33^\circ 19'$	0,07708	As cos. m. lat.	$= 45^\circ 51'$	0,14417
To diff. lat.	$= 858$	2,93349	To tang. Cou.	$= 33 19$	9,81776
So Rad	$= 90^\circ 00'$	10,00000	So diff. lat.	$= 858$	2,93349
To dist.	$= 1027$	3,01147	To diff. long.	$= 786$	2,89542

Now $22^\circ 56' W. =$ Dep. long.
60) 786 (13 06 E. $=$ Diff. long.

9 50 W. $=$ Pres. long.

BY THE TRAVERSE TABLE.

1st. With the course among the degrees, and the diff. lat. in its column, find the departure, and distance, in their columns.

2d. With the co-mid. lat. among the degrees, and departure in its column, find the diff. long. among the distances.

Thus taking $\frac{1}{20}$ of the diff. lat. 858, viz. 42,9; with it, and the course $33\frac{1}{3}^\circ$, the departure will be found 28,1, and the dist. 51,3; each being multiplied by 20, gives 562 for the depart. and 1026 for the distance.

The co-mid. lat. $45^\circ 51'$, or $45\frac{5}{8}$ deg. and 28,1 $= \frac{1}{20}$ departure, will give in the column of distances 39,5; which multiplied by 20, gives 790 for the difference of longitude.

772. BY MERCATOR'S SAILING.

Departed lat.	37° 00' N.	Mer. parts	2393
Present lat.	51 18 N.	Mer. parts	3597
Diff. lat.	14 18 = 858 m.	Mer. diff. lat.	1204

BY CONSTRUCTION. Fig. 18. Plate VIII.

Draw a meridian AB, in which take AB = 1204, the mer. diff. lat., and AD = 858, the proper diff. lat. Describe the quadrant A m p, and take m c = 33° 19', the course: Thro' B and D, draw BE, DC, parallel to A p, meeting the rhumb drawn thro' A and c, in the points E, c.

Then is AC the distance, and BE the diff. longitude.

COMPUTATION by Meridional parts.

For the distance.			For the diff. longitude.		
fin. $\angle DCA$	: AB :: R	: AC.	R	: T :: M	: L. (762)
As cos. Cou.	= 33° 19'	0,07798	As Rad	= 45° 00'	10,00000
To diff. lat.	= 858	2,93349	To tan. Cou.	= 33 19	9,81776
So Rad	= 90° 00'	10,00000	So m. diff. lat.	= 1204	3,08063
To dist.	= 1027	3,01147	To diff. long.	= 791,4	2,89839

Now 22° 56' W. = Dep. long.
60) 791 (13 11 E. = Diff. long.

Then 9 45 W. = Pref. long.

COMPUTATION by Logarithmic tangents.

Departed lat.	37° 00' N.	Co-lat.	53° 00'.	$\frac{1}{2}$ co-lat.	26° 30'.
Present lat.	51 18 N.	Co-lat.	38 42.	$\frac{1}{2}$ co-lat.	19 21.
To $\frac{1}{2}$ co-lat. 26° 30', its log. tang.		=	9,69774		
To $\frac{1}{2}$ co-lat. 19 21, its log. tang.		=	9,54552		
Difference		=	1522,2	= G.	
<i>For the distance.</i>			<i>For the diff. longitude.</i>		
fin. $\angle DCA : AB :: R : AC.$			N : G :: T : L. (764)		
			As N	=	9,89849
			To G	= 1522	3,18241
			So tang. Cou.	= 33° 19'	9,81776
The distance is found as above.			To diff. long.	= 791,8	2,89866

BY THE TRAVERSE TABLE.

With the course among the degrees, and the diff. lat. found in the column of lat. find the distance in its column.

Find the course among the degrees, and the mer. diff. lat. in the column of lat. and take the corresponding dep. for diff. long.

Now $\frac{1}{20}$ th of 858, the diff. lat., and of 1204, the mer. diff. lat., are 42,9 and 60,2. Then, under 33° 15', the diff. lat. 42,9 falls between 51 and 52 in the distances, take it 51,5; which multiplied by 20 gives 1030 for the distance.

And under 33° 15', the mer. diff. lat. falls against 39,48 of depart., which multiplied by 20 gives 789,6 for the diff. longitude.

PROBLEM

PROBLEM V.

773. Given one latitude, course and distance,
Required the other latitude and diff. longitude.

EXAM. *A ship takes her departure from a place in lat. $51^{\circ} 18' N.$ long. $9^{\circ} 50' W.$; and steers $S. 33^{\circ} 08' W.$ till she has run 1024 miles: Required her present latitude and longitude?*

BY MIDDLE LATITUDE SAILING.

BY CONSTRUCTION. Fig. 11. Plate VIII.

Draw a meridian AD , and describe the quadrant Amp ; make the arc $mc = 33^{\circ} 08'$, the course: In the rhumb drawn thro' C , take $AC = 1024$, the distance: Draw CD parallel to Ap , meeting AD in D .

BY COMPUTATION.

For the diff. latitude.
 $R : AC :: \sin. \angle C : AD.$
 As Rad. $= 90^{\circ} 00'$ 10,00000
 To dist. $= 1024$ 3,01030
 So cos. Cou. $= 33^{\circ} 08'$ 9,92293
 To diff. lat. $= 857,5$ 2,93323

For the diff. longitude.
 $As C : T :: P : L.$
 As cos. m. lat. $= 45^{\circ} 51'$ 0,14417
 To tang. Cou. $= 33^{\circ} 08'$ 9,81473
 So diff. lat. $= 857,5$ 2,93323
 To diff. long. $= 778,3$ 2,89113

Now $9^{\circ} 50' W.$ $=$ Dep. long.
 $60) 778 (12 \quad 58 \quad W.$ $=$ Diff. long.
22 48 W. $=$ Pref. long.

Now $51^{\circ} 18' N.$ $=$ Dep. lat.
 $60) 857 (14 \quad 17 \quad S.$ $=$ Diff. lat.
37 01 N. $=$ Pref. lat.
88 19 $=$ Sum lats.
44 09 $=$ Mid. lat.
45 51 $=$ Co-m. lat.

Then in the arc EF , described with the sin. Co. m. lat. $45^{\circ} 51'$, apply CD from E to F : thro' F draw AB cutting the arc mcp in B ; then mB being drawn will be the diff. long. sought.

BY THE TRAVERSE TABLE.

With the course found among the degrees, and the distance in its column, find the dep. and diff. lat.; then find the other lat.

Seek the co-mid. lat. among the degrees, and the dep. in its col. then the corresponding dist. is the diff. long. sought.

Thus. Under the course $33^{\circ} 15'$, and against the dist. 51, which is the 20th part of 1024, stands lat. 42,65; dep. 27,96.

Then $42,65 \times 20 = 853 m. = 14^{\circ} 13'$, the diff. lat.

Hence the pref. lat. is $37^{\circ} 05' N.$; and the co-mid. lat. $45^{\circ} 48'$.

Again. Over the degrees $45^{\circ} 40'$, the nearest dep. to 27,96 stands against 39 in the distances.

Then $39 \times 20 = 780 m.$ the diff. longitude.

774.

BY MERCATOR'S SAILING.

BY CONSTRUCTION. Fig. 19. Plate VIII.

Draw the meridian AB , describe the quadrant Amp ; make the arc $mc = 33^\circ 08'$, the course. In the rhumb AC drawn thro' c , take $AC = 1024$ miles, the dist.; and draw CD parallel to Ap .

COMPUTATION by Meridional parts.

For the diff. latitude.

As Rad : AC :: sin. $\angle C$: AD.	
As Rad = $90^\circ 00'$	10,00000
To dist. = 1024	3,01030
So cos. Cou. = $33^\circ 08'$	9,92293
To pro.diff. lat. = 857,5	2,93323

For the diff. longitude.

As R : T :: M : L.	
As Rad = $45^\circ 00'$	10,00000
To tang. Cou. = 33 08	9,81473
So mer. d. lat. = 1203	3,08026
To diff. long. = 785,2	2,89499

Now $9^\circ 50' W.$	= Dep. long.
60) 785 (13 05	W. = Diff. long.
22 55	W. = Pref. long.

Now $51^\circ 18' N.$	= Dep. lat.
60) 857 (14 17	S. = Diff. lat.

37 01	N. = Pref. lat.
-------	-----------------

To $51^\circ 18'$ merid. parts	= 3597
To 37 01 merid. parts	= 2394

Merid. diff. lat.	= 1203
-------------------	--------

Make $AB = 1203$; thro' B draw BE parallel to Ap , meeting AC continued in E ; then BE is the diff. longitude.

COMPUTATION by Logarithmic tangents.

The diff. lat. and present lat. being found as above.

Dep. lat. $51^\circ 18'$	Co-lat. $38^\circ 42'$	$\frac{1}{2}$ co-lat. $19^\circ 21'$
Pref. lat. 37 01.	Co-lat. 52 59.	$\frac{1}{2}$ co-lat. 26 29 $\frac{1}{2}$.

To $\frac{1}{2}$ co-lat. $26^\circ 29\frac{1}{2}'$, its log. tang.	= 9,69758
To $\frac{1}{2}$ co-lat. 19 21, its log. tang.	= 9,54552
Difference	= 1520,6 = c.

As N : G :: T : L.	
As N =	9,89849
To G = 1521	3,18213
So tang. Cou. = $33^\circ 08'$	9,81473
To diff. long. = 785,9	2,89535

Now $9^\circ 50' W.$	= Dep. long.
60) 786 (13 06	W. = Diff. long.
22 56	W. = Pref. long.

BY THE TRAVERSE TABLE.

With the course and distance, find the diff. latitude.

And then get the present lat. and the mer. diff. latitude.

With the course, and the mer. diff. lat. taken in the column of Lat. the corresponding dep. will be the diff. longitude.

Now $\frac{1}{20}$ th of the dist. 1024 is 51,2. Then under the course $33^\circ 15'$, against 51 among the distances, stands 42,65 in col. of Lat. And 42,65 $\times$ 20 gives 853; so the diff. lat. is $14^\circ 13'$; the present lat. is $37^\circ 05' N.$ and the mer. diff. lat. is 1198; $\frac{1}{20}$ th of which is 59,9.

To the course $33^\circ 15'$, against lat. 59,9, or its nearest 60,21, stands 39,48; which multiplied by 20 gives 789,6 for the diff. long. required.

PROBLEM

PROBLEM VI.

775. Given the latitudes of two places, and their distance.
Required their bearing, and difference of longitude.

EXAM. *A ship in lat. $51^{\circ} 18' N.$, long. $22^{\circ} 06' W.$, is bound to a place in the S.E. quarter, distant 1024 miles, and in lat. $37^{\circ} 00' N.$: What is her direct course, and how much must she alter her long. to arrive at the desired port?*

BY MIDDLE LATITUDE SAILING.

Lat. from	$51^{\circ} 18' N.$	Sum latitudes	$88^{\circ} 18'$
Lat. to	$37^{\circ} 00' N.$		
		Middle latitude	$44^{\circ} 09'$
Diff. lat.	$14^{\circ} 18' = 858 \text{ miles.}$	Co-middle latitude	$45^{\circ} 51'$

BY CONSTRUCTION. Fig. 12. Plate VIII.

Draw the meridian $AD = 858$ miles; describe the quadrant Amp : From A with 1024 miles, the given distance, cut DC drawn parallel to Ap , in c ; draw AC . In the arc EF described with the sine of $45^{\circ} 51'$, the co-middle lat., apply DC the departure from E to F ; thro' F draw AB , and draw the chord Bm .

Then the course is measured by the arc mc ; and the diff. long. by the line mB .

BY COMPUTATION.

For the Course.			For the diff. longitude.		
$AC : \text{Rad} :: AD : \sin. \angle c.$			$c : T :: P : L.$		
As dist.	$= 1024$	6,98970	As cof. m. lat. $= 45^{\circ} 51'$		0,14417
To Rad	$= 90^{\circ} 00'$	10,00000	To tang. Cou. $= 33^{\circ} 05'$		9,81390
So diff. lat.	$= 858$	2,93349	So diff. lat. $= 858$		2,93349
To cof. Cou. $= 33^{\circ} 05'$		9,92319	To diff. long. $= 779$		2,89156
Course SE. b. S. nearly.			Now $22^{\circ} 06' W. = \text{Long. from.}$		
			$60) 779 (12^{\circ} 59' E. = \text{Diff. long.}$		
			$9^{\circ} 07' W. = \text{Pref. long.}$		

BY THE TRAVERSE TABLE.

With the distance and diff. lat. in their columns, find the course among the degrees, and the dep. in its column.

With the co-middle lat. among the degrees, and the dep. in its column, find the diff. long. among the distances.

Thus. To the dist. 51, and the lat. 42,77, the nearest to 51,2 and 42,9, being the 20th parts of the dist. 1024 and diff. lat. 858; the course is $33^{\circ} 00'$, and the dep. 27,8.

With the co-middle lat. $45^{\circ} 51'$, or 46° , against the dep. 27,8 stands 39 in the col. of dist., which multiplied by 20 gives 780 for the diff. long.

Note, The lat. 42,77 is preferred to the lat. 42,89, which is nearer to 42,9; because taking the proportion between 51 and 51,2, the lat. in the former case comes nearer to 42,9 than in the latter case.

776. BY MERCATOR'S SAILING.

Lat. from	51° 18' N.	Mer. parts	=	3597
Lat. to	37 00 N.	Mer. parts	=	2393
Diff. lat.	14 18 = 858 m.	Mer. diff. lat.	=	1204

BY CONSTRUCTION. Fig. 20. Plate VIII.

In the meridian AB, take AB = 1204, the merid. diff. lat. and AD = 858, the proper diff. lat.: Describe the quadrant A m p; from A, with the distance 1024, cut DC, drawn parallel to A p, in c; draw AC, which continue till it meets BE, drawn parallel to A p, in E.

Then BE = diff. long. and arc m c measures the course.

COMPUTATION by Meridional parts.

For the Course.			For the diff. longitude.		
AC : Rad :: AD : f. $\angle$ ACD.			Rad : T :: M : L.		
As dist. = 1024	6,98970		As Rad = 45° 00'	10,00000	
To Rad = 90° 00'	10,00000		To tan. Cou. = 33 05	9,81390	
So p. diff. lat. = 858	2,93349		So m. diff. lat. = 1204	3,08063	
To col. Cou. = 33° 05'	9,92319		To diff. long. = 784,4	2,89453	
Course SE. by S. nearly.			Now 22° 06' W. = Long. from		
			60) 784 (13 04 E. = Diff. long.		
			9 02 W. = Long. to.		

COMPUTATION by the Logarithmic tangents.

Depart. lat. 51° 18'.	Co-lat. 38° 42'.	$\frac{1}{2}$ co-lat. 19° 21'.
Lat. bound to 37 00.	Co-lat. 53 00.	$\frac{1}{2}$ co-lat. 26 30.
To $\frac{1}{2}$ co-lat. 26° 30', its log. tang.	=	9,69774
To $\frac{1}{2}$ co-lat. 19 21, its log. tang.	=	9,54552
Difference	=	1522,2 = G.

For the Course.	For the diff. longitude.
AC : Rad :: AD : f. $\angle$ ACD.	N : G :: T : L.
	As N = 9,89849
	To G = 1522 3,18241
Course is S. 33° 05' E. as above.	So tang. Cou. = 33° 05' 9,81390
	To diff. long. = 784,9 2,89480

BY THE TRAVERSE TABLE.

With the distance and diff. lat. in their columns, find the course among the degrees.

With the course among the degrees, and the mer. diff. lat. in the col. of lat. find the diff. long. in the col. of departure.

The course found as before is 33° 00'.

And $\frac{1}{2}$ th of the mer. diff. lat. viz. 60,2, found under 33° in the col. of lat., the nearest corresponding dep. is 39,21; which multiplied by 20 gives 784,2 for the diff. longitude.

PROBLEM VII.

777- Given the latitudes of two places, and the departure.
Required the course, distance, and difference of longitude.

EXAM. *A ship from the latitude 37° 00' N., longitude 48° 20' W., sails between the N. and E., and being come to the lat. 51° 18' N., finds she has made 564 miles of departure: What was her direct course, distance run, and present longitude?*

BY MIDDLE LATITUDE SAILING.

Present lat.	51° 18' N.	Sum latitudes	88° 18'
Departed lat.	37° 00' N.		
		Mid. lat.	44° 09'
Diff. lat.	14 18 = 858 m.		
		Co. Mid. lat.	45 51

BY CONSTRUCTION. Fig. 13. Plate VIII.

In a meridian line, take $AD = 858$ m., the diff. lat. Describe the quadrant Amp , and parallel to Ap draw $DC = 564$ m., the departure; also draw the distance AC : In the arc EF , described with the sine of the co-mid. lat. $45^\circ 51'$, apply the chord EF equal to DC ; thro' F draw AB ; then the chord mB will be the diff. long., and the arc mc measures the course.

BY COMPUTATION.

To find the Course.			To find the Distance.		
$AD : DC :: \text{Rad} : \tan \angle DAC.$			$\sin \angle DAC : DC :: \text{Rad} : AC.$		
As diff. lat.	= 858	7,06651	As sin. Cou.	= $33^\circ 19'$	0,26022
To Depart.	= 564	2,75128	To Depart.	= 564	2,75128
So Rad	= $45^\circ 00'$	10,00000	So Rad	= $90^\circ 00'$	10,00000
To tang. Cou.	= 33 19	9,81779	To Dist.	= 1027	3,01150
To find the diff. longitude.					
$C : R :: D : L.$			Or $C : T :: P : L.$		
As cof. m. lat.	= $45^\circ 51'$	0,14417	Now $48^\circ 20' W.$	= Dep. long.	
To Rad	= $90^\circ 00'$	10,00000	60) 786 (13 06 E.	= Diff. long.	
So Depart.	= 564	2,75128			
				35 14 W.	= Pres. long.
To diff. long.	= 786	2,89545			

Course NE. b. N. nearly.

BY THE TRAVERSE TABLE.

The diff. lat. and dep. being found together, will give the course among the degrees, and the distance in its column.

With the co-mid. lat. among the degrees, and the dep. in its col. find the diff. longitude among the distances.

Now $\frac{1}{10}$ th of 858, the diff. lat., and of 564, the dep., are 42,9 and 28,2. The nearest numbers to these are found together under the course $33^\circ 30'$, and to the distance 51; which multiplied by 20 gives 1020.

Also the dep. 28,2 being found to the co-mid. lat. 46° , gives 39; which multiplied by 20 gives 780 for the diff. longitude.

But the parts being proportioned, gives 51,2 for the dist.; and 39,2 for the diff. long.; these multiplied by 20, give 1024 and 784.

778. BY MERCATOR'S SAILING.

Present lat.	51° 18' N.	Mer. parts	3597
Departed lat.	37 00 N.	Mer. parts	2393
Diff. lat.	14 18 = 858 m.	Mer. diff. lat.	1204

BY CONSTRUCTION. Fig. 21. Plate VIII.

In a meridian line take $AB = 1204$ m., the merid. diff. lat.; and $AD = 858$ m., the prop. diff. lat. Describe the quadrant Amp ; thro' D draw DC parallel to Ap , and make $DC = 564$ m., the dep. Thro' C draw AE meeting BE drawn parallel to Ap .

Then is AC the dist., BE the diff. long.; and the arc mc measures the course.

COMPUTATION by Meridional parts.

For the Course.				For the Distance.			
$AD : DC :: \text{Rad} : \tan \angle DAC.$				$\sin \angle DAC : DC :: \text{Rad} : AC.$			
As diff. lat.	= 858	7,06651		As sin. Cou.	= 33° 19'	0,26022	
To Depart.	= 564	2,75128		To Depart.	= 564	2,75128	
So Rad	= 45° 00'	10,00000		So Rad	= 90° 00'	10,00000	
To tang. Cou.	= 33 19	9,81779		To Dist.	= 1027	3,01150	

For the diff. long. $P : D :: M : L.$ Or $R : T :: M : L.$							
As prop. diff. lat.	= 858	7,06651		Now 48° 20' W.	= Dep. long.		
To Depart.	= 564	2,75128		60) 791 (13 11 E.	= Diff. long.		
So mer. diff. lat.	= 1204	3,08063					
To diff. long.	= 791,5	2,89842		35 09 W.	= Pref. long.		
				Course NE. b. N. nearly.			

COMPUTATION by Logarithmic tangents.

Departed lat.	37° 00' N.	Co-lat.	53° 00'.	$\frac{1}{2}$ co-lat.	26° 30'.
Present lat.	51 18 N.	Co-lat.	38 42.	$\frac{1}{2}$ co-lat.	19 21.
$10 \frac{1}{2}$ co-lat. 26° 30', its log. tang.				=	9,09774
To $\frac{1}{2}$ co-lat. 19 21, its log. tang.				=	9,54552
Difference				=	1522,2 = G.

The course and distance are found as above.

For the diff. longitude.			
$N : G :: T : L.$			
As N	=		9,89849
To G	= 1522		3,18241
So tang. Cou.	= 33° 19'		9,81776
To diff. long.	= 791,9		2,89866

BY THE TRAVERSE TABLE.

The course found as before is 30° 30'; under which against 60,2 ($= \frac{1}{20}$ th of the mer. diff. lat.) in the col. of lat., stands 39,74 in the col. of dep. Then $39,74 \times 20 = 794,8$, the diff. longitude.

PROBLEM VIII.

779. Given one latitude, the course and departure;
Required the other latitude, distance, and diff. longitude.

EXAM. A ship from the latitude $51^{\circ} 18' N.$, longitude $9^{\circ} 50' W.$, sails $S. 33^{\circ} 19' W.$ until her departure is 564 miles: Required her present latitude, longitude and distance?

BY MIDDLE LATITUDE SAILING.

BY CONSTRUCTION. Fig. 14. Plate VIII.

Draw a meridian line AD , describe the quadrant Amp ; make $Aa = 564$, the departure, and the arc $mc = 33^{\circ} 19'$, the course; thro' c draw the rhumb AC meeting ap , drawn parallel to AD , in c ; and draw CD parallel to Ap .

BY COMPUTATION.

For the diff. latitude.

$\sin. \angle DAC : CD :: \sin. \angle ACD : AD.$		
As $\sin. Cou.$	$= 33^{\circ} 19'$	0,26022
To Depart.	$= 564$	2,75128
So $\cos. Cou.$	$= 33^{\circ} 19'$	9,92202
To diff. lat.	$= 858$	2,93352

Now $51^{\circ} 18' N.$	$=$	Dep. lat.
60) 858 (14 18 S.	$=$	Diff. lat.
37 00 N.	$=$	Pres. lat.
88 18	$=$	Sum lats.
44 09	$=$	Mid. lat.
45 51	$=$	Co-m. lat.

Then in the arc EF , described with the sine of $45^{\circ} 51'$, apply CD from E to F : thro' F draw AB ; and draw the diff. long. Bm .

To find the Distance.

$\sin. \angle DAC : CD :: Rad : AC.$		
As $\sin. Cour.$	$= 33^{\circ} 19'$	0,26022
To Depart.	$= 564$	2,75128
So Rad	$= 90^{\circ} 00'$	10,00000
To Dist.	$= 1027$	3,01150

To find the diff. longitude.

$As c : Rad :: D : L.$		
As $\cos. m. lat.$	$= 45^{\circ} 51'$	0,14417
To Rad	$= 90 00$	10,00000
So Depart.	$= 564$	2,75128
To diff. long.	$= 786$	2,89545

Now $9^{\circ} 50' W.$	$=$	Dep. long.
60) 786 (13 06 W.	$=$	Diff. long.

22 56 W. $=$ Pres. long.

BY THE TRAVERSE TABLE.

With the course ($33^{\circ} 15'$ the nearest to $33^{\circ} 19'$) and the depart. (28,2 $= \frac{1}{20}$ th of 564) in its column, find in the col. of lat, the diff. lat. (42,53) and in the col. of dist. find the dist. (51).

Then $42,53 \times 20 = 850,6$; and $51 \times 20 = 1020$.

Hence the present lat. is $37^{\circ} 07' N.$, and the co-mid. lat. $45^{\circ} 48'$.

The co-mid. lat. 45,40 and dep. 28,2 gives in the column of distances 39, which multiplied by 20, gives 780, for the diff. of longitude.

But proportioning for the difference, gives 39,2; and $39,2 \times 20 = 784$, for the diff. longitude.

780.

780.

BY MERCATOR'S SAILING.

BY CONSTRUCTION. Fig. 22. Plate VIII.

Draw a meridian line AB ; describe the quadrant Amp ; make $Aa = 564$, and the arc $mc = 33^\circ 19'$; draw ac parallel to AB meeting the rhumb AC , drawn thro' c , in c ; and draw the departure CD parallel to Ap .

COMPUTATION by Meridional parts.

<i>For the diff. latitude.</i>			Now $51^\circ 18' N. =$ Dep. lat.		
$\sin. \angle DAC : CD :: \sin. \angle DCA : AD.$			60) 858 ($14 \ 18 \ S. =$ Diff. lat.		
As $\sin. Cou. = 33^\circ 19'$	0,26022		$37 \ 00 \ N. =$ Pref. lat.		
To Depart. $= 564$	2,75128		To $51^\circ 18'$ merid. parts $= 3598$		
So $\cos. Cou. = 33^\circ 19'$	9,92202		To $37 \ 00$ merid. parts $= 2394$		
To diff. lat. $= 858$	2,93352		Merid. diff. lat. $= 1204$		

Then make $AB = 1204$; draw BE parallel to Ap , meeting AC continued, in E ; and BE is the diff. of longitude.

<i>To find the Distance.</i>			<i>To find the diff. longitude.</i>		
$\text{As } \sin. \angle DAC : DC :: \text{Rad} : AC.$			$P : D :: M : L.$		
As $\sin. Cou. = 33^\circ 19'$	0,26022		As p. diff. lat. $= 858$	7,06651	
To Depart. $= 564$	2,75128		To Depart. $= 564$	2,75128	
So Rad $= 90^\circ 00'$	10,00000		So mer. diff. lat. $= 1204$	3,08063	
To Dist. $= 1027$	3,01150		To diff. long. $= 791,4$	2,89842	

Now $9^\circ 50' W. =$ Dep. long.		
60) 791 ($13 \ 11 \ W. =$ Diff. long.		
$23 \ 01 \ W. =$ Pref. long.		

COMPUTATION by Logarithmic tangents.

Find the diff. latitude and present latitude as above.

Then, Dep. lat. $51^\circ 18'$	Co-lat. $38^\circ 42'$	$\frac{1}{2}$ co-lat. $19^\circ 21'$
Pref. lat. $37 \ 00.$	Co-lat. $53 \ 00.$	$\frac{1}{2}$ co-lat. $26 \ 30.$

To $\frac{1}{2}$ co-lat. $26^\circ 30'$, the log. tang. is	$=$	9,69774
To $\frac{1}{2}$ co-lat. $19 \ 21$, the log. tang. is	$=$	9,54552
Difference	$=$	1522,2 = G.

To find the diff. longitude. N : G :: T : L.

As N $=$	9,89849	The distance and present longitude may be found as above.
To G $= 1522$	3,18241	
So tang. Cou. $= 33^\circ 19'$	9,81776	
To diff. long. $= 791,8$	2,89866	

BY THE TRAVERSE TABLE.

Find as before the diff. lat. $= 851$; and the distance $= 1020$.

Then the present lat. is $37^\circ 07' N.$; and the mer. diff. lat. is 1197; $\frac{1}{20}$ th of which is 59,85.

Now under the course $33^\circ 15'$, and against 60,21, the nearest to 59,85 in the col. of lat., stands in the col. of dep. 39,48; which multiplied by 20, gives 789,6 for the diff. longitude.

PROBLEM

PROBLEM IX.

781. Given one latitude, the distance and departure;
Required the other latitude, course and diff. longitude.

EXAM. A ship from the latitude $37^{\circ} 00' N.$, longitude $9^{\circ} 02' W.$, having sailed between the N. and W. 1027 miles, reckons she has made 564 miles of departure: What was her direct course, and the present latitude and longitude?

BY MIDDLE LATITUDE SAILING.

BY CONSTRUCTION. Fig. 15. Plate VIII.

Draw the meridian line AD; describe the quadrant A m p; make A a = 564 m.; from A with 1027, the distance, cut a c drawn parallel to AD in c; draw AC, and draw the departure CD parallel to A p.

BY COMPUTATION.

To find the Course.			
AC : Rad :: CD : sin. $\angle DAC$.			
As Dist.	= 1027	6,98843	
To Rad	= $90^{\circ} 00'$	10,00000	
So Depart.	= 564	2,75128	
To sin. Cour.	= $33^{\circ} 19'$	9,73971	
Now $37^{\circ} 00' N.$ = Dep. lat.			
60) 858 (14 18 N.	=	Diff. lat.	
51 18 N.	=	Pref. lat.	

To find the diff. latitude.			
Rad : AC :: sin. $\angle DCA$: AD.			
As Rad	= $90^{\circ} 00'$	10,00000	
To Dist.	= 1027	3,01157	
So cos. Cou.	= $33^{\circ} 19'$	9,92202	
To diff. lat.	= 858,2	2,93359	
Sum latitudes $88^{\circ} 18'$			
Mid. lat. $44^{\circ} 09'$			
Co. Mid. lat. $45^{\circ} 51'$			

Then describe the arc EF with the sine of $45^{\circ} 51'$, and therein apply CD from E to F; thro' F draw AB; and draw the diff. long. B m.

To find the diff. longitude.			
As cos. m. lat.	= $45^{\circ} 51'$	0,14417	
To tang. Cour.	= $33^{\circ} 19'$	9,81776	
So diff. lat.	= 858,2	2,93359	
To diff. long.	= 786,2	2,89552	

C : T :: P : L.			
Now $9^{\circ} 02' W.$ = Dep. long.			
60) 786 (13 06 W.	=	Diff. long.	
22 08 W.	=	Pref. long.	
Course is NW. b. N. nearly.			

BY THE TRAVERSE TABLE.

Now $\frac{1}{20}$ th of the distance, and of the dep., are 51,35, and 28,2.

Find in some column of departure, the nearest number to the given departure (28,2) standing against the given distance (51); which then gives the course ($33^{\circ} 30'$) among the degrees; and the diff. lat. (42,55) in its column.

Then $42,55 \times 20 = 850,6$: Hence the pref. lat. is $51^{\circ} 11' N.$; and the co-mid. lat. is $45^{\circ} 55'$.

With the nearest co-mid. lat. (46°) and the dep. (28,2) find among the distances (39) the diff. longitude.

Then $39 \times 20 = 780$, for the diff. longitude.

But by proportioning the parts, the diff. lat. will be found to be 856; and the diff. long. 784.

782. BY MERCATOR'S SAILING.

BY CONSTRUCTION. Fig. 23. Plate VIII.

Draw a meridian AB; describe a quadrant Amp ; make $Aa = 564$; draw ac parallel to AB; from A with 1027, the distance, cut ac in c; draw the departure CD parallel to Ap , and draw the distance AC.

COMPUTATION by Meridional parts.

To find the Course.			To find the diff. latitude.		
AC : Rad :: CD : sin. $\angle DAC$.			Rad : AC :: sin. $\angle DCA$: AD.		
As Dist.	= 1027	6,98843	As Rad	= 90° 00'	10,00000
To Rad	= 90° 00'	10,00000	To Dist.	= 1027	3,01157
So Depart.	= 564	2,75128	To cos. Cou.	= 33° 05'	9,92202
To fin. Cou.	= 33° 19'	9,73971	To diff. lat.	= 858,2	2,93359
Now 37° 00' N.	= Dep. lat.		To 51° 18', the mer. parts	= 3598	
60) 858 (14 18 N.	= Diff. lat.		To 37 00, the mer. parts	= 2394	
51 18 N.	= Pres. lat.		Mer. diff. lat. = 1204.		

Then make AB = 1024; continue AC till it meets BE drawn parallel to Ap ; and BE is the diff. longitude.

To find the diff. longitude.			Rad : T :: M : L.		
As Rad	= 45° 00'	10,00000	Now 9° 02' W.	= Dep. long.	
To tan. Cou.	= 33 19	9,81776	60) 791 (13 11 W.	= Diff. long.	
So m. diff. lat.	= 1204	3,08063			
			22 13 W. = Pres. long.		
To diff. long.	= 791,4	2,89839			
			Course NW. b. N. nearly.		

COMPUTATION by Logarithmic tangents.

Find the course and present latitude as above.

Then, Dep. lat.	37° 00'.	Co-lat.	53° 00'.	$\frac{1}{2}$ co-lat.	26° 30'.
Pres. lat.	51 18.	Co-lat.	38 42.	$\frac{1}{2}$ co-lat.	19 21.
To $\frac{1}{2}$ co-lat. 26° 30', the log. tang. is	= 9,69774				
To $\frac{1}{2}$ co-lat. 19 21, the log. tang. is	= 9,54552				
Difference is					= 1522,2 = G.

<i>To find the diff. longitude.</i>			N : G :: T : L.
As N	=	9,89849	The present longitude may be found as above.
To G.	= 1552	3,18241	
So tang. Cou.	= 33° 19'	9,81776	
To diff. long.	= 791,8	2,89866	

BY THE TRAVERSE TABLE.

Find as before, by proportioning the parts, the diff. lat. = 856, and the course N. 33° 30' W.

Hence the present lat. is 51° 16' N.; and the mer. diff. lat. = 1200, $\frac{1}{20}$ th whereof is 60.

Then in the col. of lat., under 33° 30', find 60,04, the nearest to 60; against which, in the departure, stands 39,74; which multiplied by 20, gives 794,8 for the diff. longitude.

PROBLEM

PROBLEM X.

783. Given one latitude, departure and diff. longitude;
Required the other latitude, course and distance.

EXAM. *A ship from the latitude $51^{\circ} 18' N.$, longitude $22^{\circ} 06' W.$, having sailed between the S. and E. for several days, reckons she has made 564 miles of departure, and 786 miles of longitude: What is the lat. and long. of the place arrived at, and also her direct course and distance?*

BY MIDDLE LATITUDE SAILING.

BY CONSTRUCTION. Fig. 16. Plate VIII.

Draw a meridian AD; describe the quadrant Amp ; in the arc mp apply the chord $mB = 786$, the diff. long.; draw AB; make $ma = 564$; draw aF parallel to Am , and with the radius AF describe the arc FE, and draw the chord EF.

Then is EF = departure, and AE = sine of the co-mid. lat.

BY COMPUTATION.

To find the co-mid. latitude.

L	:	D	::	Rad	:	c.
As diff. long.	=	786		7,10458		
To Depart.	=	564		2,75128		
So Rad	=	$90^{\circ} 00'$		10,00000		
To cof. m. lat.	=	$45^{\circ} 51'$		9,85586		

Now Co-mid. lat. = $45^{\circ} 51'$

Middle lat. = $44^{\circ} 09'$

Sum lats. = $88^{\circ} 18'$

Depart. lat. = $51^{\circ} 18'$

Present lat. = $37^{\circ} 00'$

Differ. lat. = $14^{\circ} 18' = 858 m.$

Make AD = 858; draw DC parallel to Ap , and equal to EF; then AC being drawn will be the distance; and $\angle DAC$ the course.

To find the Course.

P	:	L	::	C	:	T.
As diff. lat.	=	858		7,06651		
To diff. long.	=	786		2,89542		
So cof. m. lat.	=	$45^{\circ} 51'$		9,85583		
To tang. Cou.	=	$33^{\circ} 19'$		9,81776		

Now $22^{\circ} 06' W.$ = Dep. long.
 $60 \setminus 786 (13^{\circ} 06' E.$ = Diff. long.
9 00 W. = Pres. long.

To find the Distance.

fin. $\angle DAC$	:	DC	::	Rad	:	AC.
As fin. Cou.	=	$33^{\circ} 19'$		0,26022		
To Depart.	=	564		2,75128		
So Rad	=	$90^{\circ} 00'$		10,00000		
To Diff.	=	1027		3,01150		

The Course is SE. b. S. nearly.
 Distance 1027 miles.
 Present lat. $37^{\circ} 00' N.$

BY THE TRAVERSE TABLE.

Now $\frac{1}{20}$ th of the diff. long. and dep. are 39,3 and 28,2.

Seek in the table, till the given dep. 28,2 is found in its col. against the given diff. long. 39 in the col. of distances; and it gives the co-mid. lat. among the degrees.

Here 28,2 falls between 28,05 over 46° , and 27,89 over $45^{\circ} 40'$; therefore taking the co-mid. lat. = $45^{\circ} 50'$, the present lat. is $37^{\circ} 02' N.$; the diff. lat. is 856; and its $\frac{1}{20}$ th is 42,8.

Then 42,8 and 28,2 will be found together under $33^{\circ} 30'$, the course, and between 51 and 52 in the distances; suppose 51,2, which multiplied by 20, gives 1024 for the distance.

784

PROBLEM XI.

784. Given one latitude, course and difference of longitude;
Required the other latitude and distance.

EXAM. *A ship from the lat. 37° 00' N., long. 22° 56' W., runs on a course N. 33° 19' E. until she finds her diff. longitude is 786 miles: What is her present latitude and distance sailed?*

This problem cannot be resolved by Middle latitude sailing.

By MERCATOR'S SAILING.

By CONSTRUCTION Fig. 24. Plate VIII.

Draw the meridian AB; describe the quadrant A *mp*; continue A *p* till Aa = 786 miles, the diff. long., and draw a E parallel to AB. Make the arc *mc* = 33° 19'; draw the rhumb A *c* till it meets a E in E, and draw EB parallel to A *p*.

COMPUTATION by Meridional parts.

To find the Mer. diff. latitude.

T : R :: L : M.	Dep. lat. 37° 00' N.	Mer. pts. 2394
As tan. Cou. = 33° 19'		Mer d. lat. 1196
To Rad = 45 00		
So diff. long. = 786		
	Pres. lat. 51 14 N.	M. pts. 3590
To m. diff. lat. = 1196	Diff. lat. 14 14 = 854 miles.	

Make AD = 854, draw DC parallel to A *p*; then AC is the distance.

To find the distance. $\sin \angle DCA : AD :: \text{Rad} : AC.$

As cos Cou. = 33° 19'	0,67798	
To diff. lat. = 854	2,93146	
So Rad = 90° 00'	10,00000	
To Dist. = 1022	3,00944	

COMPUTATION by Logarithmic tangents.

To find the present latitude.

T : L :: N : G.	Dep. lat. 37° 00'.	Co lat. 53° 00'.
As tang. Cou. = 33° 19'	0,18224	$\frac{1}{2}$ co-lat. 26 30. L. tan. 9,69774
To diff. long. = 786	2,89542	c = 1511
So N =	10,10151	$\frac{1}{2}$ co-lat. 19 24. L. tan. 9,54664
To c = 1511	3,17917	Co-lat. 38 48.

Therefore the pres. lat. is 51° 12' N.

By THE TRAVERSE TABLE.

Seek the diff. long. in the col. of dep. to the given course, and it gives in the col. of lat. the mer. diff. lat.

Thus under 33° 15', against 39,3 (= $\frac{1}{20}$ th of 786) in the dep. stands 59,38; or by proportioning, 59,86, which multiplied by 20, gives 1197 for the mer. diff. lat. Hence the present lat. is 51° 14' N.; and the diff. lat. 854; its $\frac{1}{20}$ th is 42,7.

Then under 33° 15', 42,7 in the col. of lat. stands against 51 in the col. of distances; which multiplied by 20, gives 1020 for the distance.

SECTION V.

785. *Of compound Courses corrected by Longitude.*

How to solve compound courses, or Traverses, on the principles of plane sailing, has been already shewn: But it is necessary also, to shew how the proper allowances for longitude are to be introduced into such accounts; which is usually done by the following methods.

RULE I.

786. 1st. Complete the Traverse table to each course and distance, and find the whole difference of latitude, departure and present latitude.

2d. Then with the departed latitude, and present latitude, find the co-middle latitude; with which, and the departure, find the course, distance, and difference of longitude, as in a single course. By Prob. VII. (777)

But the longitude arrived in, is more accurately found by the following

RULE II.

787. 1st. Complete the Traverse table as before.

2d. Make a table of seven columns, title them Dep. lats., Pref. lats., Sum lats., Mid. lats., Co-mid. lats., $\frac{\text{diff. longit.}}{\text{E. | W.}}$.

3d. By the several differences of latitude in the Traverse table, fill up the first and second columns, and hence the corresponding sum lats., mid. lats., and co-mid. lats. will be obtained. (758)

4th. With each corresponding co-mid. lat. and departure, find the difference of longitude to each course, and set them in columns marked E. W., according to the name of the departure used.

5th. Add up both the east and west columns of longitude, and the difference of their sums will be the whole difference of longitude.

The first of these rules gives the method commonly practised at sea; but the latter is more to be trusted to, especially if there are several courses concerned.

788. EXAMPLE I. Suppose a ship from the latitude $52^{\circ} 20' N.$, longitude $14^{\circ} 38' W.$, has sailed the following courses, viz.

1st. ESE. 43 m. 2d. SW. 32 m. 3d. SE. b. S. 58 m. 4th. SSW. 60 m.

Required her present latitude and longitude?

By

BY THE FIRST RULE.

The TRAVERSE TABLE.					
Courses.	Diff.	N.	S.	E.	W.
ESE.	43		16,5	39,7	
SW.	32		22,6		22,6
SE. b. S.	58		48,2	32,2	
SSW.	60		55,4		23,0
			142,7	71,9	45,6
				45,6	
				26,3	

To find the diff. longitude.

BY MIDDLE LATITUDE.

Dep. lat.	=	52° 20' N.
Diff. lat.	=	2 23 S.
Present lat.	=	49 57 N.
Sum of lats.	=	102 17
Mid. lat.	=	51 08
Co-mid. lat.	=	38 52
Depart. long.	=	14 38 W.
Diff. long.	=	0 42 E. (777)
Present long.	=	13 56 W.

BY MERCATOR.

Dep. lat.	52° 20' N.	Mer. pts.	3698
Pres. lat.	49 57 N.	Mer. pts.	3470
		Mer. diff. lat.	228
P : D :: M : L.			
As prop. diff. lat.	=	142,7	7,84558
To Depart.	=	26,3	1,41996
So mer. diff. lat.	=	228	2,35793
To diff. long.	=	42,02	1,62347

BY THE SECOND RULE.

Working by middle latitude.

The Traverse table being made as above.

Dep. lats.	Present lats.	Sum lats.	Mid. lat.	Co mid lat.	Differ. longit.	
					E.	W.
52° 20'	52° 04'	104° 24'	52° 12'	37° 48'	1° 05'	0 1
52 04	51 41	103 45	51 52	38 08		0 37
51 41	50 53	102 34	51 17	38 43	0 51	
50 53	49 58	100 51	50 26	39 34		0 36
					1 50	1 13
					1 13	
					0 43	

The diff. longitude found by this method differs from the first, by only 1 min.

Here, in the col. signed Dep. lat., the lat. sailed from is entered, which is 52° 20'; from whence the first diff. lat. 16' S. being taken, leaves 52° 04' N. for the present lat., which, with regard to the next course, is a departed lat. also; therefore it is put also among the dep. lats. Then the next diff. lat. 22,6 m., or rather 23 m., being taken from

the last departed lat., leaves $51^{\circ} 41'$ for the next pref. lat.; and proceed in this manner until the present latitude is found at the end of the courses: Then take the sums of those departed and present latitudes standing against one another; also let the mid. latitudes and co-mid. latitudes be wrote in the columns as there titled: And then with these co-mid. latitudes and the corresponding departures, found before in the Traverse table, find the respective differences of longitude (777), which put in their proper columns. And hence the whole difference of longitude is readily found.

BY THE SECOND RULE.

Working by Mercator.

THE TABLE.								
Courses.	Dist.	N.	S.	Successive latit.	Mer. parts.	M. diff. lat.	Diff. long	
							E.	W.
				$52^{\circ} 20' N$	3697,8			
ESE.	43		16,5	$52^{\circ} 04'$	3671,7	26,1	63,0	
SW.	32		22,6	$51^{\circ} 41'$	3634,4	37,3		37,3
SE. b. S.	58		48,2	$50^{\circ} 53'$	3557,7	76,7	51,2	
SSW.	60		55,7	$49^{\circ} 58'$	3471,2	86,3		35,7
							114,2	73,0
							73,0	
							41,2	D lon.

Here the diff. longitude is nearly the same as before.

In this operation the departures are not wanted, therefore only the differences of latitude are found in the traverse; and with these the column of successive latitudes are easily filled: To these let the meridional parts be taken, and their differences, as in the columns; then with the courses, and the meridional diff. latitudes, the differences of longitudes are found as directed in art. 772.

From all these operations it appears, that in forming the account of a day's work, in almost any latitude, the difference of the result by either method, working by Mercator or by Middle latitude, is so very inconsiderable, that it is indifferent which of them is used; especially if these accounts are corrected by proper observations made as often as it shall be thought convenient, or as they can be obtained.

However, that the mariner may have a further proof of the agreement between the different methods proposed, the following example, in latitudes as high as most navigations are performed, is here annexed.

EXAMPLE

789. EXAMPLE II. Suppose a ship from the latitude $67^{\circ} 20' N.$, longitude $8^{\circ} 46' W.$, has sailed the following courses, viz.

NE. 64 m. NNE. 50 m. NW. b. N. 58 m. WNW. 72 m.
West 48 m. SSW. 38 m. S. b. E. 45 m. ESE. 40 m.
Required her present latitude and longitude?

BY MIDDLE LATITUDE, according to the second rule.

Courses.	Dif.	Diff. lats.		Depart.		Success. lats.	Sum lats.	Mid. lat.	Co.m. lat.	D. long.	
		N.	S.	E.	W.					E.	W.
NE.	64	45,3		45,3		67° 30'	0	0	0		
NNE.	50	46,2		19,1		68 15	135 45	67 52	22 08	120	
NW. b. N.	58	48,2			32,2	69 01	137 16	68 38	21 22	53	
WNW.	72	27,6			66,5	69 49	138 50	69 25	20 35		92
West	48				48,0	70 17	140 06	70 03	19 57		195
SSW.	38		35,1		14,5	70 17	140 34	70 17	19 43		142
S. b. E.	45		44,1	8,8		69 42	139 59	69 59	20 01		42
ESE.	40		15,2	36,0		68 58	138 40	69 20	20 40	25	
						68 42	137 41	68 50	21 10	102	
		167,3	94,5	110,1	161,2					130	471
		94,5			110,1						00
Diff. lat.		72,8		Depart		51,1		Diff. longit.		171	

BY MERCATOR, according to the second rule.

Courses.	Dif.	Diff. lat.		Depart.		Success. lats.	Mer. parts.	Mer. d. lat.	D. longit.	
		N.	S.	E.	W.				E.	W.
NE.	64	45,3		45,3		67° 30'	5551,6			
NNE.	50	46,2		19,1		68 15	5671,1	119,5	119,5	
NW. b. N.	58	48,2			32,2	69 01	5797,3	126,2	52,4	
WNW.	72	27,6				69 49	5933,9	136,6		91,1
West.	48				66,5	70 17	6016,6	82,1		198,6
SSW.	38		35,1		48,0	70 17	6016,0	00,0		142,*
S. b. E.	45		44,1	8,8	14,5	69 42	5913,7	102,3		42,4
ESE.	40		15,2	36,0		68 58	5789,0	124,7	24,8	
						68 43	5747,4	41,6	100,7	
		167,3	94,5	110,1	161,2				297,4	474,1
		94,5			110,1					297,4
Diff. lat.		72,8	Depart.		51,1			Diff. longit.		170,7

Departed lat.	$67^{\circ} 30' N.$	Depart. long.	$8^{\circ} 46' W.$
Different lat.	$1^{\circ} 13' N.$	Diff. long.	$2^{\circ} 56' W.$
Present lat.	$68^{\circ} 43' N.$	Present long.	$11^{\circ} 42' W.$

According to the first rule.

The co-mid. lat. between the first and last lats. is $21^{\circ} 54'$, with which, and the departure 51,1, the diff. long. will be 137 miles.

Also, the mer. diff. lat. between the first and last lats. is 195,8; with which, and the diff. lat. 72,8, and the dep. 51,1, gives a diff. long. 137,1.

* This diff. long. is found by art. 751.

EXAMPLE

790. EXAMPLE III. Suppose a ship from the latitude of $68^{\circ} 38' N.$, longitude $8^{\circ} 40' E.$, has sailed the following courses, viz.

NE. b. N.	63 m.	NE.	38 m.	NNE.	56 m.
North	30 m.	NW. b. N.	25 m.	NNW. $\frac{1}{2}$ W.	36 m.
N. b. E.	40 m.	NE. b. E. $\frac{1}{2}$ E.	72 m.	SE.	50 m.
ENE.	65 m.				

Required her present latitude and longitude, with her direct course and distance to the North Cape, in lat. $71^{\circ} 27' N.$, longitude $26^{\circ} 30' E.$?

Courses.	Dif.	D. lat.		Depart.		Success. lats.	Mer. parts.	Mer. diff. lat.	Diff. long.	
		N.	S.	E.	W.				E.	W.
NE. b. N.	63	52,4		35,0		$68^{\circ} 38'$	5733,7			
NE.	38	26,9		26,9		69 30	5879,2	145,5	97,2	
NNE.	56	51,7		21,4		69 57	5957,2	78,0	78,0	
North	30	30,0				70 49	6112,1	154,9	64,0	
NW. b. N.	25	20,7			13,9	71 19	6204,5	92,4		
NNW. $\frac{1}{2}$ W.	36	31,7			17,0	71 40	6270,7	66,2		44,0
N. b. E.	40	39,2		07,8		72 12	6373,9	103,2		55,0
NE. b. E. $\frac{1}{2}$ E.	72	33,9		63,5		72 51	6503,8	129,9	26,0	
SE.	50		35,3	35,3		73 25	6621,0	117,2	219,4	
ENE.	65	24,9		60,0		72 50	6500,4	120,6	120,6	
						73 15	6586,1	85,7	207,0	
		311,4	35,3	249,9	30,9				812,2	99,0
		35,3		30,9					99,0	
Diff. latit.		276,1		210,0	Dep.			Diff. longit.	712 2	

The diff. long. found by the middle latitudes is 712.

Depart. lat.	$68^{\circ} 38' N.$	Dep. long.	$8^{\circ} 40' E.$
Diff. lat.	4 36 N.	Diff. long.	11 53 E.
Present lat.	73 14 N.	Pres. long.	20 33 E.
N. Cape lat.	71 27 N.	N. Cape long.	26 30 E.

Diff. lat. 1 47 N. Mer. diff. lat. 353,0 Diff. long. 5 57 E.

Or 107 miles.

Or 357 miles.

By Middle latitude.

Sum lats. $144^{\circ} 41'$

Mid. lat. 72 21

Co-mid. lat. 17 39

With co-m. lat. and diff. long. find dep.

To diff. long. 300 the Dep. is 91,47

To diff. long. 57 Dep. is 17,38

To diff. long. 357 Dep. is 108,85

As the diff. lat. and dep. are so nearly equal, therefore

The Course is SE.

The Distance is 152 miles.

By Mercator.

With the mer. diff. lat. and the diff. long. the course will be found 4 points, or SE.

And with the course and proper diff. lat. the departure will be 107, and the distance 152.

SECTION

SECTION VI.

The construction and use of a Mercator's Chart.

The linear and numeral solutions of such questions as may be proposed concerning sailing by the principles on which Mercator's charts are founded, having been sufficiently exemplified; it follows next to shew how such questions may be solved by the Chart itself.

791. PROBLEM I.

To make a true sea chart which shall shew the known sea-coasts, islands, rocks, shoals, &c. that are contained within any proposed limits.

SOLUTION.

1st. Construct the boundaries or limits of the chart, within the proposed latitudes and longitudes, by Prop. IV. art. 725; and let the degrees of latitude and longitude be marked on these limits.

2d. On a piece of stiff paper, parchment, &c. make a scale of equal parts, like the degrees of longitude that are in the graduated parallel of latitude, and let the divisions on this scale be numbered in the same manner.

3d. From a table of the latitudes and longitudes of places, extract those places, with their latitudes and longitudes, that lie within the proposed limits.

4th. Then by sliding the outside divisions of the scale of the graduated parallel, along the graduated meridians, so that both ends of the scale, at the same time, always cut like divisions of latitude; the points corresponding to the latitudes and longitudes of the places contained within the proposed limits, may be readily marked in the chart.

5th. Lines being properly drawn from point to point, will form the outlines of the sea-coasts, islands, &c.; to which may be annexed the *depths of water, setting of currents*, and whatever else may be judged convenient for the chart to contain.

In this manner was constructed the sea-coasts, &c. in the chart in Plate VII.; containing part of the sea-coasts of Europe, Africa and America, with the intermediate islands; extending from the latitude of 23 degrees to 59 degrees north; and from the longitude of London, or 0 degrees, to 80 degrees of longitude west.

792. The Gunter's scales have drawn on them two lines, one marked *Mer.*, signifying the *nautical meridian*; and the other, which is placed directly under it, is marked *E. P.*, signifying *equal parts*.

These equal parts are degrees of longitude, to which the divisions of the nautical meridian are fitted by increasing them in their true proportion; which is readily done by the table in art. 725, taking the meridional degrees from the scale of equal parts, or degrees of longitude. The use of these two lines is to construct a Mercator's chart by, and to shew nearly the meridional diff. lat. between any two parallels of latitude.

The

The construction of the chart, or rather the limits or borders thereof, by these lines, is easily done by transferring the divisions corresponding to the degrees to be used, from the scale to the paper.

The meridional difference of latitude is found by taking with the compasses the extent between the proposed latitudes, and applying it to the line of longitude; for the number of equatorial degrees and parts of degrees being reduced to miles, will shew the mer. diff. latitude between those parallels.

It has been usual on the Gunter's hitherto made, to take so small a part for a degree of longitude, that 180 of them came within the length of two feet; and as about 85 meridional degrees answered to this length, so many were put on the line; whereby the charts constructed from them had their degrees rather too small to be of much use to seamen, whose instruments and hands are not usually so sufficiently nice, as to work by such small charts: Therefore it is apprehended, that it would be more useful to increase the divisions of these lines, and instead of 180 degrees of longitude to put 100, and then about 70 meridional degrees will come within the limits of two feet; navigations to higher latitudes being seldom undertaken. Gunter's scales with these, and other useful alterations, have been lately made by the direction of the author, and sold by their makers.

793.

P R O B L E M II.

A latitude and longitude being given, to find the corresponding place on the chart. Example. Lat. $32^{\circ} 25'$ N. long. $17^{\circ} 21'$ W. Plate VII.

SOLUTION.

1st. In the graduated meridian*, take, with a pair of compasses, the distance between the proposed latitude and the nearest parallel of latitude drawn across the chart, as between the lat. $32^{\circ} 25'$, and the parallel of 30° .

2d. With other compasses, take the distance in the graduated parallel† between the proposed longitude and the nearest meridian drawn across the chart, as between the long. $17^{\circ} 21'$ W. and the meridian passing thro' 20° of longitude.

3d. Slide the point of one compass along the meridian of 20° , and a point of the other compass along the parallel of 30° , keeping the points of each compasses perpendicular to the line they slide along; then will the meeting of the other points of the compasses shew the place corresponding to the given latitude and longitude; which in this example is the west end of the Island of Madeira.

This problem is most readily solved by the means of such a moveable scale of longitude, as is described and used in the foregoing Problem.

Or thus. A meridian drawn thro' the degrees of longitude given, and a parallel of latitude drawn thro' the given degrees of latitude, their intersection will be the place required.

* The graduated meridian, is the scale of latitudes on the right or left hand borders of the chart.

† The graduated parallel, is the scale of longitudes on the top or bottom borders of the chart.

794.

PROBLEM III.

A place being given on the chart, to find its latitude and longitude.

Suppose Palma one of the Canary isles. Plate VII.

SOLUTION.

1st. With the compasses, take the shortest distance between the given place and the nearest parallel of latitude; this distance applied to the graduated meridian, from the said parallel the same way the place lies, will give the latitude sought. Thus the distance from Palma to the parallel of 30° , applied from 30° southward, gives $28^\circ 35'$ N. for the latitude sought.

2d. In like manner, the shortest distance taken between the given place and the nearest meridian, being applied to the graduated parallel, from that meridian, will give the longitude: Thus the distance from Palma to the meridian thro' 20° , applied from 20° eastward, will give $17^\circ 41'$ W. longitude.

795.

PROBLEM IV.

Two places being given on a Mercator's chart, to find their difference of latitude and difference of longitude.

Suppose Cape Finisterre and the Island of Porto sancto. Plate VII.

SOLUTION.

Find the latitude and longitude of each place; then the required differences are readily known,

By Prob. III.	{	C. Finisterre's lat.	=	$43^\circ 15'$ N.	Long.	=	$9^\circ 20'$ W.
		Isl. Porto sancto lat.	=	$33^\circ 07'$ N.	Long.	=	$15^\circ 53'$ W.
		Diff. lat.	=	$10^\circ 08'$	D. long.	=	$6^\circ 33'$ W.

796.

PROBLEM V.

Between two given places on a Mercator's chart, to find the bearing or course.

Suppose between Cape Clear and the Island St. Michael. Plate VII.

SOLUTION.

Lay the edge of a ruler upon Cape Clear and the middle of the Island of St. Michael; and holding the ruler in this position, take with a pair of compasses the shortest distance between the edge of the ruler and the centre of the nearest compass, or flie; slide one point along the edge of the ruler, till the other point cuts the border of the flie; and the course will be found to be SW. b. S. $\frac{1}{4}$ W. nearly.

And the like for other places.

PROBLEM VI.

Two places being given in a Mercator's chart, to find their distance.

797. CASE I. *When the given places are both under the same meridian, or under the equator.*

SOLUTION.

Find the latitude of each place when under the same meridian, or their longitudes if under the equator.

Then the difference of the latitudes is the distance sought; or, the difference of longitudes is the distance sought.

798. CASE II. *When the given places are both under the same parallel of latitude.*

Suppose between Cape Roxant and the Island of St. George. Pl. VII.

SOLUTION.

Lay the edge of a ruler over the given places; take half the distance of those places with the compasses; set one point on the division cut by the ruler in the graduated meridian, and apply the other point upwards and downwards on the same meridian, noting the degrees of latitude the said point falls on: Then the difference of those latitudes will be the distance sought, very nearly.

Thus, half the distance between Cape Roxant and the Island of St. George, will reach from $38^{\circ} 45'$, the division cut by the edge of a ruler laid over those places, to $45^{\circ} 00'$ northward; and to $32^{\circ} 20'$ southward; the difference between these two latitudes is $12^{\circ} 40'$, or 760 miles nearly, for the distance required.

But if the proposed places lie in a parallel of latitude greater than 30 degrees, it will be nearer the truth to work as follows.

Take with the compasses that meridional degree wherein the parallel falls; and as many times as this distance is contained in a strait line between the two places, of so many degrees is their distance to be estimated; which is reduced to miles as before.

Thus, the distance between the latitudes of $37^{\circ} 30'$ N. and $38^{\circ} 30'$ N. will be contained between Cape Roxant and the Island of St. George about $12\frac{2}{3}$ times; or $12^{\circ} 40'$, as before *.

799. CASE III. *When the given places differ both in latitude and longitude.*

Suppose the Lizard and the Island of Corvo. Plate VII. Fig. 4.

SOLUTION.

Find the latitudes of the two places, and their diff. latitude. Take the diff. lat. in degrees and parts from the equator.

* These operations depend on the following proportion.

As mer. diff. lat. : proper diff. lat. :: diff. long. : departure; which will differ but very little from the meridional distance, unless the places are very far asunder.

Lay

Lay the edge of a ruler over the two places; slide one point of the compasses along the edge, till the other point, held perpendicular to the ruler, falls on a parallel of latitude, and take the length of the parallel intercepted between this point and the edge of the ruler.

Then this extent applied to the graduated parallel gives, in degrees, the distance sought.

Thus, the difference of latitude between Corvo and the Lizard is $10^{\circ} 09'$; then $10^{\circ} 09'$ taken from the graduated parallel, and one point of the compasses applied to the edge of the ruler laid in the direction CL , slide that point along CL till the other point, kept in a perpendicular position, cuts the parallel of 50° in B ; then the distance BL , measured on the graduated parallel, gives about $20^{\circ} 25' = 1225$ miles, the distance required.

OR THUS. Take the difference of latitude $10^{\circ} 09'$ from the graduated parallel, apply one point of the compasses to the edge of the ruler touching C and L , slide this point along till the other point touches a parallel (and not cutting it) in E ; then the other point resting at D , take the distance DL , which measured on the graduated parallel will give, in degrees, the distance sought.

These methods are in effect the same; for the triangles LBA , LDE , are congruous; as the angle at L is common, the angles A and E are right angles, and the sides AB , ED , are equal; therefore BL and DL are equal. (192)

If the difference of latitude DE be taken on a meridian thro' C , from C to F ; and the parallel of latitude FH be drawn, cutting the rhumb line CL in H ; then are the triangles LDE , HCF , congruous; and $CH = DL$. (185, 192)

Now it is evident, that the parts of the triangles CFH , CGL , bear the same proportion to one another, as have been shewn in art. 727; where CF is the proper diff. lat., CG the merid. diff. lat., GL the diff. longitude, FH the departure, and CH the distance: Consequently the distance DL , or BL , found as above, is the true distance sought.

800.

PROBLEM VII.

Given the latitude and longitude of the departed place, the course, and the latitude of the present place: Required that place on the chart.

EXAMPLE.

A ship from the Island of Flores runs SE. b. S. $\frac{3}{4}$ E. till she finds herself in the latitude of $17^{\circ} 29' N.$: Required her present place? Pl. VII. Fig. 5.

SOLUTION.

Thro' the departed place, Flores, draw the given rhumb, viz. SE. b. S. $\frac{3}{4}$ E.; and its intersection with the parallel of latitude drawn thro' the latitude of the present place, viz. $17^{\circ} 29'$, will give the ship's place on the chart; which will be near the Island of Ferro, or in lat. $27^{\circ} 48' N.$ long. $17^{\circ} 26' W.$

801. PROBLEM VIII.

Given the departed place, and also the course and distance, to find the present place on the chart.

EXAMPLE.

A ship from the Island of Ferro runs NW. b. N. $\frac{3}{4}$ W. 951 miles : What is the latitude and longitude of the present place? Plate VII. Fig. 5.

SOLUTION.

Lay the edge of a ruler over Ferro in the position of the given course NW. b. N. $\frac{3}{4}$ W. ; take the degrees $15^{\circ} 51'$ in the given distance, 951 miles, from the graduated parallel, and apply this extent from Ferro along the ruler's edge to R ; take the nearest distance between R, and a parallel PZ drawn thro' Ferro, and see how many degrees this extent makes on the graduated parallel, which will be $11^{\circ} 35'$; those degrees counted northwards from Ferro will give $39^{\circ} 23' N.$ for the ship's latitude ; then the point Q, where a parallel thro' this latitude intersects the edge of the ruler, still lying in the first position, will be the ship's present place.

802. *If the course be due east or west.*

EXAMPLE.

A ship from Cape Roxant sails due west 760 miles : Required her present place. Pl. VII.

SOLUTION.

Take half the degrees (*viz.* $6^{\circ} 20'$) in the given distance, 760 miles ; add them to, and subtract them from, the given lat. $38^{\circ} 42'$, noting the sum $45^{\circ} 02'$, and the remainder $32^{\circ} 22'$: Then the extent between those degrees taken in the graduated meridian, and applied from the given place westward in the parallel, will give the ship's present place ; which will be near the Island of St. George.

Or thus, if the latitude be above 30 degrees.

Take the length of a degree in the latitude of the given parallel, as much above the parallel as below it ; and turn this distance over in that parallel, the same way the course is, as many times as there are degrees in the given distance, and this will give the ship's place.

Thus the length of a degree taken, in the latitude $38^{\circ} 42'$, and turned over from Cape Roxant to the west $12\frac{2}{3}$ times, the compasses will fall on the Island of St. George.

803. PROBLEM IX.

Given the departed place, with the difference of latitude and distance : Required the ship's place on the chart.

EXAMPLE.

A ship from Q sails in the SE. quarter 951 miles, and then finds herself in lat. $27^{\circ} 48' N.$, having altered her latitude $11^{\circ} 35'$: Required her present place. Plate VII. Fig. 5.

SOLUTION.

Take from the graduated parallel the difference of latitude $11^{\circ} 35'$;
lay

lay this extent from Q along the edge of a ruler, lying north and south, to x; turn the edge of the ruler about x till it lies in an east and west position: From the graduated parallel take $15^{\circ} 51'$, the degrees in 951 miles the given distance, apply this extent from Q to cut the edge of the ruler in v: Let the points of the compasses rest in Q and v, and apply the edge of the ruler close to those points; then the point P, where the parallel of $27^{\circ} 48'$ is cut by the ruler's edge, is the ship's place required.

Or, by drawing lines on the chart, let the diff. lat., taken from the graduated parallel, be applied in a meridian drawn thro' Q from Q to x: From Q with the distance in degrees taken from the graduated parallel, cut, in v, a parallel drawn thro' x: Then the point P, where a rhumb thro' Q and v cuts the parallel of latitude come to, viz. $27^{\circ} 48'$, is the place sought.

804.

PROBLEM X.

Given the departed place, with the course and difference of longitude: Required the ship's place on the chart.

EXAMPLE.

A ship from P sails NW. b. N. $\frac{3}{4}$ W: Required her place when she has altered her longitude $12^{\circ} 53'$? Plate VII. Fig. 5.

SOLUTION.

Take the given difference of longitude $12^{\circ} 53'$ from the graduated parallel; slide one foot of the compasses along the edge of a ruler laid on P, in the position PQ of the given course, until the other foot just touches in T a meridian PT drawn thro' P; then will the point Q, where the foot rests against the ruler's edge, be the place required.

805. *If the course had been due east or west.*

EXAMPLE.

A ship from Cape Roxant sails due west until she has altered her longitude $16^{\circ} 40'$: Required her place on the chart.

SOLUTION.

Take the difference of longitude $16^{\circ} 40'$ from the graduated parallel; then this extent applied from the given place Cape Roxant due west, will give the place required, which is near the Island of St. George.

806.

PROBLEM XI.

Given the departed place, with the difference of latitude and departure: Required the ship's place on the chart.

EXAMPLE.

A ship from Q, in lat. $39^{\circ} 23' N.$, sails between the south and east till she comes into the lat. $27^{\circ} 48' N.$, having made by account 649 miles of departure: Required the ship's present place? Pl. VII. Fig. 5.

SOLUTION.

Take the given difference of latitude $11^{\circ} 35'$ from the graduated parallel, and apply this extent from Q, along the edge of a ruler laid in the meridian of Q, to x; take $10^{\circ} 49' = 649$ miles, the
given

given departure, from the graduated parallel; set one foot on x , and lay this extent eastward to v , so that xv be in a perpendicular position to the ruler's edge: Lay the edge of the ruler on the points Q and v , and where it cuts zP , the parallel of the ship's present latitude in P , is the ship's place required.

Then the longitude, course and distance may be found as before. See articles 794, 796, 799.

807.

PROBLEM XII.

Given the departed place, with the course and departure: Required the ship's place on the chart.

EXAMPLE.

A ship from P , in latitude $27^{\circ} 48' N.$, sails NW. b. N. $\frac{3}{4} W.$, until she has made 649 miles of departure: Required her present place? Pl.VII. Fig. 5.

SOLUTION.

Lay the edge of a ruler over the given place P , in the position of the given course NW. b. N. $\frac{3}{4} W.$; take from the graduated parallel $10^{\circ} 49' = 649$, the given departure, and slide one foot along the ruler's edge, until the other foot just touches in s a meridian PT drawn thro' P ; the distance SP applied to the graduated parallel gives the diff. latitude; and consequently the present latitude is known: Then where the ruler's edge, lying in the same position, cuts the present parallel of lat. TQ , as in Q , is the place sought.

808.

PROBLEM XIII.

Given the departed place, with the distance and departure, to find the ship's present place on the chart.

EXAMPLE.

A ship from Q , in lat. $39^{\circ} 23' N.$, in sailing between the south and east 951 miles, found she has made 649 miles of departure: Where is the present place of the ship? Plate VII. Fig. 5.

SOLUTION.

From the graduated parallel take $10^{\circ} 49' = 649$ miles, the given departure, and lay this extent from Q to o directly east; with the extent $15^{\circ} 51' = 951$ miles, the given distance, taken from the graduated parallel, one foot being set in Q , with the other foot cut the edge of a ruler laid on o in a meridional position, as in v ; transfer the distance ov from Q to x in a meridian passing thro' Q ; then Qx measured on the graduated parallel gives the diff. latitude, and consequently the present latitude $27^{\circ} 48'$ will be found: Lay a ruler's edge over Q and v , and where it cuts the parallel of $27^{\circ} 48'$, as in P , is the ship's place required.

SECTION VII.

Of sailing on a Great circle of the sphere.

809. GREAT CIRCLE SAILING is the art of finding what places a ship must go thro', and what courses to steer, so that her tract shall be in the arc of a great circle (or nearly so) passing thro' the place sailed from and that bound to.

Although Mercator's sailing resolves truly all the cases incident to a ship's course along the rhumb passing thro' two places; yet there are few cases, wherein the course performed on the direct rhumb line is the shortest distance between those two places.

For on the sphere, the shortest distance between two places is the arc of a great circle intercepted between them (404); consequently the spiral or rhumb, passing thro' those places, is not the shortest distance but when that rhumb coincides with a great circle, which can only be on a meridian, or on the equator.

Now it is chiefly on account of the shortest distance that this method of sailing has been proposed; and it is for the most part advantageous for a ship to reach her port by the shortest way she can, when and where it is practicable,

As the solutions of the cases in Mercator's sailing were performed by plane triangles; so the cases in Great circle sailing are resolved by the solution of spheric triangles.

A great variety of cases may be proposed in this kind of sailing; but as they serve rather for exercises in the solution of spheric triangles, than for any real use towards the navigating a ship, therefore those appertaining to only one problem will be here more particularly considered, as they are the most useful cases that usually occur.

810. The angle of position, is an angle which a great circle, passing over two places, makes with the meridian of one of them; and shews the angular position of one from the other.

P R O B L E M.

Given the latitudes and longitudes of two places on the earth: Required their nearest distance on the surface, together with the angles of position from either place to the other.

This problem may be branched into six cases,

811. CASE I. *When the two places lie under the same meridian.*

SOLUTION. Their difference of latitude will give their distance; and the position of one from the other will be directly north or south.

812. CASE II. *When the two places lie under the equator.*

SOLUTION. Their distance is equal to their difference of longitude; and the angle of position is a right angle, or the course from one to the other is due east or west.

813. CASE III. *When both places are in the same parallel of latitude.*

EXAMPLE. *What is the least distance between St. Maries, in latitude $37^{\circ} 00' N.$, longitude $22^{\circ} 56' W.$, and Cape Henry, in latitude $37^{\circ} 00'$, longitude $76^{\circ} 23' W.$?*

GENERAL CONSTRUCTION. Plate VIII. Fig. 25.

1st. Describe a circle PESQ representing the meridian of one of the places, suppose of the eastern one, as St. Maries; draw the line EQ representing the equator, and at right angles to it draw the line PS for the axis of the earth, whose extremity P is the north pole, and S is the south pole; and on this circle lay off from P to A the complement of the latitude of (St. Maries) the eastern place.

2d. On the equator, lay off from Q to C (446) the difference of longitude between the two places ($53^{\circ} 27'$); and thro' P, C, S, (163) describe the circle PCS, which will be the meridian of the other place (Cape Henry); on which lay from P to B the co-latitude of this place; which is done by describing the arc Aa about the pole P (456) at the distance of the co-latitude,

3d. Thro' the points A, B, D, describe a great circle ABD. (163)

Then A represents one place (St. Maries), B the other (Cape Henry); PA and PB are their co-latitudes; the angle APB, which is measured by the arc QC (389), is the diff. long. the arc AB is the nearest distance of those places; the angle PAB is the angle of position from A to B; and the angle PBA is the angle of position from B to A.

The arc AB may be measured as directed in article 446.

And the angle PAB, or PBA, as shewn in art. 448.

Now the places having the same latitude; therefore $PA = PB$, and $\angle PAB = \angle PBA$ (470). Therefore, if the arc PI be described (451) making the angle $API = 26^{\circ} 43\frac{1}{2}'$, the half of the difference of longitude; then will PI be perpendicular to AB, bisecting it.

And in the triangle AIP, right-angled at I, there will be given the hypotenuse $AP = 53^{\circ} 00'$; the $\angle API = 26^{\circ} 43\frac{1}{2}'$; to find the leg AI = half the distance sought, and the $\angle PAI$ = the angle of position.

And the solution hereof falls under art. 513, 515.

To find the distance.

As Radius	Or, As Rad	=	90° 00'	10,00000
To fin. hyp. PA,	To fin. co-lat.	=	37 00	9,90235
So fin. giv. $\angle$ API	So fin. $\frac{1}{2}$ diff. long.	=	26 43 $\frac{1}{2}$	9,65292
To fin. leg AI.	To fin. $\frac{1}{2}$ distance	=	21 03	9,55527

Which doubled, gives 42 06 for the dist. AB.

And this 42° 06' reduced to nautical miles, is 2526, which is 35 miles less than 2561, the distance given by parallel sailing. See art. 755.

To find the angle of position. (515)

As Radius	Or, As Rad	=	90° 00'	10,00000
To cos. hypoth. PA,	To fin. lat *	=	37 00	9,77946
So tan. giv. $\angle$ API	So tan. $\frac{1}{2}$ diff. lon.	=	26 43 $\frac{1}{2}$	9,70199
To cot. $\angle$ A.	To cot. $\angle$ posit.	=	73 09	9,48145

Whereby it appears, that to sail from A to B, or from B to A, the ship must first steer N. 73° 09' W. or E.; and then gradually increase her course till she comes to i, where it will be due W. or E.; and from thence the course is to be gradually diminished again till she comes to the other port, where it will be 73° 09', the same as she set out with: But how these courses are to be thus altered, will be shewn hereafter.

814. CASE IV. *When one place has latitude, and the other has none, or is under the equator.*

EXAMPLE. *What is the nearest distance between the Island of St. Thomas, in lat. 0° 00', and long. 1° 00' E., and Port St. Julian, in lat. 48° 51' S., and longitude 65° 10' W.?*

Port St. Julian, in lat. 48° 51' S.

Long. 65° 10' W.

Isl. St. Thomas, in lat. 0 00

Long. 1 00 E.

Julian's co lat. 41 09

Diff. long. 66 10

BY CONSTRUCTION. Plate VIII. Fig. 26.

Let the point A be St. Thomas under the equator, whose poles are P and s.

Make AC, the measure of the angle Asc, equal to 66° 10' = diff. long. (450)

Then as Port St. Julian is in south lat., about s the south pole, at the distance of Julian's co-lat., describe the arc aa, cutting scp, the meridian of Julian, in B. (454)

Thro' the points A, B, E, a great circle being described, then the arc AB is the distance sought. (163)

The distance AB may be measured by art. 446.

And the angles of position at A and B are to be measured as directed in art. 448.

* For the complement of a complement, is the arc itself: Thus QA, or the distance from the equator, shews the lat. But QA is the comp. of AP; and AP of AQ.

COMPUTATION. In the quadrantal triangle ASB, there is
 Given the co-lat. of St. Thomas, or $AS = 90^\circ 00'$
 the co-lat. of St. Julian, $SB = 41^\circ 09'$
 the diff. longitude, $\angle ASB = 66^\circ 10'$ } The rest are found as in art. 545.

Or, in the supplemental triangle ACB, right angled at C, there is
 Given the lat. of St. Julian's, or the leg $CB = 48^\circ 51'$
 the difference longitude, or the leg $CA = 66^\circ 10'$.

The solution falls under article 523.

To find the distance AB.

As Radius	Or, As Radius	=	$90^\circ 00'$	10,00000
To cos. either leg,	To cos. diff. long. AC	=	$66^\circ 10'$	9,60646
So cos. other leg	So cos. diff. lat. CB	=	$48^\circ 51'$	9,81825
To cos. hyp.	To cos. dist. AB	=	$74^\circ 35'$	9,42471

So that the dist. $AB = 74^\circ 35'$ is 4475 miles; which is less by 57 miles than the distance found by Mercator's sailing.

To find the angle of position at A. (522)

As Radius	Or, As Radius	=	$90^\circ 00'$	10,00000
To sin. leg AC,	To sin. diff. long. AC	=	$66^\circ 10'$	9,96129
So cot. leg CB	So cot. diff. lat. CB	=	$48^\circ 51'$	9,94146
To cot. op. $\angle A$.	To cot. $\angle$ posit. at A	=	$51^\circ 22'$	9,90275

To find the angle of position at B. (522)

As Radius	Or, As Radius	=	$90^\circ 00'$	10,00000
To sin. leg CB,	To sin. diff. lat. CB	=	$48^\circ 51'$	9,87679
So cot. leg AC	So cot. diff. long. AC	=	$66^\circ 10'$	9,64517
To cot. op. $\angle B$.	To cot. $\angle$ posit. at B	=	$71^\circ 36'$	9,52196

So that a ship sailing from the Island of St. Thomas must first shape her course S. $51^\circ 22'$ W.; and then by constantly altering her course towards the west, so as to arrive at Port St. Julian on a course S. $71^\circ 36'$ W., she will have sailed the shortest distance between those places.

815. CASE V. *When the latitudes of the given places are both north or both south.*

EXAMPLE. *What is the nearest distance between the Lizard and the Island of Bermudas; and also the angles of position at each place?*

The Lizard in lat. $49^\circ 57' N.$ and long. $5^\circ 14' W.$
 Isl. Bermudas in lat. $32^\circ 25' N.$ and long. $66^\circ 38' W.$

Diff. long. $61^\circ 24'$

BY CONSTRUCTION. Plate VIII. Fig. 27.

Make $PA = 57^\circ 35'$, the co-lat. of Bermudas; $Pa = 40^\circ 03'$, the co-lat. of the Lizard; and (456) with the tang. of Pa describe the arc aa .
 With

With the secant of $61^{\circ} 24'$, the diff. long. (452), arcs described from P and s , give the centre of the circle PCS , the meridian of the Lizard; its intersection with aa gives B , the place of the Lizard.

A great circle described thro' A , B , D (163); the intercepted arc AB is the distance sought; and the angles PAB , PBA , are the positions required.

The distance AB may be measured by art. 446; and the positions by art. 448.

Had the eastern place, the Lizard, been put on the primitive circle, the great circle AB would have been difficult to describe; and therefore was the western place put on the primitive circle; it being a matter of indifference which of the places are so taken.

COMPUTATION. In the oblique spheric triangle APB .

Given Bermudas's co-lat., $PA = 57^{\circ} 35'$
 Lizard's co-lat., $PB = 40^{\circ} 03'$
 Diff. longitude, $\angle APB = 61^{\circ} 24'$ } Required the rest.

The solution falls under art. 534.

To find the distance AB.

As Radius	Or, As Radius	=	$90^{\circ} 00'$	10,00000
To cos. giv. $\angle$,	To cos. diff. long. $\angle APB$	=	$61^{\circ} 24'$	9,68006
So tan. eith. side	So tan. Berm. co-lat. PA	=	$57^{\circ} 35'$	10,19721
To tan. M .	To tang. fourth arc	=	$37^{\circ} 01'$	9,87727

Which take from Liz. co-lat. = $40^{\circ} 03'$

Leaves a fifth arc = $3^{\circ} 02' = N$.

As cos. M	As cos. fourth arc	=	$37^{\circ} 01'$	0,09775
To cos. N ,	To cos. fifth arc	=	$3^{\circ} 02'$	9,99939
So cos. side 1st used	So sin. Bermudas lat.	=	$32^{\circ} 25'$	9,72922
To cos. side required.	To cos. distance AB	=	$47^{\circ} 54'$	9,82636

To find the angle of position PBA. (533)

As this angle is opposite to the side used in the first proportion, therefore the fourth and fifth arcs here used, are the same as above.

As sin. N	As sin. fifth arc	=	$3^{\circ} 02'$	1,27641
To sin. M ,	To sin. fourth arc	=	$37^{\circ} 01'$	9,77963
So tan. given $\angle$	So tan. diff. long. $\angle APB$	=	$61^{\circ} 24'$	10,26343
To tan. req. $\angle$	To tan. $\angle$ posit. $\angle PBA$	=	$87^{\circ} 15'$	11,31947

To find the angle of position PAB.

The other parts being found, this angle may be found by the proportion between opposite sides and angles. Thus:

As sin. PA	Or, As sin. Bermudas's co-lat.	=	$57^{\circ} 35'$	0,07357
To sin. $\angle PBA$,	To sin. $\angle$ pos. at Lizard	=	$87^{\circ} 15'$	9,99950
So sin. PB	So sin. Lizard's co-lat.	=	$40^{\circ} 03'$	9,80852
To sin. $\angle PAB$.	To sin. $\angle$ pos. at Bermudas	=	$49^{\circ} 35'$	9,88159

G g g 2

Or,

Or, working by art. 533, the same angle would be obtained.

Also, Had the two first proportions been worked with the Lizard's latitude instead of Bermudas's, the same distance would have been found.

So the shortest distance between the Lizard and Bermudas is $47^{\circ} 54'$, or 2874 sea miles; which is 178 miles less than the distance found by Mercator's sailing.

And a ship to run this shortest tract, must sail from the Lizard S. $87^{\circ} 15'$ W., and gradually to lessen the course, so as to arrive at Bermudas on the rhumb bearing S. $49^{\circ} 35'$ W.: Whereas the direct course from one place to the other, as found by Mercator's sailing, is S. $69^{\circ} 02'$ W.

816. CASE VI. *When one of the given places has north latitude and the other has south latitude.*

EXAMPLE. *What is the nearest distance from the Island of St. Helena to the Island of Bermudas; and also the angles of position at each place?*

Island of Bermudas in lat. $32^{\circ} 25'$ N. and long. $66^{\circ} 38'$ W.

Island of St. Helena in lat. $16^{\circ} 00'$ S. and long. $6^{\circ} 15'$ W.

Diff. long. $60^{\circ} 23'$

BY CONSTRUCTION. Plate VIII. Fig. 28.

Make QA = $16^{\circ} 00'$, the lat. of St. Helena; describe the arc aa about P (456) with the tangent of Pa = $57^{\circ} 35'$, the co-lat. of Bermudas.

Arcs described from P, s, with the secant of $60^{\circ} 23'$, the diff. long., will give the centre of the circle PCS, the meridian of Bermudas; and its intersection B with aa is the place of Bermudas.

Describe a great circle thro' A, B, D; the intercepted arc AB is the distance sought, which may be measured by 446: and the angles PAB, ABS, are the positions required, which are to be measured as directed in art. 448.

COMPUTATION. In the oblique spheric triangle ABP.

Given PA = $106^{\circ} 00'$, St. Helena's lat., added to 90° ; or its N. pol. dist.

PB = $57^{\circ} 35'$, Bermudas's co-lat.

$\angle APB = 60^{\circ} 23'$, their difference of longitude.

To find the rest. The solution falls under art. 533, 534.

To find the distance AB.

As Radius	Or, As Radius	=	$90^{\circ} 00'$	10,00000
To cos. $\angle P$,	To cos. diff. long.	=	$60^{\circ} 23'$	9,69390
So tang. PB	So tan. Berm. co-lat.	=	$57^{\circ} 35'$	10,19721
To tang. M.	To tang. fourth arc	=	$37^{\circ} 54'$	9,89111
	Which taken from	=	$106^{\circ} 00'$	= St. Helena's N.
				[polar dist.]
	Leaves a fifth arc	=	$68^{\circ} 06'$	= N.

As cos. M	Or, As cos. fourth arc	=	$37^{\circ} 54'$	0,10288
To cos. N,	To cos. fifth arc	=	$68^{\circ} 06'$	9,57169
So cos. PB	So sin. Bermudas lat.	=	$57^{\circ} 35'$	9,72922
To cos. AB.	To cos. distance	=	$75^{\circ} 19'$	9,40379

To find the angle of position PAB.

The fourth and fifth arcs being found as before.

As fin. N	Or, As fin. fifth arc	=	68° 06'	0,03253
To fin. M,	To fin. fourth arc	=	37 54	9,78837
So tan. $\angle P$	So tang. diff. long.	=	60 23	10,24530
To tan. $\angle A$.	To tang. $\angle$ position	=	49 20	10,06620

To find the angle of position ABP.

As Rad	Or, As Radius	=	90° 00'	10,00000
To cos. $\angle P$,	To cos. diff. long.	=	60 23	9,69390
So tan. PA	So tan. Helena's N. pol. dist.	=	106 00	10,54250
To tan. M.	To tang. fourth arc	=	59 53	10,23640

But by the determination, art. 533, this fourth arc is to be like PA, or obtuse; therefore the supplement of $59^\circ 53'$, or $120^\circ 07'$, is to be the fourth arc: Then the difference between this fourth arc and PB, is $62^\circ 32'$ for N, or the fifth arc. And

As fin. N	Or, As fin. fifth arc	=	62° 32'	0,05194
To fin. M,	To fin. fourth arc	=	120 07	9,93702
So tan. $\angle P$	So tang. diff. long.	=	60 23	10,24530
To tan. $\angle ABP$.	To tang. $\angle$ position	=	59 45	10,23426

But by the determination, art. 533, the $\angle ABP$ is to be obtuse, and is therefore $120^\circ 15'$, which is the supplement of $59^\circ 45'$.

So that was a ship to sail from St. Helena to Bermudas on the arc of a great circle, she must first shape her course N. $49^\circ 20'$ W., and gradually deflect from the north, so as to arrive at Bermudas on a course N. $59^\circ 45'$ W., after having run $75^\circ 19'$, or 4519 sea miles.

The course found by Mercator's sailing is N. $50^\circ 05'$ W., and the distance is 4527 miles.

Whereby it appears, that when the places are one in N. latitude, and the other in S. latitude, there is but a small difference between the results found by Mercator's and circle sailing: For near the equator, the rhumb lines do not greatly differ from great circles.

From the solutions of the foregoing cases it is plain, that to sail on the arc of a great circle, the ship must continually alter her course: But as this is a difficulty too great to be admitted into the practice of Navigation, therefore it has been thought sufficiently exact to effect this business by a kind of approximation; that is, by a method which nearly approaches the sailing on a great circle.

817. The principle upon which this approximation is founded, is, *that in small arcs, the difference between the arc and its chord, or tangent, is so small, that they may be taken one for the other in any nautical operations.*

Upon this principle, the great circles on the earth are supposed to be made up of short right lines, each of which is a segment of a rhumb line.

And on this supposition, the solution to the following Problem is deduced.

PROBLEM II.

818. *Having given the latitudes and longitudes of the places sailed from and bound to : To find the successive latitudes on the arc of a great circle in those places where the alteration in longitude shall be a given quantity ; together with the courses and distances between those places.*

SOLUTION.

I. Find the angle of position at each place, and their distance by one of the preceding six cases.

II. Find the greatest latitude the great circle runs thro' ; that is, find the perpendicular from the pole to that circle ; and also find the several angles at the pole, made by the given alterations of longitude between this perpendicular and the successive meridians come to.

III. With this perpendicular and the polar angles severally find as many corresponding latitudes, by saying (521) :

As Rad : tan. greatest lat. :: cos. 1st polar $\angle$: tan. 1st lat.
 :: cos. 2d polar $\angle$: tan. 2d lat.
 &c. &c.

IV. Having the several latitudes passed thro', and the difference of longitude between each, find by Mercator's sailing (770), the courses and distances between those latitudes.

And these are the several courses and distances the ship must run, to keep nearly on the arc of a great circle.

Now the smaller the alterations in longitude are taken, the nearer will this method approach the truth : But the usual way is to compute to every 5 degrees of difference of longitude ; the length of an arc of 5 degrees differing from its chord or tangent only by 0,0002 ; as may be found from the articles 320, 326, 328.

The examples to the third, fifth, and sixth of the preceding cases are hereafter resolved by this method, whereby the difference between the length of the great circular arc passing thro' the two places, and the sum of the several rhumbs sailed on between those places, may be compared.

819. QUESTION I. *A ship being bound from the Island of St. Maries to Cape Henry, proposes to sail as near the arc of a great circle as she can, by altering her course at every 5 degrees difference of longitude: Required the latitude at each time of altering the course, and also the courses and distances between those several latitudes?*

St. Maries lat. is $37^{\circ} 00' N.$ the long. $22^{\circ} 56' W.$

Cape Henry's lat. is $37^{\circ} 00' N.$ the long. $76^{\circ} 23' W.$

The diff. long. $53^{\circ} 27'$

The figure being described, and the computation made as in art. 813, the distance BA is found to be $42^{\circ} 06'$, and the $\angle A$ or $\angle B = 73^{\circ} 09'$, the angle of position. Plate VIII. Fig. 29.

Now the triangle APB being isosceles, the perpendicular PI falls in the middle of AB; and the latitudes, courses and distances being known in running the half BI, those in the half IA will also be known.

Let the points *a, b, c, d, &c.* be the places come to on each alteration of 5 degrees of longitude: then will the arcs Pa, Pb, Pc, Pd, &c. be the respective co-latitudes of those places, and are the hypotenuses of the right-angled spheric triangles PIa, PIb, PIC, PID, &c.

In the triangle PIB.		As Rad	=	$90^{\circ} 00'$	10,00000
Given PB	= $53^{\circ} 00'$	To sin. PI	=	$53^{\circ} 00'$	9,90235
$\angle PBI$	= $73^{\circ} 09'$	So sin. $\angle B$	=	$73^{\circ} 09'$	9,98094
To find PI.	Art. 513.	To sin. PI	=	$49^{\circ} 51'$	9,88329

Now $\angle IPB = \left(\frac{53^{\circ} 27'}{2}\right) = 26^{\circ} 43\frac{1}{2}'$; $\angle IPa = 21^{\circ} 43\frac{1}{2}'$; $\angle IPb = 16^{\circ} 43\frac{1}{2}'$; $\angle IPC = 11^{\circ} 43\frac{1}{2}'$; $\angle IPd = 6^{\circ} 43\frac{1}{2}'$; are the several polar angles.

	$21^{\circ} 43\frac{1}{2}'$	$16^{\circ} 43\frac{1}{2}'$	$11^{\circ} 43\frac{1}{2}'$	$6^{\circ} 43\frac{1}{2}'$
Then (521) Rad	= $90^{\circ} 00'$	10,00000	10,00000	10,00000
To cot. PI	= $49^{\circ} 51'$	9,92612	9,92612	9,92612
So cos. polar angle		9,96800	9,98123	9,99084
		9,99700		
To tang. lat.		9,89412	9,90735	9,91696
		9,92313		
Which are	$38^{\circ} 05'$	$38^{\circ} 56'$	$39^{\circ} 33'$	$39^{\circ} 57'$

The degrees and min. set over each column, are the polar angles used in that proportion, and the corresponding latitudes stand at bottom.

820. The first term of these proportions being radius, and the second term constant, the operations may be very expeditiously performed thus.

On a slip of paper let the log. of the 2d, or constant term be wrote, of the same size with the printed figures; apply this log. cot. successively to the log. cosines of the polar angles: Then the sum of the two logs. being wrote down each time, will give the log. tangents of the several latitudes arrived at.

By this method, each proportion will be worked by writing down only one line.

Hence

Hence it appears, the ship must first sail from the lat. $37^{\circ} 00' N.$ to lat. $38^{\circ} 05' N.$; thence to lat. $38^{\circ} 56' N.$; thence to lat. $39^{\circ} 33'$; thence to lat. $39^{\circ} 57' N.$; thence to lat. $40^{\circ} 09' N.$, which is the greatest latitude she must go to; and from thence she must proceed thro' the latitudes $39^{\circ} 57'$, $39^{\circ} 33'$, $38^{\circ} 56'$, $38^{\circ} 05'$, and so to $37^{\circ} 00'$, the parallel she set out from, and wherein she is to find Cape Henry.

Now between these several latitudes, with the respective differences of longitude, find by Mercator's sailing, art. 770, the courses and distances.

If the results of the several operations, in the questions of great circle sailing, be entered in such a table as the following, it will be found of some convenience to the operator.

Polar angles	Success longs.	Success. lats.	Diff. long.	Diff. lat.	Merid. parts.	Mer. diff. lat.	Cour.	Dist.
$\angle IPB$ $26^{\circ} 43\frac{1}{2}'$	$22^{\circ} 56'$	$37^{\circ} 00'$			2392,6			
$\angle IPa$ 21	$27 56$	$38 05$	300	65	2474,6	82,0	$74,43$	246,6
$\angle IPb$ 16	$32 56$	$38 56$	300	51	2539,8	65,2	$77,44$	240,0
$\angle IPc$ 11	$37 56$	$39 33$	300	37	2587,6	47,8	$80,57$	235,2
$\angle IPd$ 6	$42 56$	$39 57$	300	24	2618,8	31,2	$84,04$	232,2
	$49 39\frac{1}{2}$	$40 09$	403,5	12	2634,5	15,7	$87,46$	307,9
								1261,0

In the first column are the angles at the pole contained between the perpend. and the several meridians differing by 5° of longitude.

In the 2d col., the departed longitude $22^{\circ} 56'$ being increased by the differences of longitude, make the successive longitudes come to.

In the 3d col. are the successive latitudes passed thro', from St. Maries to the greatest latitude.

In the 4th and 5th columns, are the differences between the longitudes and latitudes in the 2d and 3d columns.

In the 6th column are the merid. parts to the successive latitudes; and in the 7th col. are the merid. diff. latitudes.

The 8th and 9th columns contain the courses and distances between the places answering to the 2d and 3d columns.

The numbers in the 3d, 8th and 9th columns, are found by working of the logarithmic proportions on a waft paper; but the work is here omitted, as it is so easily supplied.

Now the column of distances being summed up, amounts to 1261,0; which being doubled gives 2523,8 miles for the distance between St. Maries and Cape Henry; differing from the distance found (813) in a great circle by 2,2 sea miles.

And the courses the ship must steer are, 1st. N. $74^{\circ} 43' W.$; 2d. N. $77^{\circ} 44' W.$; 3d. N. $80^{\circ} 57' W.$ 4th. N. $84^{\circ} 04' W.$ 5th. N. $87^{\circ} 46' W.$ 6th. S. $87^{\circ} 46' W.$; 7th. S. $84^{\circ} 04' W.$; 8th. S. $80^{\circ} 57' W.$; 9th. S. $77^{\circ} 44' W.$; 10th. S. $74^{\circ} 43' W.$: And on these courses she must run the respective distances standing against them.

821. QUESTION II. *A ship bound from the Lizard to the Island of Bermudas, proposes to sail on a great circle, and to alter her course at every five degrees of longitude: Required the latitudes thro' which the ship is to pass, and also the course and distance between each?*

The Lizard's lat. is $49^{\circ} 57' N.$ Long. $5^{\circ} 14' W.$ } Diff. long. $61^{\circ} 24'.$
 Bermudas lat. is $32^{\circ} 25' N.$ Long. $66^{\circ} 38' W.$ }

Having described the figure, Plate VIII. Fig. 30, and found the $\angle PAB = 49^{\circ} 35'$, the $\angle PBA = 87^{\circ} 15'$, and the side $AB = 47^{\circ} 54'$, as in art. 815.

Draw PI at right angles to ABD . (449)

In the equator, lay off from the centre the tangents of $5, 10, 15, 20$ &c. to 55 degrees; and those points give the centres to the arcs of co-lat. to every 5° diff. longitude. (451)

To find the perpend. PI . (513)				To find the polar angle API . (515)			
As Rad	$= 90^{\circ} 00'$	10,00000		As Rad	$= 90^{\circ} 00'$	10,00000	
To sin. PA	$= 57^{\circ} 35'$	9,92643		To tang. $\angle A$	$= 49^{\circ} 35'$	10,06978	
So sin. $\angle A$	$= 49^{\circ} 35'$	9,88158		So cos. AP	$= 32^{\circ} 25'$	9,72922	
To sin. PI	$= 40^{\circ} 00'$	9,80801		To cot. $\angle API$	$= 57^{\circ} 48'$	9,79900	

Now the polar angle API , or the diff. long. between the perpend. PI and the meridian of Bermudas, $57^{\circ} 48'$, being taken from $61^{\circ} 24'$, the whole diff. long., leaves the diff. long. between the Lizard and I : Also 5° , the proposed alteration of longitude, being subtracted as often as can be from $57^{\circ} 48'$, leaves the several polar angles; with which, and the perpendicular PI , the several latitudes arrived at are found by art. 820.

Then having those latitudes, and the differences of longitude between them, find the successive courses and distances by art. 770.

Let the several results be placed as in the following table; the logarithmic work being performed on a waste paper, is here omitted.

Also, suppose the small letters in the figure supplied.

Polar angles	Success. longs.	Success. lats.	Diff. long.	Diff. lats.	Merid. parts.	Merid. diff. lat.	Courses	Dists.
	$5^{\circ} 14'$	$49^{\circ} 57'$						
$\angle IPB$ $3^{\circ} 36'$	8 50	50 00	216	3	3469,8	4,7	$88^{\circ} 45'$	137,5
$\angle IPa$ 2 48	11 38	49 58	168	2	3474,5	3,1	88 56	107,4
$\angle IPb$ 7 48	16 38	49 44	300	14	3471,4	21,7	85 52	194,2
$\angle IPc$ 12 48	21 38	49 17	300	27	3449,7	41,6	82 06	196,4
$\angle IPd$ 17 48	26 38	48 37	300	40	3408,1	60,9	78 31	200,6
$\angle IPe$ 22 48	31 38	47 42	300	55	3347,2	82,5	74 37	207,3
$\angle IPf$ 27 48	36 38	46 31	300	71	3264,7	104,3	70 50	216,3
$\angle IPg$ 32 48	41 38	45 03	300	88	3160,4	126,2	67 10	226,8
$\angle IPb$ 37 48	46 38	43 17	300	106	3034,2	147,8	63 46	239,8
$\angle IPi$ 42 48	51 38	41 10	300	127	2886,4	171,5	60 15	255,9
$\angle IPk$ 47 48	56 38	38 41	300	149	2714,9	194,4	57 03	273,9
$\angle IPL$ 52 48	61 38	35 46	300	175	2520,5	219,8	53 46	296,1
	66 38	32 25	300	201	2300,7	242,7	51 02	319,6
					2058,0			2871,8

Now the sum of the several distances make 2871,8 miles.

But the distance in the arc of a great circle is 2874 miles. (815)

The difference of the two methods is 2,2 miles.

In this example it was judged necessary to keep the fractional place in the meridional parts, in order to have the distances more accurate.

822. QUESTION III. *A ship being bound from the Island of St. Helena to the Island of Bermudas, it is proposed she should sail near the arc of a great circle passing thro' those places, and to alter her course at every 5° of longitude: Required the latitudes where those courses are altered, and also the course and distance between each place of alteration?*

St. Helena's lat. = 16° 00' S. Long. 6° 15' W. }
 Bermudas's lat = 32 25 N. Long. 66 38 W. } Diff. lon. = 60° 23'.

Having described the figure, Plate VIII. Fig. 31, and found the $\angle PBA = 120^\circ 15'$; $\angle PAB = 49^\circ 20'$; and the dist. $AB = 75^\circ 19'$; as in art. 816.

In the equator QR , lay off from F the tangents of $5^\circ, 10^\circ, 15^\circ$, &c. to 55° ; and those points will give the centres to the meridians passing thro' every 5 degrees of long. between St. Helena and Bermudas. (451)

To find the perpend. PI . (513)				To find the polar $\angle BPI$. (515)			
As Rad	=	90° 00'	10,00000	As Rad	=	90° 00'	10,00000
To sin. PB	=	57 35	9,92643	To cos. BP	=	57 35	9,72922
So sin. $\angle PBI$	=	59 45	9,93643	So tan. $\angle PBI$	=	59 45	10,23419
To sin. PI	=	46 49	9,86286	To cot. $\angle BPI$	=	47 25	9,96341

Now the several polar angles, longitudes, latitudes, diff. longs., diff. lats., mer. parts, mer. diff. lats., courses and distances being computed on a waft paper, their results are to be entered in a table as follows.

Polar angles	Success. longs.	Success. lats.	Diff. long.	Diff. lats.	Merid. parts	Merid. d. lat.	Courses	Dist.
	6° 15'	16° 00' S						
$\angle IP l = 102^\circ 48'$	11 15	11 45	300	255	973	264	48° 39'	386,0
$\angle IP k = 97^\circ 48'$	16 15	7 16	300	269	709	272	47 48	400,5
$\angle IP i = 92^\circ 48'$	21 15	2 38	300	278	437	279	47 05	408,3
$\angle IP h = 87^\circ 48'$	26 15	2 04 N	300	282	158	282	46 46	411,7
$\angle IP g = 82^\circ 48'$	31 15	6 43	300	279	124	280	46 58	408,8
$\angle IP f = 77^\circ 48'$	36 15	11 13	300	270	404	273	47 42	401,2
$\angle IP e = 72^\circ 48'$	41 15	15 31	300	258	677	266	48 26	388,8
$\angle IP d = 67^\circ 48'$	46 15	19 32	300	241	943	252	49 58	374,7
$\angle IP c = 62^\circ 48'$	51 15	23 13	300	221	1195	238	51 34	355,5
$\angle IP b = 57^\circ 48'$	56 15	26 34	300	201	1433	221	53 37	338,9
$\angle IP a = 52^\circ 48'$	61 15	29 34	300	180	1654	204	55 47	320,1
	66 38	32 25	323	171	1858	200	58 14	324,8
					2058			1510,3

823. The track of a ship when directed nearly in the arc of a great circle, as in the preceding articles, may be delineated on the Mercator's chart, by marking thereon, by the help of the latitudes and longitudes, the successive places where the ship is to alter her course; then those places or points being joined by right lines, will shew the path along which the ship is to sail under the proposed circumstances.

Thus in Plate VII is drawn the route proposed in Question II, art. 821, from the Lizard to Bermudas: And notwithstanding that this track on the chart appears much longer than the rhumb passing thro' those two places, yet it must be remembered, that these lines are only the representations of curves on the sphere whose absolute lengths are not hereby expressed.

SCHOLIUM.

824. The reasoning and conclusions, in the preceding sections of this book, have been delivered as if the earth was a perfect sphere: But within the last 80 years it has appeared from many observations that its figure is not strictly so, being rather like a figure altered from a sphere, by being a little flatted at the poles, somewhat of the shape of a flat turnip or bowling-green bowl; or, in the language of mathematicians, *the figure of the earth is an oblate spheroid, whose axis or polar diameter is shorter than the equatorial diameter.* The proportion of these diameters according to Sir Isaac Newton, who first discovered it, is as 229 to 230; or, according to some later observations, as 230 to 231. Now was the difference between these diameters more considerable, it would greatly affect all nautical conclusions deduced from a sphere, and the present tables of meridional parts would be very imperfect: But by the little difference of the diameters the error is scarce sensible in several days works, even in the highest latitudes where any considerable navigation is carried on, and where the error would be greatest. However, in order to put it in the power of those who are inclined to adapt their operations to the true figure of the earth, it will here be shewn how the tables of spheric meridional parts may be fitted to the spheroid.

The reader is desired to excuse the omission of the computations upon which the correcting of the meridional parts is founded; because they depend on certain elements which were thought improper to be introduced into a work of this kind: But those who are qualified to understand such computations, may find them in several books, particularly in Mac-laurin's fluxions, art. 896, and in Simpson's fluxions, art. 465. These gentlemen have in a very elegant manner deduced the same general rules for finding the meridional parts in a spheroid; but Mr. Simpson having, from some farther considerations, fitted his rules more conveniently for practice, his corrections will herein be used.

825. *To find the correction for changing a table of spheric meridional parts to spheroidal meridional parts.*

RULE.

Say, As Radius is to the sine of any given latitude,
So is the constant number 30 to the correction sought.

H h h 2

Subtract

Subtract the correction from the spheric meridional parts to the latitude used, and the remainder is the spheroidal meridional parts to that latitude.

EXAMPLE. *Required the spheroidal meridional parts for the latitude of 60 degrees?*

As Radius	10,00000	From spheric mer. parts to 60° 00'	4527,4
To f. lat. 60° 00'	9,93733	Take the correction	26,
So number 30	1,47712		
		Leaves spheroidal mer. pts. to 60° 00'	4501,4
To cor. 25,98	1,41465		

Or, the correction may be readily found by the Traverse table.

With the latitude 60° among the courses, and against 30 in the column of distances, take in the column of departure the correction 26.

826. *To find the correction, for reducing any given parallel of latitude on the sphere to that of the spheroid.*

RULE.

Say, As Radius is to the sine of double the given latitude,
So is the constant number 11,2 to the correction sought;
which subtracted from the minutes in the given latitude, reduces that parallel from the sphere to the spheroid.

Or, *By the Traverse table.*

Among the courses, seek the double of the given latitude when under 45°, or the double of the co-latitude when above 45°, and against 11 in the col. of distances take the corresponding depart., which increased by a fifth part of the diff. between that and the next greater will be the correction sought.

EXAM. I. *Find the distance between the equator and the parallel of 20° on the spheroid.*

Under 40° (the double of 20°) and against 11 in the col. of dist. stands 7,1 in the col. of dep.; the difference between it and the next greater, 7,7, is 0,6; a fifth part, or 0,1, added to 7,1 gives 7,2, the correction sought.

Now $20^\circ \times 60 = 1200$ min. Then $1200 - 7 = 1193$, the miles in the first 20 degrees of latitude on the spheroid.

EXAM. II. *What is the difference of latitude between the parallels of 60° and 40° of latitude on the spheroid?*

The complement of 60° is 30°, which doubled is 60°; and twice 40° is 80°.

Now 60 of course and 11 of dist. gives 9,5, which increased by $\frac{1}{5}$ of 0,9, the excess of the next greater, gives 9,7, or 10 m. for the correction to 60°.

And 80° of course with 11,2 of dist., gives 11 miles of corr. to 40°.

Then $60^\circ \times 60 = 3600$; and $3600 - 10 = 3590$ miles for the distance of the parallel of 60° of latitude from the equator.

Also $40^\circ \times 60 = 2400$; and $2400 - 11 = 2389$ miles for the distance of the parallel of 40° of latitude from the equator.

Then

Then $3590 - 2389 = 1201$ miles for the diff. lat. sought, which is 1 mile more than would be on the sphere.

The equation producing this latter correction was communicated both by Mr. Simpson, and Mr. Clairaut, when in London this last winter.

In order to compare the difference between the results from the sphere and the spheroid, the following example is here annexed.

EXAM. Required the course and distance from the Lizard to the Island of Barbadoes?

	Longit.	Lat.	Cor.	Com. m. p.	Cor.	Cor. m. p.
Lizard	5° 14' W.	49° 57' N. = 2997 m.	11	3469,8	23,0	3446,8
Barbados	59 50 W.	13 00 N. = 780 m.	5	786,8	6,7	780,1
Diff. lon.	54 36 60	d. lat. 36 57 = 2986 m. 60 775		2683,0 Mer. d. lat.		2666,7 Cor. m. d. l.
	3276 m.	1217 m.				1211 m.

The latitudes are corrected by art. 826, and the meridional parts by art. 825. And $2997 - 11 = 2986$. Also $780 - 5 = 775$.

To find the Course

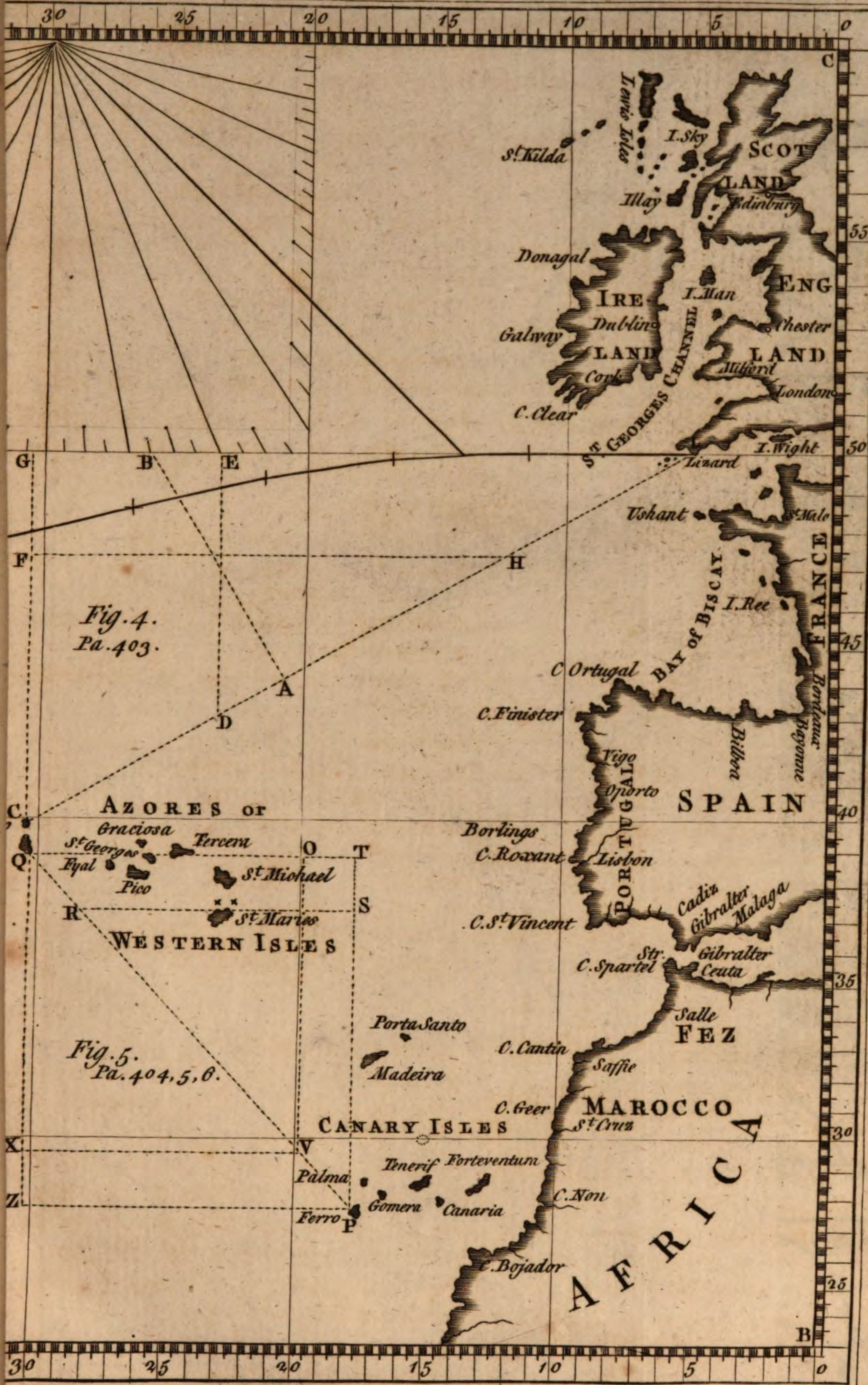
In the sphere.			In the spheroid.		
As mer. diff. lat.	=	2683 6,57138	Or =	2667	6,57398
To diff. long.	=	3276 3,51534	=	3276	3,51534
So Radius	=	Rad 10,00000	=	Rad	10,00000
To tangent Course	=	50° 41' 10,08672	=	50° 51' 10,08932	

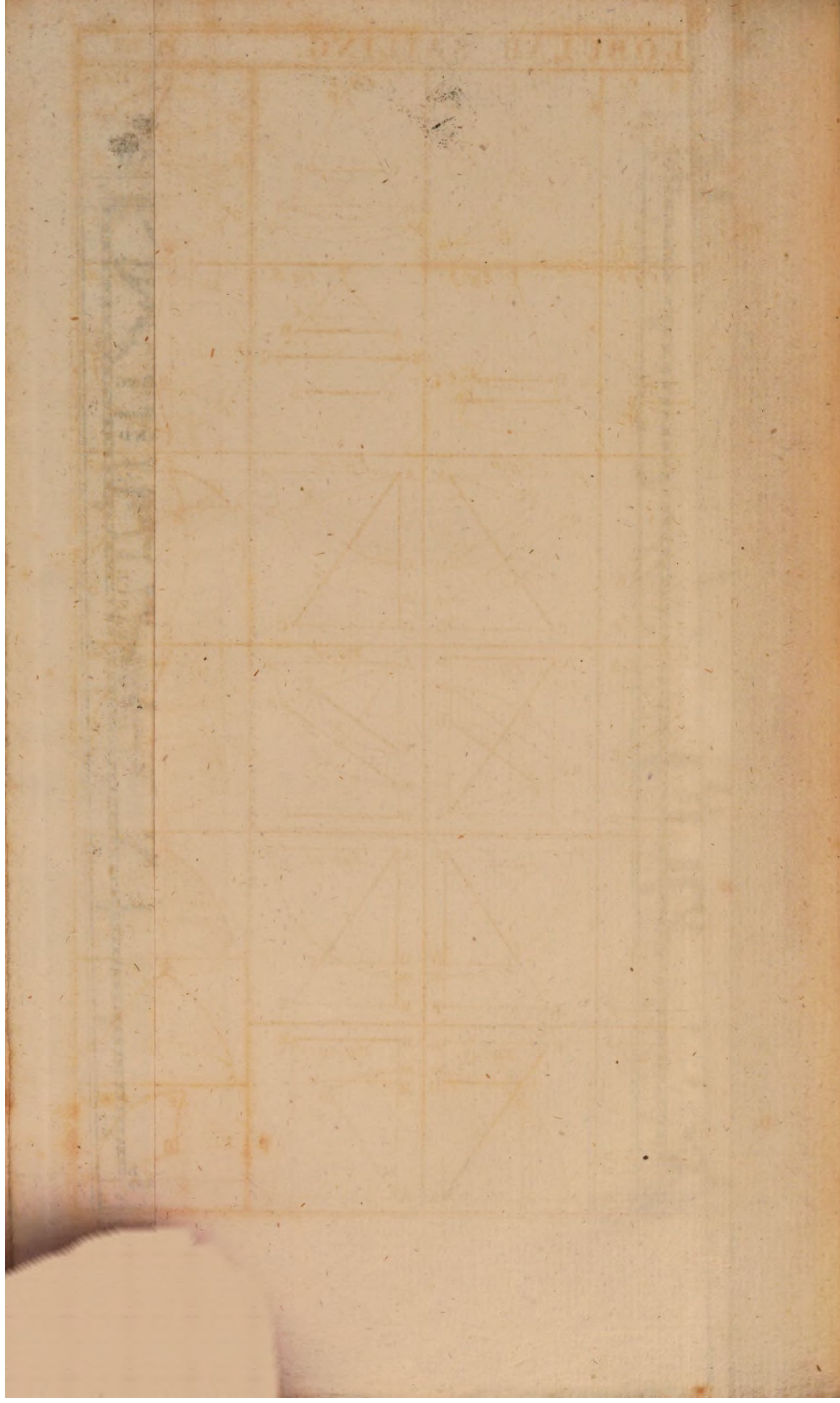
To find the Distance.

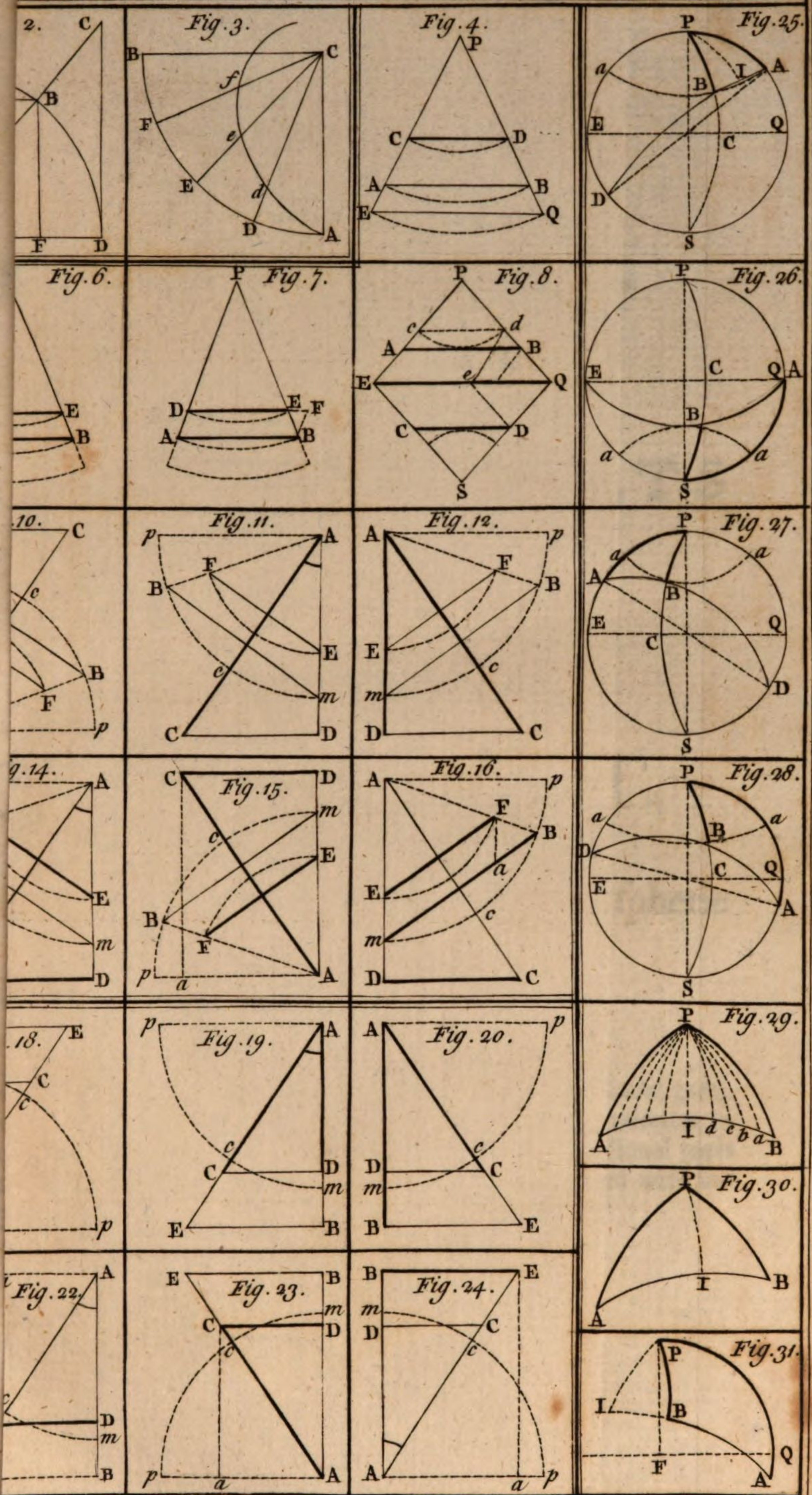
As cos. Course	=	50° 41' 0,19818	Or =	50° 51' 0,19973
To diff. lat.	=	2217 3,34577	=	2211 3,34459
So Radius	=	Rad 10,00000	=	Rad 10,00000
To Distance	=	3499 3,54395	=	3502 3,54432

Whence it appears, that the difference between the spheric and spheroidal operations amounts to 10 minutes in the course, and to 3 miles in the distance.

The foregoing corrections were made by using Sir Isaac Newton's numbers for the two diameters of the spheroid; and altho' some authors (on the supposition that the difference between the equatorial and polar diameters was greater than assigned by the illustrious Newton) have given much greater corrections, founded on the mensurations in *France* and *Lapland*; yet as those mensurations, at the time the chief of those authors wrote, had not been compared with the mensurations made in *Peru*, therefore the diameters they had chosen were defective of the truth, and consequently their corrections less accurate than could be wished: For the more this intricate problem concerning the figure of the earth has been considered, the more inclined are the mathematicians to use the diameters assigned by Newton,









STANDARD

OF THE

QUADRANT

OF THE

OF THE

OF THE

OF THE

827. A T A B L E
O F
M E R I D I O N A L P A R T S
To every DEGREE and MINUTE
O F T H E
Q U A D R A N T.

Calculated on the supposition of the spheric
figure of the EARTH.

Where D. l. stands for deg. of lat. and M. P. for meridional parts.
Seek the degrees of latitude at the top or bottom, and the minutes in
the right or left hand columns, and the corresponding meridional parts
will stand right against the minutes, and in the column signed with the
degree proposed.

D. 1.	0	1	2	3	4	5	6	7	8	9	D. 1.
min.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	min.
0	0,0	60,0	120,0	180,1	240,2	300,4	360,7	421,1	481,6	542,2	0
1	1,0	61,0	121,0	181,1	241,2	301,4	361,7	422,1	482,6	543,3	1
2	2,0	62,0	122,0	182,1	242,2	302,4	362,7	423,1	483,6	544,3	2
3	3,0	63,0	123,0	183,1	243,2	303,4	363,7	424,1	484,6	545,3	3
4	4,0	64,0	124,0	184,1	244,2	304,4	364,7	425,1	485,6	546,3	4
5	5,0	65,0	125,0	185,1	245,2	305,4	365,7	426,1	486,6	547,3	5
6	6,0	66,0	126,0	186,1	246,2	306,4	366,7	427,1	487,6	548,3	6
7	7,0	67,0	127,0	187,1	247,2	307,4	367,7	428,1	488,6	549,3	7
8	8,0	68,0	128,0	188,1	248,2	308,4	368,7	429,1	489,6	550,3	8
9	9,0	69,0	129,0	189,1	249,2	309,4	369,7	430,1	490,7	551,4	9
10	10,0	70,0	130,0	190,1	250,2	310,4	370,7	431,1	491,7	552,4	10
11	11,0	71,0	131,0	191,1	251,2	311,4	371,7	432,1	492,7	553,4	11
12	12,0	72,0	132,0	192,1	252,2	312,4	372,7	433,1	493,7	554,4	12
13	13,0	73,0	133,0	193,1	253,2	313,4	373,7	434,2	494,7	555,4	13
14	14,0	74,0	134,0	194,1	254,2	314,4	374,7	435,2	495,7	556,4	14
15	15,0	75,0	135,0	195,1	255,2	315,4	375,7	436,2	496,7	557,4	15
16	16,0	76,0	136,0	196,1	256,2	316,4	376,8	437,2	497,7	558,4	16
17	17,0	77,0	137,0	197,1	257,2	317,5	377,8	438,2	498,7	559,4	17
18	18,0	78,0	138,0	198,1	258,2	318,5	378,8	439,2	499,8	560,5	18
19	19,0	79,0	139,0	199,1	259,2	319,5	379,8	440,2	500,8	561,5	19
20	20,0	80,0	140,0	200,1	260,2	320,5	380,8	441,2	501,8	562,5	20
21	21,0	81,0	141,0	201,1	261,3	321,5	381,8	442,2	502,8	563,5	21
22	22,0	82,0	142,0	202,1	262,3	322,5	382,8	443,2	503,8	564,5	22
23	23,0	83,0	143,0	203,1	263,3	323,5	383,8	444,2	504,8	565,5	23
24	24,0	84,0	144,0	204,1	264,3	324,5	384,8	445,2	505,8	566,6	24
25	25,0	85,0	145,0	205,1	265,3	325,5	385,8	446,3	506,8	567,6	25
26	26,0	86,0	146,0	206,1	266,3	326,5	386,8	447,3	507,8	568,6	26
27	27,0	87,0	147,0	207,1	267,3	327,5	387,8	448,3	508,9	569,6	27
28	28,0	88,0	148,0	208,1	268,3	328,5	388,8	449,3	509,9	570,6	28
29	29,0	89,0	149,0	209,1	269,3	329,5	389,8	450,3	510,9	571,6	29
30	30,0	90,0	150,0	210,1	270,3	330,5	390,8	451,3	511,9	572,6	30
31	31,0	91,0	151,0	211,1	271,3	331,5	391,9	452,3	512,9	573,7	31
32	32,0	92,0	152,0	212,1	272,3	332,5	392,9	453,3	513,9	574,7	32
33	33,0	93,0	153,1	213,1	273,3	333,5	393,9	454,3	514,9	575,7	33
34	34,0	94,0	154,1	214,1	274,3	334,5	394,9	455,3	515,9	576,7	34
35	35,0	95,0	155,1	215,1	275,3	335,5	395,9	456,3	516,9	577,7	35
36	36,0	96,0	156,1	216,1	276,3	336,5	396,9	457,3	518,0	578,7	36
37	37,0	97,0	157,1	217,1	277,3	337,5	397,9	458,4	519,0	579,7	37
38	38,0	98,0	158,1	218,1	278,3	338,6	398,9	459,4	520,0	580,8	38
39	39,0	99,0	159,1	219,1	279,3	339,6	399,9	460,4	521,0	581,8	39
40	40,0	100,0	160,1	220,2	280,3	340,6	400,9	461,4	522,0	582,8	40
41	41,0	101,0	161,1	221,2	281,3	341,6	401,9	462,4	523,0	583,8	41
42	42,0	102,0	162,1	222,2	282,3	342,6	402,9	463,4	524,0	584,8	42
43	43,0	103,0	163,1	223,2	283,3	343,6	403,9	464,4	525,0	585,8	43
44	44,0	104,0	164,1	224,2	284,3	344,6	404,9	465,4	526,0	586,8	44
45	45,0	105,0	165,1	225,2	285,3	345,6	405,9	466,4	527,1	587,9	45
46	46,0	106,0	166,1	226,2	286,3	346,6	407,0	467,4	528,1	588,9	46
47	47,0	107,0	167,1	227,2	287,3	347,6	408,0	468,4	529,1	589,9	47
48	48,0	108,0	168,1	228,2	288,3	348,6	409,0	469,5	530,1	590,9	48
49	49,0	109,0	169,1	229,2	289,3	349,6	410,0	470,5	531,1	591,9	49
50	50,0	110,0	170,1	230,2	290,3	350,6	411,0	471,5	532,1	592,9	50
51	51,0	111,0	171,1	231,2	291,3	351,6	412,0	472,5	533,1	593,9	51
52	52,0	112,0	172,1	232,2	292,4	352,6	413,0	473,5	534,1	595,0	52
53	53,0	113,0	173,1	233,2	293,4	353,6	414,0	474,5	535,1	596,0	53
54	54,0	114,0	174,1	234,2	294,4	354,6	415,0	475,5	536,2	597,0	54
55	55,0	115,0	175,1	235,2	295,4	355,6	416,0	476,5	537,2	598,0	55
56	56,0	116,0	176,1	236,2	296,4	356,6	417,0	477,5	538,2	599,0	56
57	57,0	117,0	177,1	237,2	297,4	357,6	418,0	478,6	539,2	600,0	57
58	58,0	118,0	178,1	238,2	298,4	358,6	419,0	479,5	540,2	601,0	58
59	59,0	119,0	179,1	239,2	299,4	359,6	420,0	480,5	541,2	602,1	59
min.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	min.
D. 1.	0	1	2	3	4	5	6	7	8	9	D. 1.

D. l.	10	11	12	13	14	15	16	17	18	19	D. l.
min.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	min.
0	603,1	664,1	725,3	786,8	848,5	910,5	972,8	1035,3	1098,2	1161,5	0
1	604,1	665,1	726,4	787,9	849,6	911,5	973,8	1036,3	1099,3	1162,5	1
2	605,1	666,1	727,4	788,9	850,6	912,6	974,8	1037,4	1100,3	1163,6	2
3	606,1	667,1	728,4	789,9	851,6	913,6	975,9	1038,4	1101,4	1164,7	3
4	607,1	668,1	729,4	790,9	852,7	914,6	976,9	1039,5	1102,4	1165,7	4
5	608,2	669,2	730,5	792,0	853,7	915,7	978,0	1040,5	1103,5	1166,8	5
6	609,2	670,2	731,5	793,0	854,7	916,7	979,0	1041,6	1104,5	1167,8	6
7	610,2	671,2	732,5	794,0	855,8	917,7	980,0	1042,6	1105,6	1168,9	7
8	611,2	672,2	733,5	795,0	856,8	918,8	981,1	1043,7	1106,6	1170,0	8
9	612,2	673,2	734,6	796,1	857,8	919,8	982,1	1044,7	1107,7	1171,0	9
10	613,2	674,3	735,6	797,1	858,9	920,8	983,2	1045,8	1108,7	1172,1	10
11	614,2	675,3	736,6	798,1	859,9	921,9	984,2	1046,8	1109,8	1173,1	11
12	615,3	676,3	737,6	799,1	861,0	922,9	985,2	1047,9	1110,8	1174,2	12
13	616,3	677,3	738,7	800,2	862,0	923,9	986,3	1048,9	1111,9	1175,2	13
14	617,3	678,3	739,7	801,2	863,0	925,0	987,3	1049,9	1112,9	1176,3	14
15	618,3	679,4	740,7	802,2	864,1	926,0	988,4	1051,0	1114,0	1177,4	15
16	619,3	680,4	741,7	803,2	865,1	927,0	989,4	1052,0	1115,0	1178,4	16
17	620,3	681,4	742,8	804,3	866,1	928,1	990,4	1053,1	1116,1	1179,5	17
18	621,3	682,4	743,8	805,3	867,2	929,1	991,5	1054,1	1117,1	1180,5	18
19	622,4	683,4	744,8	806,3	868,2	930,1	992,5	1055,2	1118,2	1181,6	19
20	623,4	684,5	745,8	807,3	869,2	931,2	993,6	1056,2	1119,2	1182,7	20
21	624,4	685,5	746,9	808,4	870,3	932,2	994,6	1057,3	1120,3	1183,7	21
22	625,4	686,5	747,9	809,4	871,3	933,2	995,6	1058,3	1121,3	1184,8	22
23	626,4	687,5	748,9	810,4	872,3	934,3	996,7	1059,4	1122,4	1185,8	23
24	627,4	688,5	749,9	811,4	873,4	935,3	997,7	1060,4	1123,4	1186,9	24
25	628,5	689,6	751,0	812,5	874,4	936,3	998,8	1061,4	1124,5	1188,0	25
26	629,5	690,6	752,0	813,5	875,4	937,4	999,8	1062,5	1125,5	1189,0	26
27	630,5	691,6	753,0	814,5	876,5	938,4	1000,8	1063,5	1126,6	1190,1	27
28	631,5	692,6	754,0	815,5	877,5	939,4	1001,9	1064,6	1127,6	1191,1	28
29	632,5	693,6	755,1	816,6	878,5	940,5	1002,9	1065,6	1128,7	1192,2	29
30	633,5	694,7	756,1	817,6	879,6	941,5	1004,0	1066,7	1129,7	1193,2	30
31	634,6	695,7	757,1	818,6	880,6	942,5	1005,0	1067,7	1130,8	1194,3	31
32	635,6	696,7	758,1	819,6	881,6	943,6	1006,1	1068,8	1131,8	1195,4	32
33	636,6	697,7	759,2	820,7	882,7	944,6	1007,1	1069,8	1132,9	1196,4	33
34	637,6	698,7	760,2	821,7	883,7	945,6	1008,1	1070,9	1134,0	1197,5	34
35	638,6	699,8	761,2	822,7	884,7	946,7	1009,2	1072,0	1135,1	1198,5	35
36	639,6	700,8	762,2	823,7	885,8	947,7	1010,2	1073,0	1136,1	1199,6	36
37	640,7	701,8	763,3	824,8	886,8	948,7	1011,3	1074,1	1137,2	1200,7	37
38	641,7	702,8	764,3	825,8	887,8	949,8	1012,3	1075,1	1138,2	1201,7	38
39	642,7	703,8	765,3	826,8	888,9	950,8	1013,4	1076,2	1139,3	1202,8	39
40	643,7	704,9	766,3	827,9	889,9	951,9	1014,4	1077,2	1140,3	1203,9	40
41	644,7	705,9	767,4	828,9	890,9	952,9	1015,4	1078,3	1141,4	1204,9	41
42	645,7	706,9	768,4	829,9	892,0	953,9	1016,5	1079,3	1142,4	1206,0	42
43	646,8	707,9	769,4	831,0	893,0	955,0	1017,5	1080,4	1143,5	1207,1	43
44	647,8	708,9	770,4	832,0	894,0	956,0	1018,6	1081,4	1144,6	1208,1	44
45	648,8	710,0	771,5	833,0	895,1	957,1	1019,6	1082,5	1145,6	1209,2	45
46	649,8	711,0	772,5	834,1	896,1	958,1	1020,6	1083,5	1146,7	1210,2	46
47	650,8	712,0	773,5	835,1	897,1	959,2	1021,7	1084,6	1147,7	1211,3	47
48	651,8	713,0	774,5	836,1	898,2	960,2	1022,7	1085,6	1148,8	1212,4	48
49	652,9	714,1	775,6	837,2	899,2	961,3	1023,8	1086,7	1149,8	1213,4	49
50	653,9	715,1	776,6	838,2	900,2	962,3	1024,8	1087,7	1150,9	1214,5	50
51	654,9	716,1	777,6	839,2	901,2	963,4	1025,9	1088,8	1152,0	1215,5	51
52	655,9	717,1	778,6	840,3	902,3	964,4	1026,9	1089,8	1153,0	1216,6	52
53	656,9	718,2	779,7	841,3	903,3	965,5	1028,0	1090,9	1154,1	1217,7	53
54	657,9	719,2	780,7	842,3	904,3	966,5	1029,0	1091,9	1155,1	1218,7	54
55	659,0	720,2	781,7	843,4	905,4	967,6	1030,1	1093,0	1156,2	1219,8	55
56	660,0	721,2	782,7	844,4	906,4	968,6	1031,1	1094,0	1157,2	1220,9	56
57	661,0	722,3	783,8	845,4	907,4	969,6	1032,2	1095,1	1158,3	1221,9	57
58	662,0	723,3	784,8	846,5	908,4	970,7	1033,2	1096,1	1159,4	1223,0	58
59	663,0	724,3	785,8	847,5	909,5	971,7	1034,3	1097,2	1160,4	1224,1	59
min.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	min.
D. l.	10	11	12	13	14	15	16	17	18	19	D. l.

D. l.	20	21	22	23	24	25	26	27	28	29	D. l.
min.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	min.
0	1225,1	1289,2	1353,7	1418,7	1484,1	1550,0	1616,5	1683,6	1751,2	1819,5	0
1	1226,2	1290,2	1354,8	1419,7	1485,2	1551,1	1617,6	1684,7	1752,3	1820,6	1
2	1227,3	1291,3	1355,8	1420,8	1486,3	1552,2	1618,7	1685,8	1753,4	1821,7	2
3	1228,3	1292,4	1356,9	1421,9	1487,3	1553,3	1619,8	1686,9	1754,6	1822,9	3
4	1229,4	1293,5	1358,0	1423,0	1488,4	1554,4	1620,9	1688,0	1755,7	1824,0	4
5	1230,4	1294,5	1359,0	1424,1	1489,5	1555,5	1622,0	1689,1	1756,8	1825,2	5
6	1231,5	1295,6	1360,7	1425,1	1490,6	1556,6	1623,2	1690,3	1758,0	1826,3	6
7	1232,6	1296,7	1361,2	1426,2	1491,7	1557,7	1624,3	1691,4	1759,1	1827,5	7
8	1233,6	1297,8	1362,3	1427,3	1492,8	1558,8	1625,4	1692,5	1760,2	1828,6	8
9	1234,7	1298,8	1363,3	1428,4	1493,9	1559,9	1626,5	1693,6	1761,4	1829,7	9
10	1235,8	1299,9	1364,4	1429,5	1495,0	1561,0	1627,6	1694,8	1762,5	1830,9	10
11	1236,8	1301,0	1365,5	1430,6	1496,1	1562,1	1628,7	1695,9	1763,6	1832,0	11
12	1237,9	1302,0	1366,6	1431,7	1497,2	1563,2	1629,8	1697,0	1764,8	1833,2	12
13	1239,0	1303,1	1367,6	1432,8	1498,3	1564,3	1631,0	1698,1	1765,9	1834,3	13
14	1240,0	1304,2	1368,7	1433,9	1499,4	1565,4	1632,1	1699,3	1767,0	1835,5	14
15	1241,1	1305,3	1369,8	1434,9	1500,5	1566,5	1633,2	1700,4	1768,2	1836,6	15
16	1242,2	1306,3	1370,9	1436,0	1501,6	1567,6	1634,3	1701,5	1769,3	1837,8	16
17	1243,2	1307,4	1372,0	1437,1	1502,7	1568,7	1635,4	1702,6	1770,5	1838,9	17
18	1244,3	1308,5	1373,1	1438,2	1503,8	1569,8	1636,5	1703,8	1771,6	1840,1	18
19	1245,4	1309,6	1374,2	1439,3	1504,9	1571,0	1637,7	1704,9	1772,7	1841,2	19
20	1246,4	1310,6	1375,3	1440,4	1506,0	1572,1	1638,8	1706,0	1773,9	1842,4	20
21	1247,5	1311,7	1376,4	1441,5	1507,1	1573,2	1639,9	1707,1	1775,0	1843,5	21
22	1248,6	1312,8	1377,4	1442,6	1508,2	1574,3	1641,0	1708,3	1776,1	1844,6	22
23	1249,6	1313,8	1378,5	1443,7	1509,3	1575,4	1642,1	1709,4	1777,2	1845,8	23
24	1250,7	1314,9	1379,6	1444,8	1510,4	1576,5	1643,2	1710,5	1778,4	1846,9	24
25	1251,8	1316,0	1380,7	1445,8	1511,5	1577,6	1644,3	1711,6	1779,5	1848,1	25
26	1252,8	1317,1	1381,8	1446,9	1512,6	1578,7	1645,5	1712,8	1780,6	1849,2	26
27	1253,9	1318,1	1382,8	1448,0	1513,7	1579,8	1646,6	1713,9	1781,8	1850,4	27
28	1255,0	1319,2	1383,9	1449,1	1514,8	1580,9	1647,7	1715,0	1783,0	1851,5	28
29	1256,0	1320,3	1385,0	1450,2	1515,9	1582,0	1648,8	1716,1	1784,1	1852,7	29
30	1257,1	1321,4	1386,1	1451,3	1517,0	1583,2	1649,9	1717,3	1785,2	1853,8	30
31	1258,2	1322,4	1387,2	1452,4	1518,1	1584,3	1651,0	1718,4	1786,4	1855,0	31
32	1259,2	1323,5	1388,3	1453,5	1519,2	1585,4	1652,2	1719,5	1787,5	1856,1	32
33	1260,3	1324,6	1389,4	1454,6	1520,3	1586,5	1653,3	1720,7	1788,6	1857,2	33
34	1261,4	1325,7	1390,4	1455,7	1521,4	1587,6	1654,4	1721,8	1789,8	1858,4	34
35	1262,4	1326,7	1391,5	1456,8	1522,5	1588,7	1655,5	1722,9	1790,9	1859,6	35
36	1263,5	1327,8	1392,6	1457,9	1523,6	1589,8	1656,6	1724,0	1792,1	1860,7	36
37	1264,6	1328,9	1393,7	1458,9	1524,7	1590,9	1657,8	1725,2	1793,2	1861,9	37
38	1265,6	1330,0	1394,8	1460,0	1525,8	1592,0	1658,9	1726,3	1794,3	1863,0	38
39	1266,7	1331,0	1395,8	1461,1	1526,9	1593,2	1660,0	1727,4	1795,5	1864,2	39
40	1267,8	1332,1	1396,9	1462,2	1528,0	1594,3	1661,1	1728,6	1796,6	1865,3	40
41	1268,8	1333,2	1398,0	1463,3	1529,1	1595,4	1662,2	1729,7	1797,8	1866,5	41
42	1269,9	1334,2	1399,1	1464,4	1530,2	1596,5	1663,4	1730,8	1798,9	1867,6	42
43	1271,0	1335,3	1400,2	1465,5	1531,3	1597,6	1664,5	1731,9	1800,0	1868,8	43
44	1272,1	1336,4	1401,3	1466,6	1532,4	1598,7	1665,6	1733,1	1801,2	1869,9	44
45	1273,1	1337,5	1402,4	1467,7	1533,5	1599,8	1666,7	1734,2	1802,3	1871,1	45
46	1274,2	1338,6	1403,4	1468,8	1534,6	1600,9	1667,8	1735,3	1803,5	1872,2	46
47	1275,3	1339,7	1404,5	1469,8	1535,7	1602,0	1669,0	1736,5	1804,6	1873,4	47
48	1276,3	1340,7	1405,6	1470,9	1536,8	1603,1	1670,1	1737,6	1805,7	1874,5	48
49	1277,4	1341,8	1406,7	1472,0	1537,9	1604,3	1671,2	1738,7	1806,9	1875,7	49
50	1278,5	1342,9	1407,8	1473,1	1539,0	1605,4	1672,3	1739,9	1808,0	1876,8	50
51	1279,5	1344,0	1408,8	1474,2	1540,1	1606,5	1673,4	1741,0	1809,2	1878,0	51
52	1280,6	1345,0	1409,9	1475,3	1541,2	1607,6	1674,6	1742,1	1810,3	1879,2	52
53	1281,7	1346,1	1411,0	1476,4	1542,3	1608,7	1675,7	1743,2	1811,4	1880,3	53
54	1282,7	1347,2	1412,1	1477,5	1543,4	1609,8	1676,9	1744,4	1812,6	1881,5	54
55	1283,8	1348,3	1413,2	1478,6	1544,5	1610,9	1678,0	1745,5	1813,7	1882,6	55
56	1284,9	1349,4	1414,3	1479,7	1545,6	1612,0	1679,1	1746,6	1814,9	1883,8	56
57	1286,0	1350,4	1415,4	1480,8	1546,7	1613,1	1680,2	1747,8	1816,0	1884,9	57
58	1287,0	1351,5	1416,5	1481,9	1547,8	1614,2	1681,3	1748,9	1817,2	1886,1	58
59	1288,1	1352,6	1417,6	1483,0	1548,9	1615,4	1682,4	1750,0	1818,3	1887,2	59
min.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	min.
D. l.	20	21	22	23	24	25	26	27	28	29	D. l.

D. l.	30	31	32	33	34	35	36	37	38	39	D. l.
min.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	min.
0	1888,4	1958,0	2028,4	2099,6	2171,5	2244,3	2318,0	2392,7	2468,3	2545,0	0
1	1889,5	1959,2	2029,6	2100,7	2172,7	2245,5	2319,3	2393,9	2469,6	2546,2	1
2	1890,7	1960,4	2030,7	2101,9	2173,9	2246,8	2320,5	2395,2	2470,8	2547,5	2
3	1891,9	1961,6	2031,9	2103,1	2175,1	2248,0	2321,7	2396,4	2472,1	2548,8	3
4	1893,0	1962,7	2033,1	2104,3	2176,3	2249,2	2323,0	2397,7	2473,4	2550,1	4
5	1894,1	1963,9	2034,3	2105,5	2177,5	2250,4	2324,2	2398,9	2474,6	2551,4	5
6	1895,3	1965,0	2035,5	2106,7	2178,7	2251,6	2325,4	2400,2	2475,9	2552,7	6
7	1896,5	1966,2	2036,7	2107,9	2180,0	2252,9	2326,7	2401,4	2477,1	2554,0	7
8	1897,6	1967,4	2037,8	2109,1	2181,2	2254,1	2327,9	2402,7	2478,5	2555,3	8
9	1898,8	1968,5	2039,0	2110,3	2182,4	2255,3	2329,2	2403,9	2479,7	2556,6	9
10	1899,9	1969,7	2040,2	2111,5	2183,6	2256,5	2330,4	2405,2	2481,0	2557,8	10
11	1901,1	1970,9	2041,4	2112,7	2184,8	2257,8	2331,6	2406,4	2482,3	2559,1	11
12	1902,3	1972,0	2042,6	2113,9	2186,0	2259,0	2332,9	2407,7	2483,5	2560,4	12
13	1903,4	1973,2	2043,8	2115,1	2187,2	2260,2	2334,1	2409,0	2484,8	2561,7	13
14	1904,6	1974,4	2044,9	2116,3	2188,4	2261,4	2335,3	2410,2	2486,1	2563,0	14
15	1905,7	1975,6	2046,1	2117,5	2189,6	2262,7	2336,6	2411,5	2487,4	2564,3	15
16	1906,9	1976,8	2047,3	2118,7	2190,8	2263,9	2337,8	2412,7	2488,6	2565,6	16
17	1908,1	1977,9	2048,5	2119,8	2192,0	2265,1	2339,0	2414,0	2489,9	2566,9	17
18	1909,2	1979,1	2049,7	2121,0	2193,3	2266,3	2340,3	2415,2	2491,2	2568,2	18
19	1910,4	1980,3	2050,8	2122,2	2194,5	2267,6	2341,5	2416,5	2492,5	2569,5	19
20	1911,5	1981,4	2052,0	2123,4	2195,7	2268,8	2342,8	2417,8	2493,7	2570,7	20
21	1912,7	1982,6	2053,2	2124,6	2196,9	2270,0	2344,0	2419,0	2495,0	2572,0	21
22	1913,8	1983,7	2054,4	2125,8	2198,1	2271,2	2345,3	2420,3	2496,3	2573,3	22
23	1915,0	1984,9	2055,6	2127,0	2199,3	2272,5	2346,5	2421,5	2497,6	2574,6	23
24	1916,2	1986,1	2056,8	2128,2	2200,5	2273,7	2347,8	2422,8	2498,8	2575,9	24
25	1917,3	1987,3	2058,0	2129,4	2201,7	2274,9	2349,0	2424,0	2500,1	2577,2	25
26	1918,5	1988,4	2059,1	2130,6	2203,0	2276,1	2350,2	2425,3	2501,4	2578,5	26
27	1919,6	1989,6	2060,3	2131,8	2204,2	2277,4	2351,5	2426,5	2502,7	2579,8	27
28	1920,8	1990,8	2061,5	2133,0	2205,4	2278,6	2352,7	2427,8	2503,9	2581,1	28
29	1921,9	1992,0	2062,7	2134,2	2206,6	2279,8	2354,0	2429,1	2505,2	2582,4	29
30	1923,1	1993,1	2063,9	2135,4	2207,8	2281,0	2355,2	2430,3	2506,5	2583,7	30
31	1924,3	1994,3	2065,1	2136,6	2209,0	2282,3	2356,5	2431,6	2507,8	2585,0	31
32	1925,4	1995,5	2066,2	2137,8	2210,2	2283,5	2357,7	2432,9	2509,0	2586,3	32
33	1926,6	1996,6	2067,4	2139,0	2211,4	2284,7	2358,9	2434,1	2510,3	2587,6	33
34	1927,8	1997,8	2068,6	2140,2	2212,7	2286,0	2360,2	2435,4	2511,6	2588,9	34
35	1928,9	1999,0	2069,8	2141,4	2213,9	2287,2	2361,4	2436,7	2512,9	2590,2	35
36	1930,1	2000,2	2071,1	2142,6	2215,1	2288,4	2362,7	2437,9	2514,2	2591,5	36
37	1931,3	2001,3	2072,2	2143,8	2216,3	2289,7	2363,9	2439,2	2515,4	2592,8	37
38	1932,4	2002,5	2073,4	2145,0	2217,5	2290,9	2365,2	2440,4	2516,7	2594,1	38
39	1933,6	2003,7	2074,6	2146,2	2218,7	2292,1	2366,4	2441,7	2518,0	2595,4	39
40	1934,7	2004,9	2075,7	2147,4	2219,9	2293,3	2367,7	2443,0	2519,3	2596,7	40
41	1935,9	2006,0	2076,9	2148,6	2221,2	2294,6	2368,9	2444,2	2520,6	2598,0	41
42	1937,1	2007,2	2078,1	2149,8	2222,4	2295,8	2370,2	2445,5	2521,8	2599,3	42
43	1938,2	2008,4	2079,3	2151,0	2223,6	2297,0	2371,4	2446,8	2523,1	2600,6	43
44	1939,4	2009,6	2080,5	2152,2	2224,8	2298,3	2372,7	2448,0	2524,4	2601,9	44
45	1940,5	2010,7	2081,7	2153,4	2226,0	2299,5	2373,9	2449,3	2525,7	2603,2	45
46	1941,7	2011,9	2082,9	2154,6	2227,2	2300,7	2375,2	2450,6	2527,0	2604,5	46
47	1942,9	2013,1	2084,1	2155,8	2228,5	2302,0	2376,4	2451,8	2528,3	2605,8	47
48	1944,0	2014,3	2085,3	2157,0	2229,7	2303,2	2377,7	2453,1	2529,6	2607,1	48
49	1945,2	2015,4	2086,5	2158,2	2230,9	2304,4	2378,9	2454,3	2530,8	2608,4	49
50	1946,4	2016,6	2087,7	2159,4	2232,1	2305,7	2380,1	2455,6	2532,1	2609,7	50
51	1947,5	2017,8	2088,9	2160,7	2233,3	2306,9	2381,4	2456,9	2533,4	2611,0	51
52	1948,7	2019,0	2090,1	2161,9	2234,6	2308,1	2382,6	2458,1	2534,7	2612,3	52
53	1949,9	2020,2	2091,3	2163,1	2235,8	2309,4	2383,9	2459,4	2536,0	2613,6	53
54	1951,0	2021,3	2092,5	2164,3	2237,0	2310,6	2385,1	2460,7	2537,2	2614,9	54
55	1952,2	2022,5	2093,7	2165,5	2238,2	2311,8	2386,4	2461,9	2538,5	2616,2	55
56	1953,4	2023,7	2094,9	2166,7	2239,4	2313,1	2387,6	2463,2	2539,8	2617,5	56
57	1954,5	2024,9	2096,1	2167,9	2240,7	2314,3	2388,9	2464,5	2541,1	2618,8	57
58	1955,7	2026,0	2097,3	2169,1	2241,9	2315,5	2390,2	2465,8	2542,4	2620,1	58
59	1956,9	2027,2	2098,5	2170,3	2243,1	2316,7	2391,4	2467,0	2543,7	2621,4	59
min.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	min.
D. l.	30	31	32	33	34	35	36	37	38	39	D. l.

D. l.	40	41	42	43	44	45	46	47	48	49	D. l.
min.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	min.
0	2622,7	2701,6	2781,7	2863,1	2945,7	3030,0	3115,6	3202,8	3291,6	3382,1	0
1	2624,0	2702,9	2783,1	2864,5	2947,2	3031,4	3117,0	3204,2	3293,1	3383,6	1
2	2625,3	2704,3	2784,4	2865,8	2948,6	3032,8	3118,5	3205,7	3294,6	3385,2	2
3	2626,6	2705,6	2785,8	2867,2	2950,0	3034,2	3119,9	3207,2	3296,1	3386,7	3
4	2627,9	2706,9	2787,1	2868,5	2951,4	3035,6	3121,4	3208,6	3297,5	3388,2	4
5	2629,2	2708,3	2788,5	2870,0	2952,8	3037,0	3122,8	3210,1	3299,0	3389,7	5
6	2630,5	2709,6	2789,8	2871,3	2954,2	3038,4	3124,2	3211,6	3300,5	3391,3	6
7	2631,9	2710,9	2791,2	2872,7	2955,6	3039,8	3125,7	3213,0	3302,0	3392,8	7
8	2633,2	2712,2	2792,5	2874,1	2957,0	3041,3	3127,1	3214,5	3303,5	3394,3	8
9	2634,5	2713,6	2793,8	2875,4	2958,4	3042,7	3128,6	3216,0	3305,0	3395,9	9
10	2635,8	2714,9	2795,1	2876,8	2959,8	3044,1	3130,0	3217,4	3306,5	3397,4	10
11	2637,1	2716,2	2796,5	2878,2	2961,1	3045,5	3131,5	3218,9	3308,0	3398,9	11
12	2638,4	2717,5	2797,9	2879,5	2962,5	3047,0	3132,9	3220,4	3309,5	3400,4	12
13	2639,7	2718,9	2799,3	2880,9	2963,9	3048,4	3134,3	3221,9	3311,0	3402,0	13
14	2641,1	2720,2	2800,6	2882,3	2965,3	3049,8	3135,8	3223,3	3312,5	3403,5	14
15	2642,3	2721,5	2802,0	2883,7	2966,7	3051,2	3137,2	3224,8	3314,0	3405,0	15
16	2643,6	2722,9	2803,3	2885,0	2968,1	3052,6	3138,7	3226,3	3315,5	3406,6	16
17	2644,9	2724,2	2804,7	2886,4	2969,5	3054,1	3140,1	3227,7	3317,0	3408,1	17
18	2646,3	2725,5	2806,0	2887,8	2970,9	3055,5	3141,6	3229,2	3318,5	3409,6	18
19	2647,6	2726,9	2807,4	2889,2	2972,3	3056,9	3143,0	3230,7	3320,0	3411,2	19
20	2648,9	2728,2	2808,8	2890,5	2973,7	3058,3	3144,5	3232,2	3321,5	3412,7	20
21	2650,2	2729,5	2810,1	2891,9	2975,1	3059,7	3145,9	3233,6	3323,1	3414,2	21
22	2651,6	2730,8	2811,4	2893,3	2976,5	3061,2	3147,4	3235,1	3324,6	3415,8	22
23	2652,8	2732,2	2812,8	2894,7	2977,9	3062,6	3148,8	3236,6	3326,1	3417,3	23
24	2654,1	2733,5	2814,1	2896,0	2979,3	3064,0	3150,3	3238,1	3327,6	3418,8	24
25	2655,5	2734,8	2815,5	2897,4	2980,7	3065,4	3151,7	3239,5	3329,1	3420,4	25
26	2656,8	2736,2	2816,8	2898,8	2982,1	3066,9	3153,2	3241,0	3330,6	3421,9	26
27	2658,1	2737,5	2818,2	2900,2	2983,5	3068,3	3154,6	3242,5	3332,1	3423,5	27
28	2659,4	2738,8	2819,5	2901,5	2984,9	3069,7	3156,1	3244,0	3333,6	3425,0	28
29	2660,7	2740,2	2820,9	2902,9	2986,3	3071,1	3157,5	3245,5	3335,1	3426,5	29
30	2662,0	2741,5	2822,3	2904,3	2987,7	3072,6	3159,0	3246,9	3336,6	3428,1	30
31	2663,3	2742,9	2823,6	2905,7	2989,1	3074,0	3160,4	3248,4	3338,1	3429,6	31
32	2664,6	2744,2	2825,0	2907,1	2990,5	3075,4	3161,9	3249,9	3339,6	3431,2	32
33	2666,0	2745,5	2826,3	2908,4	2991,9	3076,9	3163,3	3251,4	3341,1	3432,7	33
34	2667,3	2746,9	2827,7	2909,7	2993,3	3078,3	3164,8	3252,9	3342,7	3434,2	34
35	2668,6	2748,2	2829,0	2911,2	2994,7	3079,7	3166,2	3254,4	3344,2	3435,8	35
36	2669,9	2749,5	2830,4	2912,5	2996,1	3081,1	3167,7	3255,8	3345,7	3437,3	36
37	2671,2	2750,9	2831,8	2914,0	2997,5	3082,5	3169,1	3257,3	3347,2	3438,9	37
38	2672,5	2752,2	2833,1	2915,3	2998,9	3084,0	3170,6	3258,8	3348,7	3440,4	38
39	2673,9	2753,5	2834,5	2916,7	3000,3	3085,4	3172,1	3260,3	3350,1	3442,0	39
40	2675,1	2754,9	2835,8	2918,1	3001,8	3086,9	3173,5	3261,8	3351,7	3443,5	40
41	2676,5	2756,2	2837,2	2919,5	3003,2	3088,3	3175,0	3263,3	3353,2	3445,0	41
42	2677,8	2757,6	2838,6	2920,9	3004,6	3089,7	3176,4	3264,7	3354,8	3446,6	42
43	2679,1	2758,9	2839,9	2922,3	3006,0	3091,2	3177,9	3266,2	3356,3	3448,1	43
44	2680,5	2760,2	2841,3	2923,6	3007,4	3092,6	3179,3	3267,7	3357,8	3449,7	44
45	2681,8	2761,5	2842,6	2925,0	3008,8	3094,0	3180,8	3269,2	3359,3	3451,2	45
46	2683,1	2762,9	2844,0	2926,4	3010,2	3095,5	3182,3	3270,7	3360,8	3452,8	46
47	2684,4	2764,3	2845,4	2927,8	3011,6	3096,9	3183,7	3272,2	3362,3	3454,3	47
48	2685,7	2765,6	2846,7	2929,2	3013,0	3098,3	3185,2	3273,7	3363,9	3455,9	48
49	2687,1	2766,9	2848,1	2930,6	3014,4	3099,8	3186,6	3275,2	3365,4	3457,4	49
50	2688,4	2768,3	2849,5	2932,0	3015,8	3101,2	3188,1	3276,6	3366,9	3459,0	50
51	2689,7	2769,6	2850,8	2933,3	3017,2	3102,6	3189,6	3278,1	3368,4	3460,5	51
52	2691,0	2771,0	2852,2	2934,7	3018,7	3104,1	3191,0	3279,6	3369,9	3462,1	52
53	2692,3	2772,3	2853,6	2936,1	3020,1	3105,6	3192,5	3281,1	3371,5	3463,6	53
54	2693,7	2773,7	2854,9	2937,5	3021,5	3107,0	3194,0	3282,6	3373,0	3465,2	54
55	2695,0	2775,0	2856,3	2938,9	3022,9	3108,4	3195,4	3284,1	3374,5	3466,7	55
56	2696,1	2776,4	2857,7	2940,3	3024,3	3109,8	3196,9	3285,6	3376,0	3468,3	56
57	2697,6	2777,7	2859,1	2941,7	3025,7	3111,2	3198,4	3287,1	3377,6	3469,8	57
58	2699,0	2779,0	2860,5	2943,1	3027,1	3112,7	3199,8	3288,6	3379,1	3471,4	58
59	2700,3	2780,4	2861,8	2944,4	3028,5	3114,1	3201,3	3290,1	3380,6	3473,0	59
min.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	min.
D. l.	40	41	42	43	44	45	46	47	48	49	D. l.

D. 1.	50	51	52	53	54	55	56	57	58	59	D. 1.
min.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	min.
0	3474,5	3568,8	3665,2	3763,8	3864,7	3968,0	4073,9	4182,6	4294,3	4409,2	0
1	3476,1	3570,4	3666,9	3765,5	3866,4	3969,7	4075,7	4184,5	4296,2	4411,1	1
2	3477,6	3572,0	3668,5	3767,1	3868,1	3971,5	4077,5	4186,3	4298,1	4413,1	2
3	3479,2	3573,6	3670,1	3768,8	3869,8	3973,2	4079,3	4188,2	4300,0	4415,0	3
4	3480,7	3575,2	3671,7	3770,4	3871,5	3975,0	4081,1	4190,0	4301,9	4417,0	4
5	3482,3	3576,8	3673,4	3772,1	3873,2	3976,7	4082,9	4191,8	4303,8	4418,9	5
6	3483,9	3578,4	3675,0	3773,8	3874,9	3978,5	4084,7	4193,7	4305,7	4420,8	6
7	3485,4	3580,0	3676,6	3775,4	3876,6	3980,2	4086,5	4195,5	4307,6	4422,8	7
8	3487,0	3581,6	3678,2	3777,1	3878,3	3982,0	4088,3	4197,4	4309,5	4424,7	8
9	3488,5	3583,2	3679,9	3778,8	3880,0	3983,7	4090,1	4199,2	4311,4	4426,7	9
10	3490,1	3584,8	3681,5	3780,4	3881,7	3985,5	4091,9	4201,1	4313,2	4428,6	10
11	3491,7	3586,4	3683,1	3782,1	3883,4	3987,2	4093,7	4202,9	4315,1	4430,6	11
12	3493,2	3588,0	3684,8	3783,8	3885,1	3989,0	4095,5	4204,7	4317,0	4432,5	12
13	3494,8	3589,5	3686,4	3785,5	3886,8	3990,7	4097,3	4206,6	4318,9	4434,5	13
14	3496,3	3591,1	3688,0	3787,1	3888,6	3992,5	4099,1	4208,4	4320,8	4436,4	14
15	3497,9	3592,7	3689,7	3788,8	3890,3	3994,2	4100,9	4210,3	4322,7	4438,4	15
16	3499,5	3594,3	3691,3	3790,5	3892,0	3996,0	4102,7	4212,1	4324,6	4440,4	16
17	3501,0	3595,9	3692,9	3792,1	3893,7	3997,7	4104,5	4214,0	4326,5	4442,3	17
18	3502,6	3597,5	3694,6	3793,8	3895,4	3999,5	4106,3	4215,8	4328,4	4444,3	18
19	3504,2	3599,1	3696,2	3795,5	3897,1	4001,3	4108,1	4217,7	4330,3	4446,2	19
20	3505,7	3600,7	3697,8	3797,2	3898,8	4003,0	4109,9	4219,5	4332,2	4448,2	20
21	3507,3	3602,3	3699,5	3798,8	3900,5	4004,8	4111,7	4221,4	4334,2	4450,2	21
22	3508,9	3603,9	3701,1	3800,5	3902,3	4006,5	4113,5	4223,2	4336,1	4452,1	22
23	3510,5	3605,5	3702,7	3802,2	3904,0	4008,3	4115,3	4225,1	4338,0	4454,1	23
24	3512,0	3607,1	3704,4	3803,9	3905,7	4010,0	4117,1	4227,0	4339,9	4456,0	24
25	3513,6	3608,7	3706,0	3805,5	3907,4	4011,8	4118,9	4228,8	4341,8	4458,0	25
26	3515,1	3610,3	3707,7	3807,2	3909,1	4013,6	4120,7	4230,7	4343,7	4460,0	26
27	3516,7	3611,9	3709,3	3808,9	3910,9	4015,3	4122,5	4232,5	4345,6	4461,9	27
28	3518,3	3613,6	3710,9	3810,6	3912,6	4017,1	4124,3	4234,4	4347,5	4463,9	28
29	3519,8	3615,2	3712,6	3812,3	3914,3	4018,9	4126,1	4236,2	4349,4	4465,9	29
30	3521,4	3616,8	3714,2	3813,9	3916,0	4020,6	4127,9	4238,1	4351,3	4467,8	30
31	3523,0	3618,4	3715,9	3815,6	3917,7	4022,4	4129,7	4240,0	4353,3	4469,8	31
32	3524,6	3620,0	3717,5	3817,3	3919,5	4024,2	4131,6	4241,8	4355,2	4471,8	32
33	3526,1	3621,6	3719,2	3819,0	3921,2	4025,9	4133,4	4243,7	4357,1	4473,8	33
34	3527,7	3623,2	3720,8	3820,7	3922,9	4027,7	4135,2	4245,6	4359,0	4475,7	34
35	3529,3	3624,8	3722,4	3822,3	3924,6	4029,5	4137,0	4247,4	4360,9	4477,7	35
36	3530,9	3626,4	3724,1	3824,0	3926,4	4031,2	4138,8	4249,3	4362,8	4479,7	36
37	3532,4	3628,0	3725,7	3825,7	3928,1	4033,0	4140,6	4251,2	4364,8	4481,7	37
38	3534,0	3629,6	3727,4	3827,4	3929,8	4034,8	4142,5	4253,0	4366,7	4483,6	38
39	3535,6	3631,3	3729,0	3829,1	3931,5	4036,6	4144,3	4254,9	4368,6	4485,6	39
40	3537,2	3632,9	3730,7	3830,8	3933,3	4038,3	4146,1	4256,8	4370,5	4487,6	40
41	3538,8	3634,5	3732,3	3832,5	3935,0	4040,1	4147,9	4258,6	4372,5	4489,6	41
42	3540,3	3636,1	3734,0	3834,2	3936,7	4041,9	4149,7	4260,5	4374,4	4491,6	42
43	3541,9	3637,7	3735,6	3835,8	3938,5	4043,6	4151,6	4262,4	4376,3	4493,5	43
44	3543,5	3639,3	3737,3	3837,5	3940,2	4045,4	4153,4	4264,3	4378,2	4495,5	44
45	3545,1	3640,9	3738,9	3839,2	3941,9	4047,2	4155,2	4266,1	4380,2	4497,5	45
46	3546,7	3642,5	3740,6	3840,9	3943,7	4049,0	4157,0	4268,0	4382,1	4499,5	46
47	3548,2	3644,2	3742,2	3842,6	3945,4	4050,8	4158,8	4269,9	4384,0	4501,5	47
48	3549,8	3645,8	3743,9	3844,3	3947,1	4052,5	4160,7	4271,8	4385,9	4503,5	48
49	3551,4	3647,4	3745,6	3846,0	3948,9	4054,3	4162,5	4273,6	4387,9	4505,5	49
50	3553,0	3649,0	3747,2	3847,7	3950,6	4056,1	4164,3	4275,5	4389,8	4507,5	50
51	3554,6	3650,6	3748,9	3849,4	3952,3	4057,9	4166,2	4277,4	4391,7	4509,4	51
52	3556,1	3652,3	3750,5	3851,1	3954,1	4059,7	4168,0	4279,3	4393,7	4511,4	52
53	3557,7	3653,9	3752,2	3852,8	3955,8	4061,4	4169,8	4281,1	4395,6	4513,4	53
54	3559,3	3655,5	3753,8	3854,5	3957,6	4063,2	4171,7	4283,0	4397,5	4515,4	54
55	3560,9	3657,1	3755,5	3856,2	3959,3	4065,0	4173,5	4284,9	4399,5	4517,4	55
56	3562,5	3658,7	3757,2	3857,9	3961,0	4066,8	4175,3	4286,8	4401,4	4519,4	56
57	3564,1	3660,4	3758,8	3859,6	3962,8	4068,6	4177,2	4288,7	4403,4	4521,4	57
58	3565,7	3662,0	3760,5	3861,3	3964,5	4070,4	4179,0	4290,6	4405,3	4523,4	58
59	3567,3	3663,6	3762,2	3863,0	3966,3	4072,1	4180,8	4292,5	4407,2	4525,4	59
min.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	min.
D. 1.	50	51	52	53	54	55	56	57	58	59	D. 1.

D. 1.	60	61	62	63	64	65	66	67	68	69	D. 1.
min.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	min.
0	4527,4	4649,2	4775,0	4905,0	5039,4	5178,8	5323,6	5474,0	5630,9	5794,6	0
1	4529,4	4651,3	4777,1	4907,2	5041,7	5181,2	5326,0	5476,6	5633,5	5797,4	1
2	4531,4	4653,4	4779,3	4909,4	5044,0	5183,6	5328,5	5479,2	5636,2	5800,2	2
3	4533,4	4655,5	4781,4	4911,6	5046,3	5186,0	5330,9	5481,7	5638,9	5803,0	3
4	4535,4	4657,5	4783,5	4913,8	5048,6	5188,3	5333,4	5484,3	5641,5	5805,8	4
5	4537,4	4659,6	4785,7	4916,0	5050,8	5190,7	5335,9	5486,9	5644,2	5808,6	5
6	4539,4	4661,7	4787,8	4918,2	5053,2	5193,1	5338,3	5489,4	5646,9	5811,4	6
7	4541,4	4663,7	4790,0	4920,4	5055,5	5195,4	5340,8	5492,0	5649,6	5814,2	7
8	4543,4	4665,8	4792,1	4922,6	5057,7	5197,8	5343,3	5494,6	5652,3	5817,0	8
9	4545,4	4667,9	4794,2	4924,8	5060,0	5200,2	5345,7	5497,1	5655,0	5819,8	9
10	4547,5	4669,9	4796,4	4927,1	5062,3	5202,6	5348,2	5499,7	5657,6	5822,6	10
11	4549,5	4672,0	4798,5	4929,3	5064,6	5205,0	5350,7	5502,3	5660,3	5825,4	11
12	4551,5	4674,1	4800,7	4931,5	5066,9	5207,3	5353,2	5504,9	5663,0	5828,2	12
13	4553,5	4676,2	4802,8	4933,7	5069,2	5209,7	5355,6	5507,4	5665,7	5831,0	13
14	4555,5	4678,2	4804,9	4935,9	5071,5	5212,1	5358,1	5510,0	5668,4	5833,9	14
15	4557,5	4680,3	4807,1	4938,1	5073,8	5214,5	5360,6	5512,6	5671,1	5836,7	15
16	4559,5	4682,4	4809,2	4940,4	5076,1	5216,9	5363,1	5515,2	5673,8	5839,5	16
17	4561,5	4684,5	4811,4	4942,6	5078,4	5219,3	5365,6	5517,8	5676,5	5842,3	17
18	4563,6	4686,6	4813,5	4944,8	5080,7	5221,7	5368,1	5520,4	5679,2	5845,2	18
19	4565,6	4688,6	4815,7	4947,0	5083,0	5224,1	5370,5	5523,0	5681,9	5848,0	19
20	4567,6	4690,7	4817,8	4949,3	5085,3	5226,5	5373,0	5525,6	5684,6	5850,8	20
21	4569,6	4692,8	4820,0	4951,5	5087,7	5228,9	5375,5	5528,2	5687,3	5853,7	21
22	4571,6	4694,9	4822,2	4953,7	5090,0	5231,3	5378,0	5530,8	5690,0	5856,5	22
23	4573,7	4697,0	4824,3	4956,0	5092,3	5233,7	5380,5	5533,4	5692,8	5859,3	23
24	4575,7	4699,1	4826,5	4958,2	5094,6	5236,1	5383,0	5536,0	5695,5	5862,2	24
25	4577,7	4701,2	4828,6	4960,4	5096,9	5238,5	5385,5	5538,6	5698,2	5865,0	25
26	4579,7	4703,2	4830,8	4962,7	5099,2	5240,9	5388,0	5541,2	5700,9	5867,9	26
27	4581,8	4705,3	4832,9	4964,9	5101,5	5243,3	5390,5	5543,8	5703,6	5870,7	27
28	4583,8	4707,4	4835,1	4967,1	5103,9	5245,7	5393,0	5546,4	5706,3	5873,5	28
29	4585,8	4709,5	4837,3	4969,4	5106,2	5248,1	5395,5	5549,0	5709,1	5876,4	29
30	4587,8	4711,6	4839,4	4971,6	5108,5	5250,5	5398,0	5551,6	5711,8	5879,3	30
31	4589,9	4713,7	4841,6	4973,9	5110,8	5252,9	5400,5	5554,2	5714,5	5882,1	31
32	4591,9	4715,8	4843,8	4976,1	5113,1	5255,3	5403,0	5556,8	5717,3	5885,0	32
33	4593,9	4717,9	4845,9	4978,3	5115,5	5257,7	5405,6	5559,5	5720,0	5887,8	33
34	4596,0	4720,0	4848,1	4980,6	5117,8	5260,1	5408,1	5562,1	5722,7	5890,7	34
35	4598,0	4722,1	4850,3	4982,8	5120,1	5262,6	5410,6	5564,7	5725,5	5893,6	35
36	4600,1	4724,2	4852,5	4985,1	5122,5	5265,0	5413,1	5567,3	5728,2	5896,4	36
37	4602,1	4726,3	4854,6	4987,3	5124,8	5267,4	5415,6	5569,9	5731,0	5899,3	37
38	4604,1	4728,4	4856,8	4989,6	5127,1	5269,8	5418,1	5572,6	5733,7	5902,2	38
39	4606,2	4730,5	4859,0	4991,8	5129,5	5272,3	5420,7	5575,2	5736,4	5905,1	39
40	4608,2	4732,6	4861,2	4994,1	5131,8	5274,7	5423,2	5577,8	5739,2	5907,9	40
41	4610,3	4734,7	4863,3	4996,3	5134,1	5277,1	5425,7	5580,5	5741,9	5910,8	41
42	4612,3	4736,9	4865,5	4998,6	5136,5	5279,5	5428,2	5583,1	5744,7	5913,7	42
43	4614,3	4739,0	4867,7	5000,9	5138,8	5282,0	5430,8	5585,7	5747,5	5916,6	43
44	4616,4	4741,1	4869,9	5003,1	5141,2	5284,4	5433,3	5588,4	5750,2	5919,5	44
45	4618,4	4743,2	4872,1	5005,4	5143,5	5286,8	5435,8	5591,0	5753,0	5922,4	45
46	4620,5	4745,3	4874,3	5007,6	5145,9	5289,3	5438,4	5593,7	5755,7	5925,2	46
47	4622,5	4747,4	4876,4	5009,9	5148,2	5291,7	5440,9	5596,3	5758,5	5928,1	47
48	4624,6	4749,5	4878,6	5012,2	5150,6	5294,2	5443,5	5599,0	5761,3	5931,0	48
49	4626,6	4751,7	4880,8	5014,4	5152,9	5296,6	5446,0	5601,6	5764,0	5933,9	49
50	4628,7	4753,8	4882,9	5016,7	5155,3	5299,0	5448,5	5604,3	5766,8	5936,8	50
51	4630,7	4755,9	4885,2	5019,0	5157,6	5301,5	5451,1	5606,9	5769,6	5939,7	51
52	4632,8	4758,0	4887,4	5021,2	5160,0	5303,9	5453,6	5609,6	5772,3	5942,6	52
53	4634,8	4760,1	4889,6	5023,5	5162,3	5306,4	5456,2	5612,2	5775,1	5945,5	53
54	4636,9	4762,3	4891,8	5025,8	5164,7	5308,8	5458,7	5614,9	5777,9	5948,5	54
55	4639,0	4764,4	4894,0	5028,1	5167,0	5311,3	5461,3	5617,5	5780,7	5951,4	55
56	4641,0	4766,5	4896,2	5030,3	5169,4	5313,7	5463,8	5620,2	5783,5	5954,3	56
57	4643,1	4768,6	4898,4	5032,6	5171,8	5316,2	5466,4	5622,9	5786,2	5957,2	57
58	4645,1	4770,8	4900,6	5034,9	5174,1	5318,6	5468,9	5625,5	5789,0	5960,1	58
59	4647,2	4772,9	4902,8	5037,2	5176,5	5321,1	5471,5	5628,2	5791,8	5963,0	59
min.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	min.
D. 1.	60	61	62	63	64	65	66	67	68	69	D. 1.

D. l.	70	71	72	73	74	75	76	77	78	79	D. l.
min.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	min.
0	5966,0	6145,7	6334,9	6534,5	6745,7	6970,3	7210,1	7467,2	7744,6	8045,7	0
1	5968,9	6148,8	6338,1	6537,9	6749,4	6974,2	7214,2	7471,7	7749,4	8051,0	1
2	5971,8	6151,9	6341,4	6541,3	6753,0	6978,1	7218,3	7476,1	7754,2	8056,2	2
3	5974,7	6155,0	6344,6	6544,7	6756,6	6981,9	7222,5	7480,6	7759,0	8061,5	3
4	5977,7	6158,0	6347,8	6548,2	6760,3	6985,8	7226,6	7485,0	7763,9	8066,8	4
5	5980,6	6161,1	6351,1	6551,6	6763,9	6989,7	7230,8	7489,5	7768,7	8072,0	5
6	5983,5	6164,2	6354,3	6555,0	6767,6	6993,6	7234,9	7494,0	7773,5	8077,3	6
7	5986,5	6167,3	6357,6	6558,5	6771,2	6997,5	7239,1	7498,5	7778,4	8082,6	7
8	5989,4	6170,4	6360,9	6561,9	6774,9	7001,4	7243,3	7502,9	7783,2	8087,9	8
9	5992,4	6173,5	6364,1	6565,4	6778,5	7005,3	7247,5	7507,4	7788,1	8093,2	9
10	5995,3	6176,6	6367,4	6568,8	6782,2	7009,2	7251,6	7511,9	7793,0	8098,5	10
11	5998,3	6179,7	6370,6	6572,3	6785,8	7013,1	7255,8	7516,4	7797,8	8103,8	11
12	6001,2	6182,8	6373,9	6575,7	6789,5	7017,0	7260,0	7520,9	7802,7	8109,2	12
13	6004,2	6185,9	6377,2	6579,2	6793,2	7020,9	7264,2	7525,4	7807,6	8114,5	13
14	6007,1	6189,0	6380,5	6582,6	6796,9	7024,8	7268,4	7530,0	7812,5	8119,8	14
15	6010,1	6192,1	6383,7	6586,1	6800,5	7028,7	7272,6	7534,5	7817,4	8125,2	15
16	6013,0	6195,2	6387,0	6589,5	6804,2	7032,7	7276,8	7539,0	7822,3	8130,6	16
17	6016,0	6198,3	6390,3	6593,0	6807,9	7036,6	7281,0	7543,6	7827,2	8135,9	17
18	6019,0	6201,4	6393,6	6596,5	6811,6	7040,5	7285,2	7548,1	7832,2	8141,3	18
19	6021,9	6204,6	6396,9	6600,0	6815,3	7044,5	7289,4	7552,7	7837,1	8146,7	19
20	6024,9	6207,7	6400,2	6603,4	6819,0	7048,4	7293,7	7557,2	7842,0	8152,1	20
21	6027,9	6210,8	6403,5	6606,9	6822,7	7052,4	7297,9	7561,8	7847,0	8157,5	21
22	6030,8	6213,9	6406,8	6610,4	6826,4	7056,3	7302,1	7566,3	7841,9	8162,9	22
23	6033,8	6217,1	6410,1	6613,9	6830,1	7060,3	7306,4	7570,9	7856,9	8168,3	23
24	6036,8	6220,2	6413,4	6617,4	6833,8	7064,2	7310,6	7575,5	7861,9	8173,7	24
25	6039,8	6223,3	6416,7	6620,9	6837,6	7068,2	7314,9	7580,1	7866,8	8179,2	25
26	6042,7	6226,5	6420,9	6624,4	6841,3	7072,2	7319,1	7584,7	7871,8	8184,6	26
27	6045,7	6229,6	6423,3	6627,9	6845,0	7076,2	7323,4	7589,3	7876,8	8190,1	27
28	6048,7	6232,7	6426,6	6631,4	6848,7	7080,1	7327,7	7593,9	7881,8	8195,5	28
29	6051,7	6235,9	6429,9	6635,0	6852,5	7084,1	7332,0	7598,5	7886,8	8201,0	29
30	6054,7	6239,0	6433,2	6638,5	6856,2	7088,1	7336,2	7603,1	7891,8	8206,5	30
31	6057,7	6242,2	6436,6	6642,0	6860,0	7092,1	7340,4	7607,7	7896,8	8212,0	31
32	6060,7	6245,3	6439,9	6645,5	6863,7	7096,1	7344,8	7612,3	7901,9	8217,5	32
33	6063,7	6248,5	6443,2	6649,1	6867,5	7100,1	7349,1	7617,0	7906,9	8223,0	33
34	6066,7	6251,7	6446,6	6652,6	6871,2	7104,1	7353,4	7621,6	7911,9	8228,5	34
35	6069,7	6254,8	6449,9	6656,1	6875,0	7108,2	7357,7	7626,3	7917,0	8234,1	35
36	6072,7	6258,0	6453,3	6659,7	6878,7	7112,2	7362,0	7630,9	7922,1	8239,6	36
37	6075,7	6261,2	6456,6	6663,2	6882,5	7116,2	7366,4	7635,6	7927,1	8245,1	37
38	6078,8	6264,4	6460,0	6666,8	6886,3	7120,2	7370,7	7640,2	7932,2	8250,7	38
39	6081,8	6267,5	6463,3	6670,3	6890,1	7124,3	7375,0	7644,9	7937,3	8256,3	39
40	6084,8	6270,7	6466,7	6673,9	6893,8	7128,3	7379,4	7649,6	7942,4	8261,8	40
41	6087,8	6273,9	6470,0	6677,4	6897,6	7132,3	7383,7	7654,3	7947,5	8267,4	41
42	6090,8	6277,1	6473,4	6681,0	6901,4	7136,4	7388,0	7659,0	7952,6	8273,0	42
43	6093,9	6280,3	6476,8	6684,6	6905,2	7140,4	7392,4	7663,7	7957,7	8278,6	43
44	6096,9	6283,5	6480,1	6688,1	6909,0	7144,5	7396,8	7668,4	7962,8	8284,2	44
45	6099,0	6286,6	6483,5	6691,7	6912,8	7148,6	7401,1	7673,1	7968,0	8289,9	45
46	6103,0	6289,8	6486,9	6695,3	6916,6	7152,6	7405,5	7677,8	7973,1	8295,5	46
47	6106,0	6293,0	6490,3	6698,9	6920,4	7156,7	7409,9	7682,6	7978,2	8301,1	47
48	6109,1	6296,2	6493,6	6702,4	6924,2	7160,8	7414,2	7687,3	7983,4	8306,8	48
49	6112,1	6299,4	6497,0	6706,0	6928,1	7164,9	7418,6	7692,0	7988,5	8312,4	49
50	6115,1	6302,7	6500,4	6709,6	6931,9	7169,0	7423,0	7696,8	7993,7	8318,1	50
51	6118,2	6305,9	6503,8	6713,2	6935,7	7173,0	7427,4	7701,5	7998,9	8323,8	51
52	6121,2	6309,1	6507,2	6716,8	6939,5	7177,1	7431,8	7706,3	8004,0	8329,4	52
53	6124,3	6312,3	6510,6	6720,4	6943,4	7181,2	7436,2	7711,0	8009,2	8335,1	53
54	6127,4	6315,5	6514,0	6724,0	6947,2	7185,3	7440,6	7715,8	8014,4	8340,8	54
55	6130,4	6318,7	6517,4	6727,6	6951,1	7189,5	7445,0	7720,6	8019,6	8346,6	55
56	6133,5	6322,0	6520,8	6731,2	6954,9	7193,6	7449,5	7725,4	8024,8	8352,3	56
57	6136,5	6325,2	6524,2	6734,9	6958,8	7197,7	7453,9	7730,2	8030,0	8358,0	57
58	6139,6	6328,4	6527,6	6738,5	6962,6	7201,8	7458,3	7735,0	8035,3	8363,7	58
59	6142,7	6331,7	6531,0	6742,1	6966,5	7205,9	7462,8	7739,8	8040,5	8369,5	59
min.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	min.
D. l.	70	71	72	73	74	75	76	77	78	79	D. l.

D. 1.	80	81	82	83	84	85	86	87	88	89	D. 1.
min.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	min.
0	8375,3	8739,1	9145,5	9605,9	10137,0	10764,7	11532,6	12522,3	13916,6	16299,8	0
1	8381,0	8745,5	9152,7	9614,1	10146,6	10776,2	11547,0	12541,4	13945,4	16357,5	1
2	8386,8	8751,9	9159,9	9622,4	10156,2	10787,7	11561,4	12560,7	13974,4	16416,3	2
3	8392,6	8758,3	9167,2	9630,6	10165,8	10799,3	11575,9	12580,0	14003,7	16476,1	3
4	8398,3	8764,8	9174,4	9638,9	10175,4	10810,0	11590,5	12599,5	14033,2	16537,0	4
5	8404,1	8771,2	9181,6	9647,2	10185,1	10822,6	11605,0	12619,1	14063,0	16594,9	5
6	8409,9	8777,7	9188,9	9655,5	10194,8	10834,2	11619,7	12638,8	14093,0	16662,0	6
7	8415,8	8784,1	9196,2	9663,8	10204,6	10845,9	11634,5	12658,6	14123,3	16726,2	7
8	8421,6	8790,6	9203,5	9672,2	10214,4	10857,7	11649,3	12678,6	14153,9	16791,7	8
9	8427,4	8797,1	9210,8	9680,6	10224,2	10869,6	11664,1	12698,6	14184,7	16858,5	9
10	8433,3	8803,6	9218,1	9689,0	10234,0	10881,4	11679,1	12718,8	14215,8	16926,5	10
11	8439,1	8810,1	9225,4	9697,4	10243,8	10893,3	11694,0	12739,1	14247,2	16990,0	11
12	8445,0	8816,6	9232,7	9705,8	10253,7	10905,2	11709,1	12759,5	14278,9	17066,9	12
13	8450,9	8823,2	9240,2	9714,2	10263,6	10917,2	11724,2	12780,0	14310,9	17130,3	13
14	8456,8	8829,7	9247,6	9722,7	10273,5	10929,1	11739,4	12800,7	14343,2	17213,2	14
15	8462,6	8836,3	9255,0	9731,2	10283,5	10941,2	11754,7	12821,5	14375,8	17288,7	15
16	8468,6	8842,8	9262,4	9739,7	10293,5	10953,3	11770,0	12842,5	14408,7	17366,0	16
17	8474,5	8849,4	9269,9	9748,3	10303,5	10965,5	11785,4	12863,5	14441,9	17444,0	17
18	8480,4	8856,0	9277,3	9756,8	10313,6	10977,7	11800,9	12884,7	14475,4	17525,9	18
19	8486,3	8862,6	9284,8	9765,4	10323,7	10989,9	11816,4	12906,0	14509,3	17608,7	19
20	8492,3	8869,3	9292,3	9774,0	10333,8	11002,2	11832,0	12927,4	14543,5	17693,6	20
21	8498,2	8875,9	9299,8	9782,7	10344,0	11014,5	11847,6	12948,9	14578,1	17780,5	21
22	8504,2	8882,6	9307,3	9791,3	10354,1	11026,9	11863,4	12970,6	14613,0	17879,9	22
23	8510,2	8889,2	9314,8	9800,0	10364,3	11039,3	11879,2	12992,5	14648,3	17961,6	23
24	8516,2	8895,9	9322,4	9808,6	10374,5	11051,7	11895,1	13014,4	14683,9	18055,8	24
25	8522,2	8902,6	9330,0	9817,3	10384,8	11064,2	11911,0	13036,6	14719,9	18152,6	25
26	8528,2	8909,3	9337,6	9826,1	10395,0	11076,8	11927,1	13058,8	14756,3	18252,3	26
27	8534,2	8916,0	9345,2	9834,8	10405,3	11089,3	11943,1	13081,2	14793,0	18354,9	27
28	8540,2	8922,7	9352,8	9843,6	10415,7	11102,0	11959,3	13103,8	14830,2	18460,7	28
29	8546,2	8929,5	9360,4	9852,4	10426,2	11114,6	11975,6	13126,5	14867,8	18569,8	29
30	8552,3	8936,2	9368,1	9861,3	10436,6	11127,4	11991,9	13149,3	14905,8	18682,5	30
31	8558,4	8943,0	9375,8	9870,1	10447,1	11140,1	12008,4	13172,3	14944,2	18799,1	31
32	8564,4	8949,8	9383,5	9879,0	10457,5	11152,9	12024,9	13195,5	14983,0	18919,7	32
33	8570,5	8956,6	9391,2	9887,8	10468,0	11165,8	12041,5	13218,8	15022,3	19044,7	33
34	8576,6	8963,4	9398,9	9896,7	10478,5	11178,7	12058,2	13242,3	15062,1	19174,4	34
35	8582,7	8970,2	9406,6	9905,7	10489,1	11191,7	12074,9	13265,9	15102,3	19309,2	35
36	8588,9	8977,1	9414,4	9914,6	10499,7	11204,7	12091,7	13289,7	15143,0	19449,5	36
37	8595,0	8983,9	9422,1	9923,6	10510,4	11217,7	12108,6	13313,7	15184,2	19595,8	37
38	8601,1	8990,8	9429,9	9932,7	10521,1	11230,8	12125,6	13337,8	15225,8	19748,6	38
39	8607,3	8997,7	9437,8	9941,7	10531,8	11244,0	12142,7	13362,1	15268,0	19908,5	39
40	8613,5	9004,6	9445,6	9950,8	10542,6	11257,2	12159,9	13386,6	15310,7	20076,4	40
41	8619,6	9011,5	9453,4	9959,8	10553,3	11270,5	12177,1	13411,2	15353,8	20252,5	41
42	8625,8	9018,4	9461,3	9968,9	10564,1	11283,8	12194,4	13436,1	15397,8	20433,3	42
43	8632,0	9025,4	9469,1	9978,0	10574,9	11297,1	12211,8	13461,1	15442,1	20635,1	43
44	8638,2	9032,3	9477,0	9987,2	10585,8	11310,5	12229,3	13486,3	15486,9	20843,5	44
45	8644,5	9039,3	9484,9	9996,3	10596,7	11324,0	12246,9	13511,6	15532,6	21065,4	45
46	8650,7	9046,3	9492,9	10005,5	10607,6	11337,6	12264,6	13537,0	15578,7	21302,5	46
47	8656,9	9053,3	9500,8	10014,8	10618,6	11351,1	12282,4	13562,8	15625,5	21557,3	47
48	8663,2	9060,3	9508,8	10024,0	10629,7	11364,8	12300,2	13588,9	15672,7	21832,5	48
49	8669,5	9067,3	9516,8	10033,3	10640,8	11378,4	12318,1	13615,1	15721,0	22131,6	49
50	8675,7	9074,4	9524,8	10042,6	10651,9	11392,2	12336,3	13641,4	15769,8	22459,3	50
51	8682,0	9081,4	9532,9	10051,9	10663,0	11406,0	12354,4	13667,8	15819,3	22821,5	51
52	8688,3	9088,5	9540,9	10061,3	10674,1	11419,8	12372,7	13694,5	15869,4	23226,4	52
53	8694,6	9095,6	9548,9	10070,6	10685,3	11433,7	12391,0	13721,5	15920,4	23685,4	53
54	8701,0	9102,7	9557,0	10080,0	10696,6	11447,7	12409,5	13748,9	15972,1	24215,3	54
55	8707,3	9109,8	9565,1	10089,4	10707,7	11461,7	12428,0	13776,1	16024,6	24842,1	55
56	8713,6	9116,9	9573,2	10098,9	10719,1	11475,8	12446,5	13803,7	16077,9	25609,2	56
57	8720,0	9124,0	9581,4	10108,4	10730,4	11489,9	12465,3	13831,3	16132,0	26598,2	57
58	8726,4	9131,2	9589,5	10117,9	10741,8	11504,1	12484,2	13859,6	16187,0	27992,1	58
59	8732,7	9138,4	9597,7	10127,4	10753,3	11518,3	12503,1	13887,8	16242,9	30375,0	59
min.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	M. P.	min.
D. 1.	80	81	82	83	84	85	86	87	88	89	D. 1.



THE
ELEMENTS
OF
NAVIGATION.

BOOK VIII.
OF ASTRONOMY.

SECTION I.

Of Solar Astronomy.

828. **A**STRONOMY is a science which treats of the motions and distances of the heavenly bodies, and of the appearances thence arising.

There have been a great variety of opinions among the philosophers of the preceding ages concerning the disposition of the great bodies in the universe; or of the positions of the bodies which appear in the heavens: But the notion now embraced by the most judicious astronomers, is, that the universe is composed of an infinite number of systems or worlds; in every system there are certain bodies moving in free space, and revolving at different distances around a SUN placed in or near the centre of the system; and that these *suns* and other bodies are the stars which are seen in the heavens.

829. That system in which our earth is placed, is by astronomers called the SOLAR SYSTEM; and herein the notion of the sun being fixed in or near the centre, with several bodies revolving round him at different distances, is confirmed by all the observations hitherto made: This opinion is called the COPERNICAN SYSTEM; from *Nicholas Copernicus*,

a Polish philosopher, who about the year 1473 revived this notion from the oblivion it had been buried in for many ages.

Stars are distinguished into two kinds, namely, *fixed* and *wandering*.

830. The FIXED STARS are the suns in the centres of their systems shining by their own light; and which are observed to preserve always the same situations in respect to one another.

831. The fixed stars appear of various sizes, doubtless occasioned by their different distances; these sizes are usually distinguished into six or seven classes, called Magnitudes: The largest, or brightest, are said to be of the FIRST MAGNITUDE; those in the next class, or degree of brightness, are called of the SECOND MAGNITUDE; and so on to the least, or those just discernible to the naked eye: But beside these, there are scattered throughout the heavens a great number of other stars, called TELESCOPIC STARS, they being only to be seen through that instrument. And indeed, it is to the assistance of that most admirable machine, that a great part of the modern astronomy owes its rise, and which will undoubtedly transmit, with the greatest honour to the latest posterity, the name of GALILEO, among whose many inventions the telescope is to be ranked.

832. Altho' the visible fixed stars are probably at very unequal distances from the centre of the *solar system*, yet astronomers, for their ease of computation, reckon them as tho' they were all equally distant from our sun, forming the surface of a sphere inclosing our system, and called the CELESTIAL SPHERE: Which supposition may be strictly admitted, considering the immense distance of even the nearest of the fixed stars.

833. A CONSTELLATION is a number of stars lying in the neighbourhood of one another on the surface of the *celestial sphere*, and which astronomers, for their easy remembrance, suppose to be circumscribed by the outlines of some *animal*, or other *figure*; whereby the motions of the wandering stars are more readily described and compared.

The number of these constellations are 80, each containing several stars of different magnitudes: The number of stars of each magnitude, and also the constellation wherein they are ranged, are contained in the following table; where it may be observed, that the constellations are distinguished under three heads, namely, in the *zodiac*, and in the northern and southern hemispheres.

834.

CONSTELLATIONS IN THE ZODIAC.

Names. Northern.	Number	Magnitudes						Names. Southern.	Number	Magnitudes					
		I	II	III	IV	V	VI			I	II	III	IV	V	VI
Aries	46	C	1	1	3	5	36	Libra	33	0	2	1	8	3	19
Taurus	109	1	1	3	9	24	71	Scorpio	41	1	3	6	14	5	15
Gemini	94	1	2	4	8	13	66	Sagittarius	48	C	1	5	9	11	22
Cancer	75	C	C	0	6	8	61	Capricornus	58	0	0	2	5	9	42
Leo	91	2	2	6	13	11	47	Aquarius	93	0	0	4	7	8	4
Virgo	93	1	C	5	1	24	52	Pisces	110	0	0	1	7	8	74

835. NOR-

835.

NORTHERN CONSTELLATIONS.

Names	Number	Magnitudes						Names	Number	Magnitudes					
		I	II	III	IV	V	VI			I	II	III	IV	V	VI
Little Bear	12	0	2	1	4	3	2	Camelopardalus	23	0	0	0	5	7	11
Great Bear	105	0	5	5	16	30	49	Serpent	50	0	1	7	6	5	31
Dragon	49	0	1	7	8	10	23	Serpentarius	67	0	1	6	12	17	31
Cepheus	40	0	0	3	7	10	20	Sobieski's Shield	8	0	0	0	2	3	3
Greyhounds	24	0	0	0	1	7	16	Eagle	29	1	0	5	1	4	18
Bootes	53	1	0	7	10	12	23	Antinous	34	0	0	5	2	7	20
Mons Mænalus	11	0	0	0	1	0	10	Dolphin	18	0	0	6	0	2	10
Berenice's Hair	24	0	0	0	6	8	10	Colt	12	0	0	0	4	1	7
Charles's Heart	3	0	1	0	0	0	2	Arrow	13	0	0	0	4	1	8
Northern Crown	11	0	1	0	6	3	1	Andromeda	66	0	3	2	10	16	35
Hercules	92	0	0	12	2	28	40	Perseus	67	1	1	5	10	14	36
Cerberus	9	0	0	0	3	1	5	Pegasus	81	0	3	4	9	11	54
Harp	24	1	0	3	2	8	10	Auriga	46	1	1	1	9	9	25
Swan	73	0	1	5	5	20	32	Lynx	55	0	0	1	8	21	25
Fox	29	0	0	0	6	11	12	Little Lion	20	0	0	1	6	5	8
Goose	10	0	0	0	0	2	8	Great Triangle	10	0	0	0	3	1	6
Lizard	12	0	0	0	3	5	4	Little Triangle	5	0	0	0	0	0	5
Cassiopeia	52	0	0	0	7	7	33	Musca	6	0	0	1	2	2	1

836.

SOUTHERN CONSTELLATIONS.

Names	Number	Magnitudes						Names	Number	Magnitudes					
		I	II	III	IV	V	VI			I	II	III	IV	V	VI
Whale	80	0	2	8	13	10	47	Peacock	14	0	1	3	5	4	1
Eridanus	72	1	0	10	24	19	18	Southern Crown	12	0	0	0	1	3	8
Phenix	13	0	1	5	5	0	2	Crane	14	0	2	1	2	9	0
American Goose	9	0	0	4	2	3	0	Southern Fish	15	1	0	2	9	2	1
Orion	93	2	4	3	19	15	50	Hare	25	0	0	4	9	4	8
Monoceros	32	0	0	1	10	10	11	Noah's Dove	10	0	2	0	1	6	1
Little Dog	14	1	0	1	0	2	10	Charles's Oak	13	0	1	2	6	4	0
Hydra	53	0	1	3	14	13	22	Ship Argo	48	1	6	1	13	14	3
Sextan's Urania	4	0	0	0	0	0	4	Great Dog	29	1	5	1	4	10	8
Cup	11	0	0	0	8	2	1	Bee	4	0	0	0	2	2	0
Crow	8	0	0	2	2	2	2	Swallow	11	0	0	0	4	3	4
Centaur	36	2	6	6	14	8	0	Indus	12	0	0	0	4	6	2
Wolf	36	0	0	3	6	18	9	Chamelion	10	0	0	0	0	9	1
Altar	9	0	0	1	6	1	1	Flying Fish	7	0	0	0	0	6	1
Southern Triangle	5	0	1	2	0	2	0	Sword Fish	7	0	0	2	2	1	2

Constellations in the zodiac 12, contains
 Northern constellations 36, contains
 Southern constellations 32, contains

Number of stars in the 80 constellations

Stars	I	II	III	IV	V	VI
894	6	12	38	100	169	569
1243	5	21	92	200	291	634
706	9	32	75	185	188	217
2843	20	55	205	485	648	1420

As these stars are found not to alter their situation in respect to one another, they serve astronomers as fixed points, whereby the motions of other

other bodies may be compared; and therefore their relative positions have with great care been sought after for many ages, and catalogues of the observations have from time to time been published by those who have been at the pains to make them: Among these catalogues, the most copious, and, as generally esteemed, the best, is that called the *Historia Caelestis* of FLAMSTEED.

837. The positions of the stars being obtained, their relative places may be delineated on a sphere or plane, as well as the parts of the earth, by knowing their latitudes and longitudes (662); and thus are the maps or charts of the heavens made, and the constellations drawn, inclosing their respective stars. There are two maps, usually called Celestial hemispheres, made by *Senex*, and *others*, that are to be had in many places; by the help whereof, a person may readily become acquainted with the positions and names of some of the principal fixed stars, thus.

On a clear night, let these prints be laid so as to correspond to the north and south parts of the heavens; then the observer sometimes looking on the stars and then on the hemispheres, will with a little practice know some of the stars in the heavens, whose like positions and names he has observed on the prints.

838. The WANDERING STARS are those bodies within our system or celestial sphere, which revolve round the sun; they appear luminous or bright, only by reflecting the light they receive from the sun, and are of three kinds, namely, *primary planets*, *secondary planets*, and *comets*.

839. The PRIMARY PLANETS are those bodies which, in revolving round the sun, respect him only as the centre of their courses; wherein their motions are regularly performed in tracks or paths, that are, by observations found to be, nearly circular and concentric to one another.

840. A SECONDARY PLANET, commonly called a SATELLITE or MOON, is a body which, while it is carried round the sun, does also revolve round a primary planet which it respects as its centre.

841. COMETS, vulgarly called *Blazing stars*, are bodies which also revolve round the sun; probably in as regular order as the planets: Their appearances near the sun are very seldom; they are in number about 40, and their tracks or courses pass among the paths of the planets in a great variety of directions.

842. The ORBIT of a planet or comet is that track or path along which it moves.

There are six *primary planets*, which reckoned in order from the sun, their names and marks are MERCURY ☿, VENUS ♀, the EARTH ♂ or ⊕, MARS ♂, JUPITER ♃, SATURN ♄.

Mars, *Jupiter*, and *Saturn*, are called SUPERIOR PLANETS, as their orbits include that of the *Earth*: But *Venus* and *Mercury*, whose orbits are contained within the *Earth's*, are called INFERIOR PLANETS.

843. There are found, by observations with telescopes, to be ten *secondary planets*; the *Earth* is attended by one called the MOON, *Jupiter* by four, and *Saturn* by five.

Saturn is also observed to have a kind of circle, called his RING, which furrounds the planet at some distance from his surface: And *Jupiter*

Jupiter has certain appearances, which seem like zones or girdles round him; and these are called JUPITER'S BELTS.

Every primary planet is supposed to have two motions, namely, *annual* and *diurnal*.

844. The ANNUAL MOTION of a planet is that whereby the planet is carried in its orbit round the sun; which in every one is found by observation to be from west to east.

This motion is discovered by the planets changing their places in the celestial sphere, upon whose surface they appear to move among the fixed stars; and in certain times return to the same stars from whence they were seen to depart; and so on continually.

845. The DIURNAL MOTION of a planet is that by which it turns or spins about its axis, and is also from west to east.

This motion is discovered by the spots that are seen by telescopes on the surfaces of the planets: These spots appear first on the eastern margin, or side of the planet, and gradually move from thence across it, till they disappear on the western side, or *limb*; and after a certain time appear again on the eastern side, and so on.

846. Each planet is observed always to pass thro' the constellations *Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricornus, Aquarius, Pisces*; and it also appears that every one has a track peculiar to itself; whereby, the paths of the six planets form among the stars a kind of road, which is called the ZODIAC, the middle path whereof, called the ECLIPTIC, is the orbit described by the Earth, with which the orbits of the other planets are compared.

As the ecliptic runs thro' twelve constellations, it is supposed to be divided into twelve equal parts, of 30 degrees each, called signs, having the same names with the twelve constellations they run thro'.

847. The *Plane of the Ecliptic* is supposed to divide the celestial sphere into two equal parts, called the *northern* and *southern celestial hemispheres*; and any body in either of these hemispheres is said to have north or south latitude, according to the hemisphere it is in: So that the LATITUDE of a celestial object is its nearest distance from the ecliptic.

The Planes of the other five orbits are observed to lie partly in the northern and partly in the southern hemisphere; so that every one cuts the ecliptic in two opposite points, called NODES: One called the ASCENDING NODE, is that thro' which the planet passes when it moves out of the southern into the northern hemisphere; and the other called the DESCENDING NODE, is that thro' which the planet must pass in going out of the northern into the southern hemisphere.

The right line joining the two nodes of any planet, is called the LINE OF THE NODES.

848. The names to most of the constellations were given by the ancient astronomers, who reckoned that star in Aries, now marked γ , (according to *Bayer*) to be the first point in the ecliptic, this star being next the sun when he entered the *Vernal equinox* (633); and at that time each constellation was in the sign by which it was called: But observations shew, that the point marked in the heavens by the *Vernal equinox* has been constantly going backwards by a small quantity every year, whereby the stars appear to have advanced as much forwards; so that

that the constellation *Aries* is now almost removed into the sign *Taurus*; the said first star γ being got almost 30 degrees forwards from the equinox; which difference is called the PRECESSION OF THE EQUINOXES, whereof the yearly alteration is about 50 seconds of a degree, or about a degree in 72 years.

849. It was said in art. 839, that the planets revolved round the sun in orbits nearly circular and concentric; for the several phænomena arising from their motions shew they are not strictly so; and the only curve they can move in, to reconcile all the various appearances, is found to be an ELLIPSIS (vulgarly called an *oval*): Which is a curve that cannot like the circle be described on a plane from one centre, but requires two; about which two, if there be moved a stretched doubled thread, whereof each of the two folds is longer than the distance of the two centres, the bight or loop of the thread will by such motion describe the curve called the ELLIPSIS. The closer the centre pins are brought together, the nearer will the ellipse approach to a circle; so that when the centres coincide, then a circle will be described by the motion of the thread.

The two centres of the ellipse are called Focus's: A right line drawn thro' the two focus's, and terminated at each end by the curve, is called the *Transverse*, or *longer diameter*: Another right line, drawn at right angles to the *transverse diameter* thro' its middle, and terminated by the curve, is called the *Conjugate diameter*: The intersection of the two diameters is called the *Centre of the ellipse*; and the distance of this centre from either focus is called the EXCENTRICITY.

850. All the planets have one common focus, in which the sun is placed: For as no other supposition can solve all the appearances that are observed in the motion of the planets, and as it also agrees with the strictest physical and mathematical reasoning; therefore it is now received as an elementary principle.

851. The Line of the nodes of every planet passes thro' the sun: For as the motion of every planet is in a plane passing thro' the sun, consequently the intersections of these planes, that is the lines of the nodes, must also pass thro' the sun.

852. All the planets in their revolutions are sometimes nearer, sometimes farther from the sun: This is a consequence of the sun not being placed in the centre of each orbit, and they being ellipses.

853. The APHELION, or SUPERIOR APSIS, is that point of the orbit where the planet is farthest from the sun: And the PERIHELION, or INFERIOR APSIS, is that point where it is nearest the sun: And the transverse diameter of the orbit, or the line joining the two apses, is called the LINE OF THE APSSES, or APSIDES.

854. The planets move faster as they approach the sun, or come nearer to the perihelion, and slower as they recede from the sun, or come nearer the aphelion: This is not only a consequence from the nature of the planets motions about the sun, but is confirmed by all good observations.

855. If a right line drawn from the sun thro' any planet, and called by some the *Vector radius*, be supposed to revolve round the sun with the planet, then this line will describe or pass thro' every part of the plane

plane of the orbit; so that the *vector radius* may be said to describe the area of the orbit.

856. There are two *chief laws* observed in the solar system which regulate the motions of all the planets; namely,

I. *The planets describe equal areas in equal times*: That is, in equal portions of time the *vector radius* describes equal areas or portions of the space contained within the planet's orbit.

II. *The squares of the periodical times of the planets are as the cubes of their mean distances from the sun*: That is, the square of the time which a planet A takes to revolve in its orbit, is to the square of the time taken by any other planet B to run thro' its orbit; so is the cube of the mean distance of A from the sun, to the cube of the mean distance of B from the sun.

857. The MEAN DISTANCE of a planet from the sun, is its distance from him when the planet is at either extremity of the conjugate diameter; and is equal to half the transverse diameter.

858. The foregoing laws are the two famous laws of KEPLER, a great astronomer, who flourished in Germany about the beginning of the 17th century, and who deduced them from a multitude of observations: But the first who demonstrated these laws was the incomparable Sir ISAAC NEWTON.

By the second law, the relative distances of the planets from the sun are known; and was the real distance of any one known, the absolute distances of all the others would thereby be obtained.

859. Every thing already said of the planets, is found in a great measure to be applicable also to the comets, as well from the observations that have been made of them, as from the physical and mathematical considerations of their motions.

860. Was the motions of the planets to be observed from the sun, each of them would be ever seen to move the same way, though with different velocities; those nearer the sun running their courses thro' the zodiac in less time than those at greater distances: And hence it would happen, that some of them overtaking of the others would, in their passing by them, appear to be sometimes above, sometimes below, and sometimes as if they touched one another, according to the parts of the orbits those planets happened to be in with respect to their nodes.

861. When two planets are seen together in the same sign equally advanced therein, they are said to be in CONJUNCTION: But when they are in direct opposite parts of the zodiac, they are said to be in OPPOSITION.

862. As the planes of the orbits are inclined to one another, therefore when two planets happen to be in conjunction at the time they come near their nodes, they would be seen from the sun apparently to touch one another; and the farthest of those planets from the sun would for some time have its light obscured by the nearer planet, it being then directly between the sun and the remoter planet. Also, should an opposition of two planets happen near their nodes, the sun being then directly

rectly between them, would hide the light of one from the other. And these obscurations, or interceptions of the light of the planets one from the other, are called ECLIPSES.

863. The place that any planet appears to occupy in the celestial sphere, when seen by an observer supposed in the sun, is called its *Heliocentric place*: And indeed all celestial appearances, as seen from the sun, are called *Heliocentric phenomena*.

864. The following table exhibits at once some of the most material conclusions that have been deduced from the observations hitherto made, the mean distance of the earth from the sun being reckoned at 1000.

TABLE OF THE SOLAR SYSTEM.

Planets Names	Marks	Mean dist. compared to that of the earth	Eccentricity	Inclination of the orbits to the eclipt.	Times of the periodical revolutions	Diurnal ro- tations	True diameters	N ^o of moons
Mercury	☿	387	81	6° 52'	3 m. or 87 ^d 23 ^h	uncertain	0,32	0
Venus	♀	724	5	3 23	8 m. or 224 17	23 ^h 0 ^m 0 ^s	0,87	0
Earth	♁	1000	17	0 00	1 y. or 365 6	23 56 4	1,00	1
Mars	♂	1524	141	1 52	2 y. or 686 23	24 40 0	0,73	0
Jupiter	♃	5201	250	1 20	12 y. or 4232 12	9 56 0	7,70	4
Saturn	♄	9538	543	2 20	30 y. or 10759 8	uncertain	4,19	5

865. By all the observations made on the secondary planets, it appears,

1st. That the satellites revolve round their superior planets from west to east, in curve lined orbits like ellipses, the primary planet being in the focus, and one of the orbit's diameters directed towards the sun.

2d. The planes of the satellites orbits are inclined to the plane of their respective planet's orbit.

3d. That, like the primary planets, they describe equal areas in equal times; and the squares of the times of their revolutions are as the cubes of the distances from their primary planet.

As the orbits of the secondary planets surround their primary, therefore in every revolution of a moon it must, in respect to its primary and the sun, be twice in conjunction, and may be eclipsed in either case, according as the moon's nodes happen to be posited at those times: For the planes of a moon's orbit is inclined to that of its primary, and so makes two nodes; in whose line, whenever the sun, planet and moon happen at the same time, there must be an eclipse; which would occur at every conjunction, but for the inclination of the orbit.

One of the conjunctions, namely that made by the moon's going beyond the primary, is called the SUPERIOR CONJUNCTION; and the other, made by the satellite on that side of the planet next the sun, is called the INFERIOR CONJUNCTION.

The following table shews the times taken by each satellite in its revolution ; and also its distance from the primary in semidiameters thereof.

866.

	I	II	III	IV	V
	d h m	d h m	d h m	d h m	d h m
Saturn's fatellites	1 21 18 $\frac{1}{2}$	2 17 41 $\frac{1}{3}$	4 12 25 $\frac{1}{5}$	15 22 41 $\frac{1}{4}$	79 7 48
Dist. from Saturn	8 $\frac{7}{8}$ f. diam.	11 $\frac{1}{4}$ f. diam.	15 f. diam.	36 f. diam.	108 f. diam.
Jupiter's fatellites	1 18 28 $\frac{3}{5}$	3 13 18 $\frac{5}{8}$	7 3 59 $\frac{2}{3}$	16 18 5 $\frac{1}{10}$	
Dist. from Jupiter	5 $\frac{2}{3}$ f. diam.	9 f. diam.	14 $\frac{1}{3}$ f. diam.	25 $\frac{1}{3}$ f. diam.	
The Moon	29 12 44 $\frac{1}{2}$	and is dist. from the earth 60 $\frac{1}{5}$ semidiameters.			

867. *Of the figure and light of the Planets.*

That the sun and planets are spherical bodies, is evident from all the observations that have been made of them ; and that the earth is of like figure, is not only deduced by analogy, but sufficiently confirmed by observation (553, 713). Now altho' astronomers generally say that the planets are spherical, yet they do not mean a Geometrical sphere, but a figure called an oblate spheroid, which is something like the figure that a flexible sphere would be formed into by gently pressing it at its poles : Observations have determined this in Jupiter, and it is known that the earth is of this figure both from observations and actual mensuration.

That the planets must have this oblate figure, is evident from this consideration ; that as they are of matter, and violently whirled on their axes, all the parts would endeavour to fly off, like water from a trundled mop ; those in the equator moving swiftest, have the greatest tendency to depart : And altho' the parts are retained in the sphere by the superior force of gravity, yet the equatorial diameters will be somewhat increased, and the polar lessened.

868. The planets are all opaque or dark bodies, and consequently shine only by the light they receive from the sun : This is known, by observing that those bodies are not visible when they are in such parts of their orbits as are between the sun and earth, or partly so. Now as all the planets appear with a strong light, therefore the rays they receive from the sun must convey to them a degree of warmth proportional in some measure to their distance, which is reciprocally as the squares of the distances, and this must be readily inferred from the heat which the inhabitants of the earth receive from the sun.

869. As a planet revolves on its axis, every part of its surface will be turned towards the sun, and thereby enjoy its light and heat.

S E C T I O N II.

Of Terrestrial Astronomy.

870. The motions described in the preceding section were such as would appear to an observer viewing the heavens from the sun: But was he placed in one of the planets, suppose the Earth, and there observed the motions of the rest, it would appear to him that the sun, and other planets, revolved round the earth as a centre; but the sun would be the only one that moved uniformly the same way: For the other planets would seem to move sometimes from west to east, and then to stand still; then they would seem to run from east to west, and after standing some time they would again move from west to east, and so on continually.

871. The place in the celestial sphere that any planet appears in, when seen from the centre of the earth, is called its *GEOCENTRIC PLACE*.

872. The *DIRECT MOTION* of a planet is that by which it appears to move from west to east, and this motion is said to be *according to the order of the signs*, or in *consequentia*. When the planet appears to stand still, it is said to be *STATIONARY*; and when its motion is apparently from east to west, it is then called *RETROGRADE*, or has a motion in *antecedentia*, or contrary to the order of the signs. These different appearances follow partly in consequence of the observer being himself in motion while he is viewing those of the planets, and partly because he is not in the centre of the motions which he observes.

873. *The phenomena of the Inferior planets.* See Fig. 1. Plate IX.

Let ABC represent an arc of the celestial sphere; EOP the earth's orbit; LNIG the orbit of an inferior planet, as of *Venus*; and s the sun: Let the earth at first be supposed to stand still in its orbit at E: Now it is evident that the sun will appear at the point B, and the planet always within the arc AC. Whilst the planet moves in its orbit from I thro' Q to N, it will seem to move from B to A in *consequentia*: But passing from N to L, it will seem to an eye at E to return back from A to B, or be *retrograde*. While the planet is at or near the point N, being betwixt moving on from B to A, and returning from A to B, and moving as it were in a right line towards the earth, it will for some time seem to stand still near A, that is, to be *stationary*.

874. When the planet is in that part of her orbit N or G, which is contiguous to the tangent EA or EC, she will then appear at A or C, her greatest distance from the sun, and is said to be in her greatest *ELONGATION*: This elongation is measured by the angle SEG. The more distant a planet is from the sun, the greater will its angle of elongation be; that of *Venus* is about 48 degrees, and of *Mercury* about 28 degrees.

In the space of a revolution, the two inferior planets will with respect to the earth, undergo two conjunctions; one when it is beyond the sun at I, the other when it is at L between the sun and the earth; the former is called the *superior*, and the latter the *inferior conjunction*.

875.

875. Whilst Venus goes from her superior conjunction she appears in the arc AB, always to the eastward of the sun, and therefore sets some time after the sun, and is called the *evening star*: But during the time she is going from her inferior to her superior conjunction, she will be seen somewhere in the arc BC to the westward of the sun, and so will set before him in the evening, and rise before him in the morning; hence she is called the *morning star*.

Hitherto the planet only has been supposed to move while the earth stood still: But when both move, the foregoing phenomena will be much the same, only the planet will be more direct in the farthest part of the orbit, and less retrograde; the former arising from the sum of their motions, and the latter from the difference.

876. *Of the phenomena of the Superior planets.*

The *direct*, *stationary* and *retrograde* motions of the superior planets are explained much after the same manner as those of the inferior ones, but with these differences.

1st. The retrograde motions of the superior planets happen oftner the slower their motions are, as the retrogradations of the inferior planets happen oftner the swifter their angular motions: Because the retrograde motions of the superior planets depend upon the motions of the earth, but those of the inferior on their own angular motions. A superior planet is retrograde once in each revolution of the earth, an inferior one in every revolution of its own.

2d. The superior planets do not always accompany the sun as the inferior do, but are often in opposition; which necessarily follows from the orbit of the earth being included within the orbits of the superior planets.

877. *Of the apparent Motion of the Sun.*

As a spectator in the sun would see the earth revolve thro' the signs in the ecliptic, so to a spectator in the earth, the sun apparently revolves the same way, but is always in the opposite point of the ecliptic: (For it is well known to every one, especially to those who use the sea, that fixed objects appear to change their place by the motion of the observer): So that the heliocentric place of the earth, and the geocentric place of the sun, are always in direct opposite points of the ecliptic.

Now altho' it is the motion of the earth that really causes a great variety in the apparent motions of the other planets; yet as that of the sun being known, gives that of the earth, therefore it is that astronomers speak of the motion of the sun, and in their computations use the quantities of those motions as if they were real.

878. Beside the various appearances that arise from the annual motion of the earth, there are many resulting from its diurnal motion: For that the earth has a daily motion round its axis, must necessarily be inferred from the most strict reasoning on the motions of the planets: Besides the notion of bodies, so immensely distant as are the stars, really revolving round the earth in 24 hours, is now treated as a great ab-

furdity by every one who has rightly considered these things: However, as the motions are apparent, and the speaking of them as real is customary, and no way affecting the conclusions; therefore astronomers treat of these motions as they appear.

879. Any sphere revolving as on an axis, must have two points on its surface, at the extremities of its axis, that do not revolve at all; these points, with respect to the earth, are called its *poles*.

880. By the earth's rotation on its axis from west to east in a day, the surface of the celestial sphere appears to move from east to west in the same time; and all the celestial objects appear to describe circles in the heavens, which are greater or less, according as they are farther from, or nearer to, the apparent centres of those motions: For there are two points in the heavens which are apparently fixed, and the nearer any stars are to these points, the slower are their motions. These points are called the **CELESTIAL POLES**; the right line joining them is called the **AXIS OF THE SPHERE**, and passes thro' the poles of the earth. That circle in the heavens, equally distant from the poles of the celestial sphere, is called the **EQUINOCTIAL**; the corresponding circle on the earth is called the **EQUATOR**, which is equally distant from both the poles.

881. As the sun's rays falling on any sphere enlightens one half of its surface; therefore one half of the earth is always illuminated at once, and consequently the enlightned part is bounded by a great circle, which may be called the **TERMINATOR**, from its property of terminating, or bounding the verges of light and darkness: Now by the rotation of the earth on its axis once in 24 hours, there will be a constant succession of light in all parts as they are turned towards the sun, and of darkness in these parts as they move out of his rays; and hence arise the vicissitudes of **DAY** and **NIGHT**.

882. Did the plane of the equator coincide with the plane of the ecliptic, and the axis of the earth stand perpendicular to it, the terminator would always pass thro' the poles of the earth, and there would be a constant equality of day and night in every part of its surface, except at the two poles, where there would be constant day: But the contrary of this is known to every one, and observations shew that the earth's axis is inclined to the plane of the ecliptic in an angle of about $66\frac{1}{2}$ degrees; therefore the poles of the ecliptic and equator are about $23\frac{1}{2}$ degrees distant from one another; consequently the ecliptic and equinoctial, which in the heavens intersect one another in the opposite points of Aries and Libra, make at those intersections angles of about $23\frac{1}{2}$ degrees (413): This angle is called the **OBLIQUITY OF THE ECLIPTIC**.

The axis of the earth being thus inclined to the plane of the ecliptic, and moving parallel to itself in all points of the **ANNUAL ORBIT**, or ecliptic, is the occasion of the inequalities of days and nights, and of the different seasons of the year; which two phenomena are explained as follows.

883. It must be observed, that the sun will appear to be vertical to that part of the earth, which is cut by a strait line joining the centres of the sun and earth.

884. Now when the earth is at ϖ , the sun appearing then in ϖ , will be vertical to that point of the terrestrial ecliptic, it lying in the right line joining the centres of the sun and earth: And this point being in the earth's northern hemisphere, all those who live therein will enjoy summer, or the hottest time of the year, the solar rays falling more copiously, and more perpendicularly upon their hemisphere at that time. Fig. 3. Pl. IX.

885. At the same time the inhabitants of the southern hemisphere will have winter, the rays of the sun falling more obliquely, and in less quantity on them, and consequently affording them less heat.

886. Again, the inhabitants of the northern hemisphere will have their days longer than their nights, in proportion as they are more distant from the equator; whilst those who live under the equator will have an equal share of day and night all the year round: For in this position the *terminator*, which is always at right angles to the plane of the ecliptic, will pass $23\frac{1}{2}$ degrees beyond the north pole, and consequently will cut all the circles parallel to the equator, it meets with, into two unequal parts, the greater portion being in the enlightened hemisphere: But the terminator being a great circle, will cut the equator into two equal parts; therefore half the equator is always illuminated.

887. Hence it necessarily follows, that those who live under the equator will have their days and nights equal: The inhabitants of the north *temperate zone* (567) will have their nights shorter than their days; while those who live in the north *frigid zone* enjoy perpetual day. In the mean time, those who live in the southern hemisphere have their nights longer than their days, in proportion as they approach nearer the south pole; and those contained within the south *frigid zone* will have constant night.

888. Suppose the earth now to move in its orbit from ϖ thro' the signs φ , χ to $\vargamma$, and the sun will seem to run thro' the signs Ω , π to ϵ ; and this will be the place of the sun in autumn.

While the earth is in $\vargamma$, the days and nights will be equal in both hemispheres, and the season is a medium between summer and winter: For at that time the sun will appear vertical to the equator, because a right line joining the centres of the sun and earth will then cut the surface of the earth in the equator; so that the *terminator*, whose plane is always at right angles to the said line, will pass thro' the poles; consequently, all the earth will then have an equal share of day and night. And because the rays of the sun then fall perpendicularly upon the axis of the earth, it will follow, that they must fall with an equal obliquity, and in equal number upon either hemisphere; therefore they must enjoy an equal degree of heat and cold.

Now suppose the earth to move from $\vargamma$ to ϖ , and the sun will seem to move from ϵ to ϖ , where it will be in its nearest approach to the south pole; and at this time of the year it will be winter in the northern hemisphere. For to this hemisphere the like phenomena will now happen, as did before to the southern, when the earth was in ϖ ; and by a parity of reason, when the earth has got as far as ϵ , and the sun is apparently in φ , the northern hemisphere will enjoy spring, and the southern will have autumn.

889. The four points of the ecliptic, wherein the earth has been considered in summer, autumn, winter and spring, are called the four **CARDINAL POINTS**: ♈ and ♎ are called **SOLSTITIAL POINTS**; ♊ and ♋, **EQUINOCTIAL POINTS**.

890. The first point of Cancer is called the **SUMMER SOLSTICE**; because when the sun enters it, which is about the 21st of June, he is then got to his greatest extent northwards, and being about to return towards the equator, he seems for a day or two to be at a stand. And for the same reason, the first point of Capricorn, which the sun enters about the 21st of December, is called the **WINTER SOLSTICE**, with respect to the northern hemisphere.

891. The first points of Aries and Libra are called the **VERNAL** and **AUTUMNAL EQUINOCTIAL POINTS**, from the equality of days and nights all over the surface of the earth, when the sun enters those points.

892. *Of the Rising and Setting of the Stars.*

There is only one half of the celestial sphere visible at one time to any observer on the surface of the earth, the other half being hid by the earth itself: Now the apparent plane on which the observer stands, seems to be extended to, and cuts the heavens, and there marks out a circle that divides the visible from the invisible hemisphere; this circle is called the **HORIZON**, above which all the celestial motions are seen. When this horizon is a great circle of the celestial sphere, it is called the **RATIONAL HORIZON**: But when by the particular situation of the observer, he sees more or less than half the celestial sphere, then the circle bounding his view is called the **SENSIBLE HORIZON**.

The horizon is one of the most useful circles in astronomy, for to this circle, which is the only apparent one, almost all the celestial motions are referred. It is the common termination of day and night; it marks out the times of the rising and setting of the sun and stars, and many other particulars, of which hereafter.

893. *Of Parallaxes.*

The parallax of any object is the difference between the places it is referred to in the celestial sphere, when seen at the same time from two different places within that sphere: Or, it is the angle under which any two places in the inferior orbits are seen from a superior planet, or even from the fixed stars: But the parallaxes used by astronomers are those arising from seeing the object from the centres of the earth and sun; from the surface and centre of the earth; and from all three compounded.

894. The difference between the heliocentric and geocentric place of a planet, is called the parallax of the annual orbit (namely that of the earth); that is, the angle at any planet, subtended by the distance between the sun and earth, is called the parallax of the earth's, or annual orbit.

895. The difference in the two longitudes is called the parallax of longitude; and that of the two latitudes, is called the parallax of latitude.

896. In the *SIZYGIES*, that is, in the *oppositions* or *conjunctions*, the sun and planet being either equally advanced in the same sign, or in like places in opposite signs, the parallax of latitude is then greatest.

897. And when the planet is in its *QUADRATURES*, that is, when it is 90 degrees distant from the sun, the parallax of longitude is then the greatest.

898. To explain the parallaxes which respect the earth only. Fig. 2. Plate IX.

Let *HSW* represent the earth, where *T* is the centre; *ORG* part of the moon's orbit, *Pr g* part of a planet's orbit, and *z a A* part of a great circle in the celestial sphere: Now to a spectator at *s* upon the surface of the earth, let the moon appear in *G*, that is, in the sensible horizon of *s*, and it will be referred to *A*; but if viewed from the centre *T*, it will be referred to the point *D*, which is its true place.

The arc *AD* will be the moon's parallax; the angle *SGT* the parallactic angle: Or the parallax is expressed by the angle under which the semidiameter *TS* of the earth is seen from the moon.

If the parallax be considered with respect to different planets, it will be greater or less, as those objects are more or less distant from the earth. Thus the parallax *AD* of *G* is greater than the parallax *A d* of *g*. If it be considered with respect to the same planet, it is evident that the horizontal parallax (or the parallax when the object is in the horizon) is greatest of all; and diminishes gradually as the body rises above the horizon, until it comes to the zenith, where the parallax vanishes, or becomes equal to nothing. Thus *AD* and *A d*, the horizontal parallaxes of *G* and *g*, are greater than *a B* and *a b*, the parallaxes of *R* and *r*; and the objects *O* or *P*, seen from *s* or *T*, appear in the same place *z*, or the zenith.

899. By knowing the parallax of any celestial object, its distance from the centre of the earth may be easily obtained by Trigonometry. Thus if the distance of *G* from *T* be sought; in the triangle *STG*, the side *ST* being known, and the angle *SGT* determined by observation, the side *TG* is thence known.

The parallax of the moon may be determined by two persons observing her from different stations at the same time, she being vertical to the one, and horizontal to the other; and it is generally concluded to be about 57 minutes of a degree; consequently her distance *TG* is about 60 semidiameters of the earth, or 60 times *TS*.

But the parallax the most wanted, is that of the sun, which if well obtained would give the distance of all the rest; and from a transit of Venus over the sun's disk or face, which will happen on the 26th of May, 1761, the distance of the centres of the sun and earth may be determined to within a 500th part of the whole, as has been shewn by the excellent Dr. Halley.

900. *Of the Measure of the Earth.*

The relative distances of the planets are discovered by the 2d *Keplerian law*, and their relative magnitudes are gathered from the angles which they appear under (when viewed with very accurate instruments)

com-

compounded with their distances : Now as these distances and magnitudes can by means of the parallaxes be compared with the diameter of the earth, consequently this diameter being accurately known, would serve as a measure wherewith the magnitudes and distances of all the other planets might be compared.

To find the measure of the earth is a problem of such importance in astronomy, that it has been attempted by some of the most considerable men in almost all the preceding ages : But its solution was not brought to any degree of accuracy till the year 1635, when it was happily perfected by our countryman RICHARD NORWOOD, an eminent mathematician at that time. The principle he proceeded upon was, that as 360 degrees were contained in every great circle, both of the celestial sphere and of the earth, and that these circles being considered as concentric to the centre of the earth ; therefore was the measure of the length known on a great circle of the earth, corresponding to one or more degrees of a great circle of the heavens, then, by analogy, the whole circumference of a great circle of the earth would be known in that measure, and consequently its diameter would be obtained. (271)

NORWOOD solved this problem in the following manner : He chose two distant places which were known to lie nearly north and south one of the other, as London and York ; and by the method of Traverse sailing he found their difference of latitude, or the distance between the parallels of latitude passing thro' those places ; or which is the same thing, the length of that arc of the terrestrial meridian : He also with a good instrument found the distance between the zeniths of those places, and consequently he thence knew the quantity of the celestial arc answering to the measured terrestrial one : Then saying, As that celestial arc, is to a great circle of the celestial sphere, or 360 degrees ; so is the arc of the terrestrial great circle measured in feet, to the circumference of a great circle on the earth, in feet measure.

And thus he found that about $69\frac{1}{2}$ English miles answered to one degree ; whereby the circumference of the earth appears to be 25020 miles ; and its diameter about 8000 miles.

901.

Of the Moon.

The moon revolves in her orbit from west to east round the earth, and with it is carried perpetually thro' the annual orbit round the sun, making in the space of one year 13 *periodical*, and 12 *synodical revolutions*.

902. A PERIODICAL MONTH, OR REVOLUTION, is the time the moon takes up in revolving from one point of her orbit to the same point again, and consists of 27 d. 7 h. 43 m.

903. A SYNODICAL MONTH, OR REVOLUTION, is the time the moon spends in passing from one conjunction with the sun to another, which is 29 d. 12 h. 44 m. ; being 2 d. 5 h. 1 m. longer than the *Periodical month* : For whilst the moon is passing from her former conjunction with the sun round to it again, the earth has proceeded forwards in its annual course, as it were leaving the moon behind it ; so that, in order

order to compleat her next conjunction with the sun, she must not only come round to her former point again, but also go beyond it.

904. Besides this monthly motion of the moon round the earth, she has also a motion round her axis, which is performed exactly in the same time with her periodical revolution: From whence it comes to pass, that the same face of the moon is always turned towards the earth, her diurnal motion turning just as much of her face to us, as her periodical from us.

905. Tho' the same side of the moon be ever turned towards us, yet it is not always visible, but daily seems to put on different appearances, called PHASES: For the moon being an opaque body like the rest of the planets, it borrows its light from the sun, having always one hemisphere enlightned by the solar rays.

When the enlightned hemisphere is wholly turned from the earth, as at her change, the planet then being betwixt us and the sun, the moon's whole enlightned face or *disk* must needs be invisible to the earth. When passing from this state, and turning some little portion of the illuminated half to us, it must appear horned, the CUSPS or points being turned from the sun towards the east. When the moon is in her quadratures, or at 90 degrees from the sun, then half the illuminated face becomes visible: She afterwards continues to shew more than half the enlightned disk, until she comes in opposition to the sun, when the whole of the illumined orb is presented to us; from whence receding, she must put on the like phases as before, but in an inverse order, the cusps being now turned towards the west.

906. *Of solar and lunar Eclipses.*

Eclipses of the sun and moon can only happen about the times of the conjunctions and oppositions: Those of the sun fall out at the conjunctions, when the moon intercepts the light of the sun from the earth; and those of the moon occur in the oppositions, when the earth getting between the sun and moon, the latter loses her light during the time of that interposition.

The reason why there is not an eclipse in every syzigie is, because of the inclination of the plane of the moon's orbit to that of the ecliptic, which is about $5^{\circ} 18'$: For it is certain, that unless the sun, earth and moon be all in the plane of the ecliptic, or nearly so, the shadows of the earth and moon can never fall on one another, but must be directed either above or below. Now they can never be in the same plane, and in one right line, but when the moon is in her nodes, and the nodes and sun's centre are in the same right line.

907. The solar and lunar eclipses do not happen every year in the same places of the zodiac, but in succeeding years, they fall in places gradually removed backwards, or towards the antecedent signs: For since the nodes are found to go continually backwards, the eclipses must also observe the same order.

908. Eclipses of the moon are either *total* or *partial*: The partial are either *central*; that is, when the node happens to be exactly in the centre of the shadow; or *excentrical*, when the node happens to be on either side the centre, within or without the shadow. Now the longer

the duration is of a partial eclipse, so much the greater is that part of the moon which enters into the shadow of the earth.

909. Hence it is usual to conceive the moon's diameter as divided into 12 parts, called DIGITS, by which the greatness of partial eclipses are measured, they being said to be of so many digits as there are parts covered by the earth's shadow: Thus if 5 of the 12 parts are covered, it is called an eclipse of 5 digits.

910. As the planet Mars is never eclipsed by the Earth, it is plain the shadow of the latter does not reach so far as the orbit of the former, but tapers to a point at a less distance; and consequently the earth's shadow must be a cone, whose vertex is extended beyond the orbit of the moon. It naturally follows from hence, that the sun is a much larger body than the earth.

911. If a person was placed just at the vertex, or point of this shadow, he would see nothing of the sun but a small rim of light round his disk; and the farther the observer was removed from the vertex, the larger would the rim of light appear, and consequently the fewer rays would be intercepted by the opaque body, till at last it would appear only as a spot in the sun; in like manner as the planets Venus and Mercury appear when they are seen to pass over the sun's disk.

912. What has hitherto been said of the shadow, includes that of the atmosphere surrounding the earth; for in lunar eclipses, the shadow of the atmosphere is to be considered: And hence it is that the moon is visible in eclipses, the shadow cast by the atmosphere being not near so dark as that cast by the earth.

913. The moon always enters the western side of the shadow with her eastern limb, and quits it with her western limb; and in her approach to, and recess from the shadow, she must pass thro' a PENUMBRA, or imperfect shade, which is caused by the earth itself.

914. In the same manner, in which it has been shewn that the moon must come into the shadow of the atmosphere when she is at full, and at or near a node, it may also be shewn that her shadow must fall upon the earth at the time of new moon, provided she be in or near a node: But the penumbra of the moon's shadow is much more sensible in solar eclipses, than that accompanying the shadow of the earth in lunar ones.

915. It is observed, that to determinate parts of the earth, solar eclipses do not happen so oft as lunar ones, which is owing to the shadow of the moon being less than that of the earth; for the earth's shadow often covers all the moon, but that of the moon cannot cover all the earth; and as it sometimes falls on one part, sometimes on another, it causes solar eclipses in general to be more frequent than lunar ones; yet to any determinate place on the earth, there are more eclipses of the moon visible, than of the sun.

What has hitherto been said, may suffice to give beginners a general idea of the motions of the bodies in the solar system, and of some of the phenomena thence arising; those who desire to be farther acquainted with particulars, may find them fully treated of in M. le Caille's Elements of Astronomy, published in English a few years since.

SECTION

SECTION III.

The Astronomy of the sphere.

916. *Definitions and Principles.*

By the Astronomy of the sphere, is meant the finding, from proper things given, the measure of certain arcs and angles formed on the surfaces of the celestial and terrestrial spheres, by the apparent motions of the bodies which are seen in the heavens.

The surfaces of those spheres are supposed to be concentric to the centre of the earth, and to have correspondent circles described on both spheres.

917. GREAT CIRCLES are those which divide either sphere into two equal parts.

LESSER CIRCLES, those which divide either into unequal parts.

The POLES of a circle are those points on the sphere equally distant from that circle.

An AXIS is a right line supposed to connect the poles.

The CELESTIAL AXIS is that right line about which the heavens seem to revolve.

The NORTH and SOUTH POLES of the world are those two points where the axis cuts the celestial sphere.

918. The EQUINOCTIAL, or EQUATOR, is that great circle of the sphere equally distant from the poles of the world.

919. MERIDIANS, or HOUR CIRCLES, or CIRCLES OF RIGHT ASCENSION, or CIRCLES OF TERRESTRIAL LONGITUDE, are great circles perpendicular to the equator, and passing thro' the poles of the world.

920. The ECLIPTIC is a great circle inclined to the equator in an angle of about $23\frac{1}{2}^{\circ}$, and cutting it in two points diametrically opposite.

The ecliptic is supposed to be divided into 12 equal parts, called SIGNS, beginning from one of its intersections with the equator; each sign containing 30 degrees, named and noted thus.

<i>Aries</i>	<i>Taurus</i>	<i>Gemini</i>	<i>Cancer</i>	<i>Leo</i>	<i>Virgo</i>
♈	♉	♊	♋	♌	♍
<i>Libra</i>	<i>Scorpio</i>	<i>Sagittarius</i>	<i>Capricornus</i>	<i>Aquarius</i>	<i>Pisces</i>
♎	♏	♐	♑	♒	♓

921. The first six are called *northern*, and the latter six *southern* signs.

The CARDINAL POINTS of the ecliptic are the four first points of the signs ♈, ♋, ♎, ♑; those of ♈ and ♎ are called EQUINOCTIAL POINTS, and those of ♋ and ♑ are called SOLSTITIAL POINTS.

922. The EQUINOCTIAL COLURE is a meridian passing thro' the equinoctial points; and the SOLSTITIAL COLURE is another meridian passing thro' the *solstitial* points. The colures cut one another at right angles in the poles of the world.

923. CIRCLES OF CELESTIAL LONGITUDE are great circles perpendicular to the ecliptic.

924. The LATITUDE of any point in the heavens is an arc of a circle of longitude intercepted between that point and the ecliptic, and

is called north or south latitude, as the point is on the north or south side of the ecliptic.

925. PARALLELS OF CELESTIAL LATITUDE are small circles parallel to the ecliptic.

926. The LONGITUDE of any point in the heavens is an arc of the ecliptic intercepted between the first point of Aries and a circle of longitude passing thro' that point.

927. The RIGHT ASCENSION of any point of the celestial sphere is an arc of the equator, contained between the first point of Aries and a meridian passing thro' that point: Or, it is the angle formed by the equinoctial colure, and the meridian passing over that point.

928. The DECLINATION of any point, is an arc of a meridian contained between that point and the equinoctial: If the point is on the north side of the equinoctial, it is called *north declination*; but if on the south side, it is called *south declination*.

929. The OBLIQUITY OF THE ECLIPTIC is the angle made by the intersection of the equator and ecliptic, and is measured by the sun's greatest declination; which, according to the modern observations, is about $23^{\circ} 28\frac{1}{2}'$, but is usually reckoned at $23^{\circ} 29'$.

930. PARALLELS OF DECLINATION are small circles parallel to the equinoctial. The TROPIC OF CANCER is a parallel of declination at $23^{\circ} 29'$ distant from the equinoctial in the northern hemisphere; and the TROPIC OF CAPRICORN is that parallel of declination as far distant in the southern hemisphere.

931. The ARTIC POLAR CIRCLE is a parallel of declination at $23^{\circ} 29'$ distant from the north pole; and the ANTARTIC POLAR CIRCLE is that parallel of declination as far distant from the south pole.

932. The ZENITH is that point of the heavens directly over a place; and the NADIR is the point directly underneath.

933. The HORIZON is that great circle of the sphere which is equally distant from the *zenith* and *nadir* of any place, and divides the sphere into the upper and lower hemispheres.

934. The RISING of a celestial object, is when its centre appears in the eastern part of the horizon; and its SETTING, is when its centre disappears in the western quarters of the horizon.

935. AZIMUTH, or VERTICAL CIRCLES, are great circles perpendicular to the horizon, passing thro' its poles, which are the *zenith* and *nadir*.

936. The PRIME VERTICAL is that vertical circle which passes thro' the east and west points of the horizon, and is at right angles to the *meridian of the place*, which is a vertical circle passing thro' the north and south points of the horizon.

937. As the meridian of a place is called the *twelve o'clock hour circle*, so the hour circle at right angles to the meridian is called the *six o'clock hour circle*.

938. The AZIMUTH of any celestial object, is an angle at the zenith formed by the meridian of any place, and a vertical circle passing thro' that object when it is above the horizon: Or, it is measured by the arc of the horizon intercepted between those vertical circles.

939. The **AMPLITUDE** of any point in the heavens, is an arc of the horizon contained between that point at its rising or setting, and the intersection of the meridian of the place with the horizon: Or, it is the angle at the zenith included between the meridian of a place and a vertical circle passing thro' the point at its rising or setting.

940. The **ALTITUDE** of any point in the heavens, is an arc of a vertical circle intercepted between that point and the horizon.

941. The **ZENITH DISTANCE** of any object, is an arc of a vertical circle contained between that point and the zenith.

The altitude and zenith distance are complements one of the other.

942. The **MERIDIAN ALTITUDE**, or **MERIDIAN ZENITH DISTANCE**, is the altitude or zenith distance when the object is on the meridian of the place.

943. The **CULMINATING** of any celestial object, is the time it *transits*, or comes to the meridian.

944. **ALMICANTHERS**, or **PARALLELS OF ALTITUDE**, are small circles parallel to the horizon.

945. A **PARALLEL SPHERE**, is that position of the sphere wherein the circles, apparently described by the diurnal rotation, are parallel to the horizon; which can happen only at the poles.

946. A **RIGHT SPHERE**, is that wherein the diurnal motions are at right angles to the horizon: Thus it appears in all places under the equator.

947. An **OBLIQUE SPHERE** has all the diurnal motions oblique to the horizon: And thus the motions appear to all parts of the earth, except under the poles and equator.

948. **DIURNAL ARCS**, are those parts of the parallels of declination of celestial objects apparently described between the times of the rising and setting of those objects: And **NOCTURNAL ARCS**, are the parts of those parallels apparently described from the time of setting to the time of rising.

949. **SEMI-DIURNAL** and **SEMI-NOCTURNAL ARCS**, or the halves of diurnal and nocturnal arcs, are the parts of the parallels between the meridian and their intersection with the horizon. The corresponding part of the equator answering to the semi-diurnal arc, gives the time between noon and the rising or setting; and the equatorial part answering to the semi-nocturnal arc, shews the time between midnight and the time of setting or rising.

950. The **OBLIQUE ASCENSION** of any point in the heavens, is the degree of the equator which rises with that point in an oblique sphere: And the **OBLIQUE DESCENSION**, is the degree which sets with that point.

951. The **ASCENSIONAL DIFFERENCE** belonging to any celestial object, is the difference between its *right* and *oblique ascensions*. In the sun, it is the time that he rises or sets before or after the hour of six.

952. The **LATITUDE** of any place on the earth, is an arc of a terrestrial meridian contained between that place and the equator; being north or south, according to the side of the equator it is on.

953. The **LONGITUDE** of any place on the earth, is an arc of the equator contained between the meridian of that place and the meridian which

which is chose for the first, where the reckoning of longitude begins : Or, it is the angle at the pole formed by the first meridian, and that of the place.

954. REFRACTION, in an astronomical sense, is the difference between the true and apparent altitudes of an object : For the atmosphere furrounding the earth (602), causes all celestial objects to appear more elevated above the horizon than they really are.

955. The TWILIGHT is that medium between light and darkness, which happens in the morning before sun-rise, and in the evening after sun-set.

This is occasioned by the atmosphere's refracting the solar rays upon any place, altho' the sun is below the horizon of that place; and by observation it is found to begin and end at about 18° below the horizon.

956. The CREPUSCULUM is a small circle parallel to the horizon at 18° below it, where the twilight begins and ends.

957. The LATITUDE OF A PLACE is expressed by an arc of the meridian, shewing the distance between the zenith of that place and the equinoctial; or, by an arc of the meridian, shewing the height of the pole above the horizon.

For under the pole, or in the latitude of 90 degrees, the pole is in the zenith, or is 90 degrees above the horizon; so that, in this case, the horizon coincides with the equinoctial.

And as many degrees as the observer goes from the pole towards the equator, so many degrees does his horizon go below the equator on one side, and approach the pole on the other side.

Therefore the pole approaches the horizon just as much as the zenith approaches the equator; that is, the height of the pole above the horizon is equal to the distance of the zenith from the equinoctial, which is equivalent to the distance of the observer from the equator, or is equal to the latitude.

958. *To convert time into degrees, and degrees into time.*

Note	15°	$00'$	$00''$	$\equiv$	1^h	00^m	00^s
	5	00	00	$\equiv$	0	20	00
	1	00	00	$\equiv$	0	04	00
	0	15	00	$\equiv$	0	01	00
	0	05	00	$\equiv$	0	00	20
	0	01	00	$\equiv$	0	00	04
	0	00	15	$\equiv$	0	00	01

When time is given.

959. RULE. Multiply the given hours by 15 gives degrees, to which add 1° for every 4 minutes; for every overplus minute reckon $15'$ of a degree; and for every second of time, take $15''$ of a degree.

When degrees are given.

960. RULE. Divide the given degrees by 15 for hours; multiply the remainder by 4 for min., adding to the product 1 min. for every $15'$ of a degree; the overplus minutes of a degree multiplied by 4 gives seconds of time.

EXAM

EXAM. I. Reduce $4^h 37^m 23^s$ to degrees, &c.?

$$\begin{array}{r}
 4^h 37^m 23^s \\
 15 \\
 \hline
 60^\circ 00' 00'' \text{ for } 4 \text{ hours.} \\
 9 \quad 15 \quad 00 \text{ for } 37 \text{ min.} \\
 \quad 5 \quad 45 \text{ for } 23 \text{ seconds.} \\
 \hline
 69 \quad 20 \quad 45
 \end{array}$$

EXAM. II. Reduce $43^\circ 29'$ to time?

$$\begin{array}{r}
 15 \mid 43^\circ 29' (2^h 53^m 56^s. \\
 30 \\
 \hline
 13 \\
 4 \\
 \hline
 52 + 1 = 53 \text{ minutes.} \\
 \hline
 \text{And } 14' \times 4 = 56 \text{ seconds.}
 \end{array}$$

961. *Of the Tables of Declination and Right ascension.*

These tables are here disposed into three parts.

I. *Tables of the Sun's declination*, at art. 1052, for every day in the year, as well for leap-year, as for the first, second and third years after: They are contained in four pages, each being a year; the months whereof are at top and bottom, and the days in the right and left hand columns.

These tables are fitted to the meridian of London for each day at noon, but by a proper correction they may be adapted to any other meridian and intermediate hour. For which see art. 964, 965.

II. *A Table of the Sun's right ascension*, at art. 1053. This table, like the former, is made for every day in the year; but only for one year, because the alteration in right ascension is so small in several years, that it is not necessary to be considered in nautical operations.

III. *A Table of the Right ascensions and declinations of some of the principal fixed stars* is at art. 1054. The left hand column shews the situations of the constellations that are marked in the 2d column, whether in the zodiac, or a northern or southern constellation. The 3d column shews what part of the constellations the stars are in, whose names are in the 4th column. The 5th column shews the marks that are annexed to stars in the figures of the constellations, for the ready comparing of celestial observations. The figures in the 6th column shew the relative magnitude or size of the stars. The other two columns are according to their titles.

The use of the Tables of Declination and Right ascension.

962. I. Of the Sun's declination.

To find it. Seek the year and month at the head of the table, and the day in the right or left hand column, against which, in the proper column, is the declination sought.

EXAM. *Required the declination for the 10th of May, in the year 1753, for the meridian of London?*

In the table for the year 1753, in the column signed at top with May, and against the 10th day, is $17^\circ 44' N.$, which is the declination sought; and will serve also for the same month and day in the years 1757, 1761.

963. When these tables of declination are used in places more than 15° to the eastward or westward of London, they should be fitted to the meridian of that place: For the tables which give the declination every day at noon, have a daily variation arising gradually from noon to noon, and

and divided among the intermediate hours; and as every 15° of longitude answers to an hour, consequently places to the eastward or westward of London will have their declinations at noon, or other times of the day, different from what they are at London.

A table of the corrections for declination is at art. 1055: Where the left hand column is the daily variation, and the following columns contain the correction to the longitudes at the head of those columns.

964. *To find the correct declination for any meridian.*

RULE. If the decl. be increasing; then in W. longit. add the correction (1055) to the tabular declination; but in E. longit. subtract it. If the decl. be decreasing; then in W. longit. subtract the correction from the tabular declin.; but in E. longit. add it. And in either case the correct declination will be obtained.

NOTE. The decl. is increasing from 21st of March to 21st of June; and from 22d of September to the 21st of December: And decreasing from 21st of June to the 22d of September; and from the 21st of December to the 21st of March.

EXAM. I. *What is the sun's declination at noon on the 1st of June, 1753, at a place 120° to the eastward of the meridian of London?*

June 1st, 1753, the decl. at London is $22^\circ 07'$; the increase to next day is $8'$. In the tab. art. 1055, under 120° lon. stands 3 , against $8'$ of daily diff.

Here the decl. is increas. and E. long. so $22^\circ 04'$, the diff. is the decl. sought. [sought.]

But had it been W. longitude, then $22^\circ 10'$, the sum, would be the decl.

EXAM. II. *On March 15, 1753, what was the sun's declination at noon, in a place 105° to the westward of the meridian of London?*

March 15, 1753, the decl. at London is $1^\circ 57'$ S. the decr. to next day is $24'$. In tab. art. 1055, under 105 lon. stands 7 against $24'$ of daily diff.

Here the decl. is decr. and W. long. so $1^\circ 50'$ S. the diff. is the decl. sought.

But had it been E. longitude, then $2^\circ 04'$ S. the sum would be the decl.

In like manner may the tables of declination be fitted for any time of the day, by increasing or decreasing the declination by a proportional part of the difference between the declinations of that day and the next, thus.

965. *To find the declination at any hour under any meridian.*

RULE 1st. Correct the declination for the given longitude as above.

2d. If the declin. be increasing; in the forenoon subtr. the correction; and in the afternoon add it.

But if the declin. be decreasing; in the forenoon add the correction; and in the afternoon subtract.

This rule is the same as at art. 964, reckoning the time A. M. or before noon, in degrees of E. longitude, and the time P. M. or afternoon, in degrees of W. longitude.

EXAM.

EXAM. I. *What is the sun's declination at 4 o'clock in the morning on the 1st of June, 1753, at London?*

4 o'clock, is 8 h. A. M. which is $\frac{1}{3}$ of the day before noon; the daily diff. is 8'. The decl. is incr. and in forenoon; therefore $\frac{1}{3}$ of 8', or 3' being subtracted from $22^{\circ} 07'$, the decl. at noon, gives $22^{\circ} 04'$, the decl. at 8 o'clock in the morning.

Or, 8 h. A. M. is the same as 120° to the eastward of London; and so the Exam. becomes the same as the 1st to art. 964.

EXAM. II. *In a place 70° to the westward of the meridian of London, what is the sun's declination at 2 h. 20 m. P. M. on the 15th of March, 1753?*

2 h. 20 m. P. M. is 35° west; and 70° and 35° make 105° west: So the exam. is the same as Exam. II. art. 964.

When a star is to be observed, it is convenient to know at what time of the night it may be expected on the meridian; or at any known time of the night, to find what stars will then be on the meridian. For the performing of this, there is at art. 1053 a table of the sun's right ascension; and at art. 1054 is a table of some of the chief stars, with their right ascension and declination.

To find the culminating of the stars.

966. CASE I. *To find the time when any star of the table will be upon the meridian.*

RULE. Subtract the sun's right ascension from the right ascension of the star (increased if necessary by 24 hours); the remainder is the hours after mid-day, when the star comes to the meridian. Those hours, if less than 12, is the time after noon; or their overplus above 12, is the time after midnight.

EXAM. I. *Required the time when the star Arcturus comes to the meridian on the 1st of September, 1753?*

Arcturus's right ascen. = $14^{\text{h}} 03^{\text{m}}$

The sun's right ascen. = $10 \quad 42$

The remain. is * culm. = $3 \quad 21$

That is, the star Arcturus will be on the meridian of London at 21 minutes after 3 in the afternoon.

EXAM. II. *On the 25th of February, 1753, at what hour will the star Virgin's spike be on the meridian?*

Virgin's spike right ascen. = $13^{\text{h}} 12^{\text{m}}$

Which added to 24^{h} gives $37 \quad 12$

The sun's right ascen. = $22 \quad 34$

The remain. is * culm. = $14 \quad 38$

Or, the culm. abating 12 is $2 \quad 38$

That is, the star will be on the meridian at 38 min. after 2 in the morning.

967. CASE II. *To find if any star in the table will be on or near the meridian at a proposed time.*

RULE. Add the given time, reckoned from noon, to the sun's right ascension; seek the sum (rejecting 24 h. if the sum is greater) among the right ascensions of the stars, which if found there, the star answering it will be on the meridian at the time proposed: Or, the star, whose right ascension is nearest that sum, will be on the meridian the nearest to the given time.

EXAM. *January 25th, 1754: What star will be on the meridian about 10 o'clock at night?*

Jan. 25, at noon, the sun's right ascension is 20 h. 32 m.; and at 10 hours after it may be reckoned 20 h. 34 m.; to which the given time being added, makes 30 h. 34 m.; and abating 24 h. is 6 h. 34 m. which is the right ascension of the star Sirius; So Sirius will be on the meridian at 10.

SECTION IV.

Of the Projection of the sphere.

968.

PROBLEM I.

To project the sphere upon the plane of the solstitial colure, or upon the plane of the meridian of any place; those planes being supposed to coincide.

For this projection, the eye is supposed to be in the first point of ϖ , or the common intersection of the equator, ecliptic, and equinoctial colure; that being the pole of the plane of projection, or primitive circle. Plate IX. Fig. 4.

1st. With the chord of 60 degrees describe a circle PESQ to represent the solstitial colure, whose centre ϖ is its pole. (442)

2d. A diameter EQ will be the equator, and another Ps at right angles to it will shew the equinoctial colure (440), or the axis of the world, whose extremities P, s, will be the north and south poles.

3d. *For the parallels of declination.* On the primitive circle, beginning at E and Q, lay off the chords of the given degrees of declination, suppose every 10 degrees, and also the distances of the *tropics* and *polar circles* from the equator, namely, $23\frac{1}{2}^\circ$ and $66\frac{1}{2}^\circ$: Then from the centre ϖ in the axis Ps produced, apply the respective secants of the complements of the degrees laid on the primitive, and these will give the centres of the corresponding parallels of declination; from which centres, with the extents to the several divisions in the circumference, describe the small circles (456) 10, 10; 20, 20; &c.; and these will be the parallels of declination required: Among which *a* ϖ , *b* ϖ , are the tropics of Cancer and Capricorn; and *c* *c*, *d* *d*, the arctic and antartic polar circles.

4th. *For the circles of right ascension, or hour circles.* In the diameter EQ produced, lay off from the centre ϖ , both ways, the tangents of 15° , 30° , 45° , 60° , 75° , respectively, and they will give the centres of circles to be described thro' P and s, and cutting the equator in the points representing the 24 hours; the solstitial colure being the 12 o'clock, and the equinoctial colure, Ps, the 6 o'clock hour circles. And in like manner may any other of these kind of circles be drawn. (451)

5th. *The ecliptic* ϖ ϖ is drawn, making with the equator an angle of $23\frac{1}{2}^\circ$; whose poles *c*, *d*, are the intersections of the polar circles with the solstitial colure.

6th. *Parallels of celestial latitude* are drawn parallel to the ecliptic, in the same manner as the circles of declination are drawn parallel to the equator.

7th. *Circles of celestial longitude* are described thro' *c*, *d*, the poles of the ecliptic, in the same manner as the circles of right ascension were described thro' P, s, the poles of the equator; and thus were the divisions of the ecliptic found that are marked with the signs.

8th. *The horizon* is represented by drawing a diameter HR making an angle with the axis Ps, equal to the latitude of the place; and the poles of the horizon Z, N, the zenith and nadir, are at 90° dist. from the circle.

9th. *Azimuth, or vertical circles*, making any angle with the meridian, are described like circles of right ascension: Thus ZN is the prime vertical, and ZAN is another azimuth, 45° from the south.

10th. *Almicanters, or parallels of altitude*, are in this projection drawn parallel to the horizon, in like manner as were the circles of declination drawn parallel to the equator, 969.

969.

PROBLEM II.

To project the sphere upon the plane of the horizon.

In this projection the eye is supposed in one of the poles of the horizon, or plane of projection. Plate IX. Fig. 5.

1st. *The horizon* is represented by the primitive circle, where the upper XII is the south, the lower XII the north, E the east, and Q the west points.

2d. *The azimuth circles* are represented by diameters drawn thro' z, the centre or pole of the horizon: Thus the diameter XII, XII is for the meridian, and EZQ for the prime vertical; and other azimuth circles, forming any angle with the meridian, are readily drawn by laying off their distances in the primitive from the north or south points.

3d. *Parallels of altitude* are concentric to the primitive, and are described about the pole z with the half tangent of their distance from it: Thus the small circle, whose diameter is *ab*, is a parallel of altitude 10° above the horizon, or at 80° distant from its pole z.

4th. *The equinoctial's* distance from the zenith, is equal to the latitude of the place, and therefore this circle makes with the horizon an angle, which is measured by the complement of the latitude; then setting off from the centre z, in z XII continued, the tangent of 50° (the latitude in this example being 40°), it will give the centre of the circle EAQ, representing the equinoctial; and the half tangent of 50° , set the same way from z, will give P, the pole of the world.

5th. *The six o'clock hour circle* passes thro' the poles of the world, making with the horizon an angle equal to the measure of the latitude; therefore taking in the meridian from z towards A, the tangent of the latitude 40° , it gives G, the centre of the 6 o'clock hour circle EPQ.

6th. *The hour circles* pass thro' the poles of the world, and make with one another angles of 15 degrees: Therefore (435) in a line DE, drawn thro' G at right angles to the meridian, set off on both sides of G the tangents 15° , 30° , 45° , 60° , 75° , to the radius PG, and they will give the centres of the several hour circles passing thro' P, cutting the horizon and equinoctial in the hour points.

7th. *The polar circles, tropics, and other circles of declination*, are described parallel to the equinoctial, about its pole P, at given distances from it, either by finding the centres of such parallels as shewn in art. 454; or by setting off on each side of z the half tangents of their greatest and least distances from z; then the middles of those intervals are the centres sought. Thus; the artic circle is dist. from P $23\frac{1}{2}^\circ$; then to, and from, $zP = 50^\circ$, add and take $23\frac{1}{2}^\circ$, gives $73\frac{1}{2}^\circ$ and $26\frac{1}{2}^\circ$; the half tangents of these set off from z, gives p and q; then a circle being described on the diameter p q is the artic circle.

In like manner will the centres of the tropic of Cancer *c* ∞ *c*, and of Capricorn *d* ∞ *d*, be obtained.

8th. *The northern portion of the ecliptic* $\gamma \infty \alpha$ is described from a centre distant from z towards P, the tang. of $73\frac{1}{2}^\circ$, $= \angle$ the ecliptic makes with the horizon.

9th. *Circles of longitude* $\gamma p \infty$, $p \gamma$, $p \Pi$, $p \Omega$, $p \mathfrak{M}$, are described thro' p, the pole of the ecliptic, from centres in the line BFC; in like manner as the hour circles were described thro' P, the pole of the equator.

10th. *Circles of celestial latitude*, VIII *q* VI, are described about *p*, as the circles of declination were described about *P*, the pole of the equinoctial.

970.

PROBLEM III.

To project the sphere upon the plane of the equator.

In this projection the eye is supposed to be in one of the poles of the equator, suppose in the south pole, and projecting the north hemisphere. Plate IX. Fig. 6.

1st. *The equator* is represented by the primitive circle, whose centre and pole is *P*.

2d. *The hour circles* are expressed by diameters making angles of 15° with one another; of which XII *P* XII is the meridian, or solstitial colure, and VI *P* VI the 6 o'clock circle, or equinoctial colure.

3d. *Circles of declination* are circles parallel and concentric to the equator, described from its centre with radii equal to the half tangents of their several distances from the pole *P*, or half co-tangents of their degrees of declination: Thus *p q* the arctic circle, and *a ∞* the tropic of Cancer, are described with the half tangents of $23\frac{1}{2}^\circ$ and $66\frac{1}{2}^\circ$ respectively; and so of the others.

4th. *The ecliptic* making an angle of $23\frac{1}{2}^\circ$ with the equator; therefore the tangent of these degrees laid from *P* towards *a* will give the centre for describing the ecliptic *r ∞ ∞*, whose pole *p* is in the polar circle.

5th. *Circles of longitude* are described thro' *p*, the pole of the ecliptic, in like manner as the hour circles were described thro' *P*, the pole of the equator in the last problem; and thus were the divisions γ , Π , Ω , $\mathcal{M}$, obtained.

6th. *Circles of celestial latitude* are projected in the same manner as the circles of declination in the last problem.

7th. *The horizon of any place*, suppose of London, being inclined to the equator in an angle equal to the co-latitude, $38^\circ 28'$; the tangent of this laid from *P* towards ∞ , and the half tangent laid from *P* to *z*, will give the centre, and *z* the pole, of the horizon HOR.

8th. *The prime vertical* HZR making an angle with the equator equal to $51^\circ 32'$, the latitude of the place, its centre is found by laying the tangent of $51^\circ 32'$ from *P* towards *o*.

9th. *Azimuth circles*, making given angles with the meridian *zo*, are thus described: In a line drawn thro' the centre of the prime vertical, at right angles to the meridian, take distances from that centre, equal to the tangents of the proposed azimuth angles, the semidiameter of the prime vertical being the radius, those distances give the centres sought; and thus was the azimuth circle *zA* described.

10th. *Parallels of altitude* are described about *z*, the pole of the horizon, at the distances of the co-altitudes, in the same manner as were the circles of declination, in the last problem, described about *P*, the pole of the equator; and thus was the small circle *v b vii* described at 10° distant from the horizon, or 80° distant from its pole *z*.

971.

PROBLEM IV.

To project the sphere upon the plane of the ecliptic.

The eye is here supposed to be in one of the poles of the ecliptic, and thence seeing the northern hemisphere projected. Plate IX. Fig. 7.

1st. *The ecliptic* is here represented by the primitive circle, whose centre p is the pole of the ecliptic.

2d. *Circles of longitude* are herein shewn by diameters; those making angles of 30° with one another, being drawn thro' the divisions marked with the signs of the zodiac.

3d. *Parallels of celestial latitude*, are circles described about p concentric to the ecliptic; such is the small circle whose diameter is ab , representing the parallel of 10° of latitude.

4th. *The equator* making an angle with the ecliptic of $23\frac{1}{2}^\circ$; therefore the tangent of this inclination laid from p towards ∞ , will give the centre of the equator $\gamma XII \triangle$; and the half tangent of $23\frac{1}{2}^\circ$ laid from p the same way, gives P for the pole of the equator.

5th. *The equinoctial colure*, which here makes *the six o'clock circle*, makes an angle with the ecliptic of $66\frac{1}{2}^\circ$; therefore the tangent of $66\frac{1}{2}^\circ$ laid from p towards ∞ , gives the centre of the 6 o'clock circle $\gamma P \triangle$.

6th. *Hour circles* passing thro' P , and making angles of 15° with one another, are described from centres, found in a right line passing thro' the centre of $\gamma P \triangle$, and drawn at right angles to the solstitial colure $\infty p \infty$; by laying off in that line the tangents of $15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ$, reckoned from the centre of $\gamma P \triangle$, on both sides, the semidiameter of this circle being the radius. These hour circles cut the equator in the hour points.

7th. *Parallels of declination*, such as the tropic of Cancer, and arctic circle, whose diameters are $12, 12$, and pq , are described by laying off from p the half tangents of their greatest and least distances: Thus q being distant from p 47° , make $pq = \frac{1}{2} \text{ tang. of } 47^\circ$, the middle of pq will be the centre of the polar circle.

8th. *The horizon* HOR is to make an angle with the ecliptic equal to the diff. between the co-lat. and the obliquity of the ecliptic, when P is projected to the north of p ; otherwise, that angle is equal to the sum of those quantities. And for London, where the said diff. $= (38^\circ 28' - 23^\circ 29' =) 14^\circ 59'$, the tangent of $14^\circ 59'$ gives the centre of HOR, and the half tang. gives z the zenith.

9th. *The prime vertical* HZR is described, by laying from p towards o the cotangent of pz for a centre.

10th. *Azimuth circles* are described thro' z , making given angles with the meridian zo , by finding their centres in a line drawn thro' the centre of HZR, in the manner described for the hour circles, art. 969.

11th. *Parallels of altitude* are represented, by describing small circles parallel to the horizon HOR, at given distances from it; or, which comes to the same, describing small circles about the pole z , at distances equal to the complements of the given altitudes: And thus the circle cdc was described for a parallel of 33° of altitude.

SECTION

SECTION V.

Problems of the sphere.

972.

PROBLEM V.

Given the sun's longitude, and the obliquity of the ecliptic;
Required the sun's right ascension and declination.

EXAM. Let the obliquity of the ecliptic, or the sun's greatest declination be $23^{\circ} 29'$, and the sun's place be $13^{\circ} 16'$ in Taurus: Required the rest?

CONSTRUCTION.

In the primitive circle PESQ representing the plane of the solstitial colure, whose centre is γ , draw a diameter EQ for the equator, and at right angles to EQ draw a diameter PS for the equinoctial colure: Make $\angle EQS = 23^{\circ} 29'$, and draw a diameter $\gamma\omega$ for the ecliptic, in which (447) take $\gamma\odot = 43^{\circ} 16'$ for the sun's distance from the point γ : Thro' $\odot$ describe (163) a circle of right ascension.

COMPUTATION. See art. 513, 514.

In the right angled spheric triangle $\gamma\odot B$.

Given sun's longitude $\gamma\odot = 43^{\circ} 16'$ } Req. right ascen. γB .
greatest declinat. $\angle\odot\gamma B = 23^{\circ} 29'$ } pref. declin. $B\odot$.

To find the present declination.

As Radius	= R	10,00000
To s. sun's lon.	= $43^{\circ} 16'$	9,83594
So s. great. decl.	= $23^{\circ} 29'$	9,60041
To s. pref. decl.	= $15^{\circ} 51'$	9,43635

To find the right ascension.

As Radius	= R	10,00000
To t. sun's lon.	= $43^{\circ} 16'$	9,97371
So cos. obl. ecl.	= $23^{\circ} 29'$	9,96245
To t. rt. ascen.	= $40^{\circ} 48'$	9,93616

973. While the sun is moving from γ to ω , or is in the first quadrant of the ecliptic, the given longitude is the hypotenuse in the triangle $\gamma\odot B$, the declination $B\odot$ is north, and γB is the right ascension.

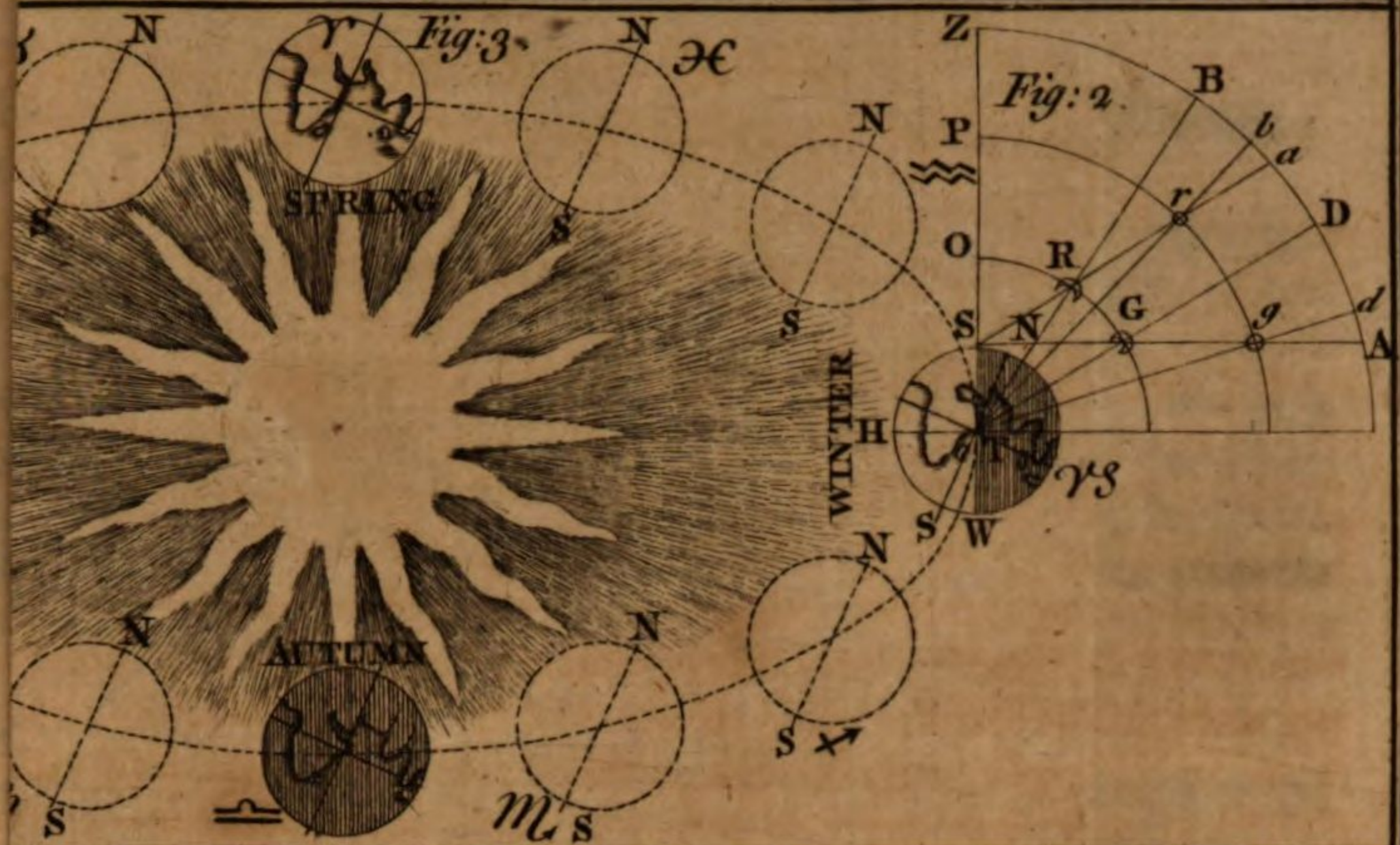
974. When the sun has past the solstice ω , and is descending towards ω , he is then said to be in the second quadrant, and his longitude or distance from γ being taken from 180° , the remainder $\omega\odot$ becomes the hypotenuse, and the declination is still north; but the arc $B\omega$ found for the right ascension, is only the supplement, and must therefore be taken from 180° .

975. The sun having past the point ω , and descending towards ω , is got into the third quadrant; the longitude then, reckoned from γ , will be greater than 180° : In this case the excess above 180° , or the distance the sun is removed from ω , will be the hypotenuse $\omega\odot$; the declination will be south; and the arc ωA , found for the right ascension, must be added to 180° , to give the right ascension estimated from γ .

976. When the sun has past the solstice ω , and is ascending towards γ , he is then in the fourth quadrant; therefore the longitude is greater than 270° , and must be taken from 360° , to give the hypotenuse $\omega\odot$. Herein the declination is south, and the right ascension ωA , found by the proportion, must be taken from 360° , to give the right ascension from γ .

977. At equal distances from the equinoctial points γ or ω , the sun will have equal quantities of declination; but will be of different names, according as it is on the north or south sides of the equinoctial.

978.

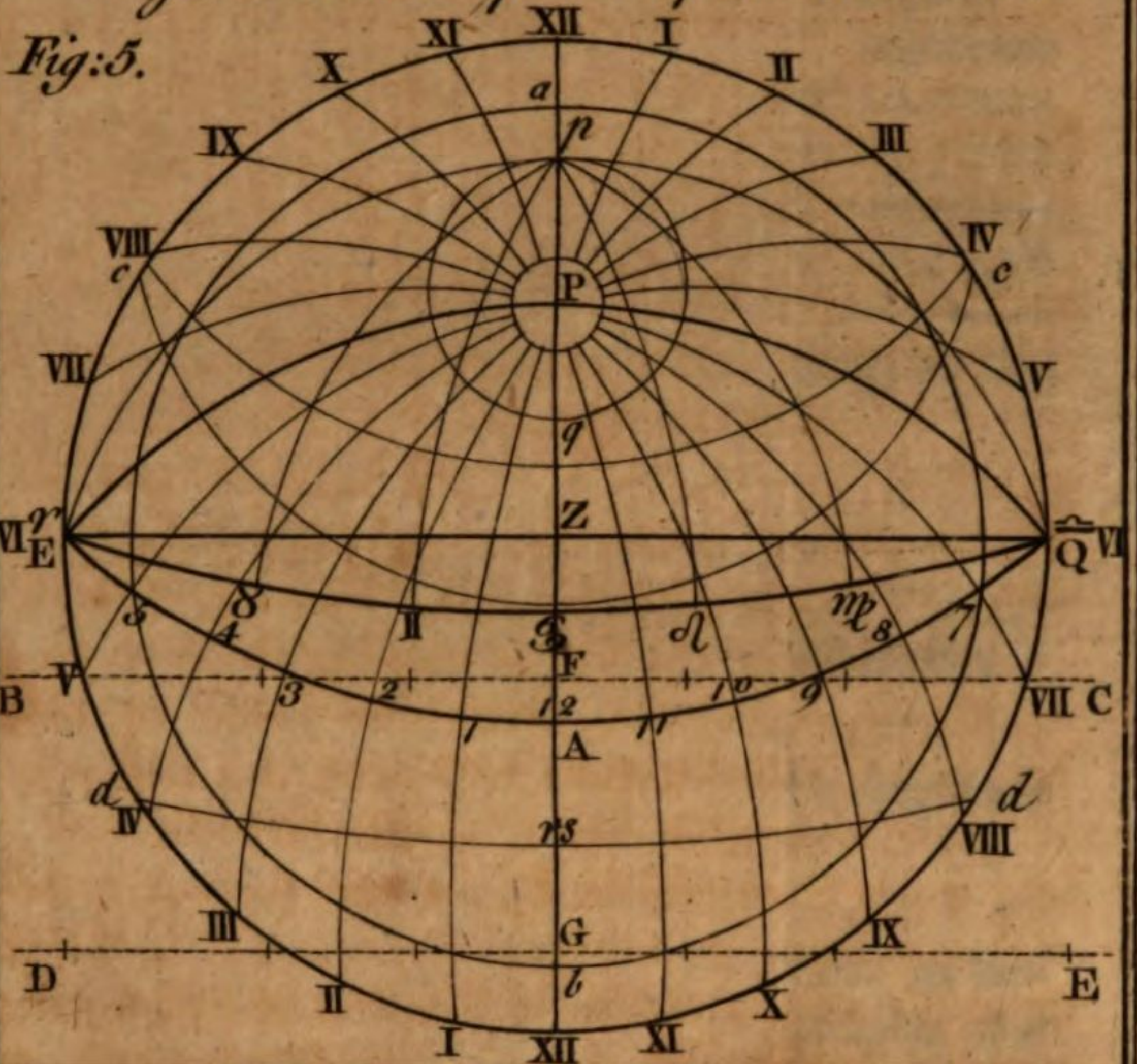


or Meridian



Projection on the plane of the Horizon.

Fig: 5.



Equator:



Projection on the plane of the Ecliptic.

Fig: 7.

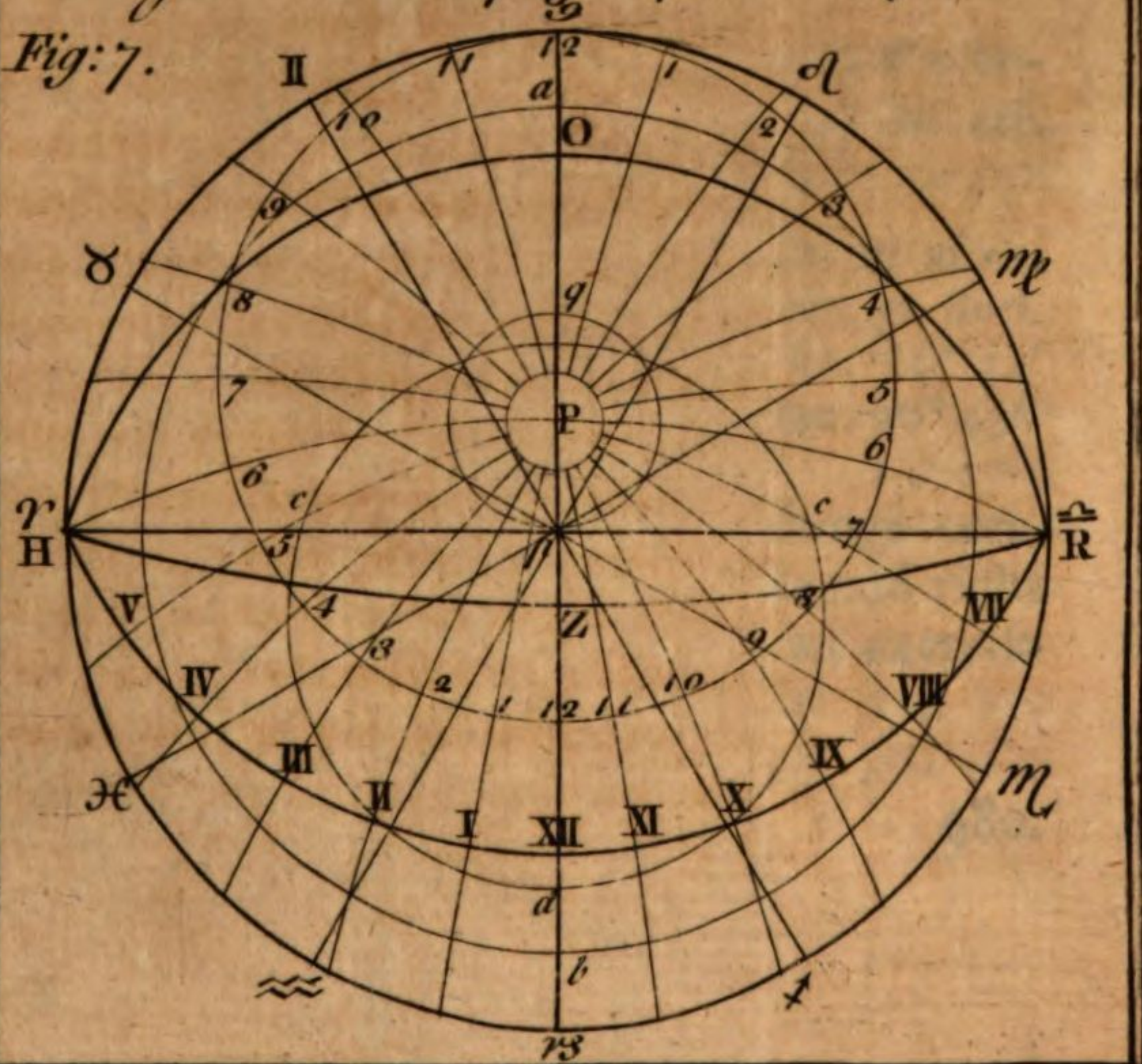


Table 1	
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
11	121
12	144
13	169
14	196
15	225
16	256
17	289
18	324
19	361
20	400
21	441
22	484
23	529
24	576
25	625
26	676
27	729
28	784
29	841
30	900
31	961
32	1024
33	1089
34	1156
35	1225
36	1296
37	1369
38	1444
39	1521
40	1600
41	1681
42	1764
43	1849
44	1936
45	2025
46	2116
47	2209
48	2304
49	2401
50	2500
51	2601
52	2704
53	2809
54	2916
55	3025
56	3136
57	3249
58	3364
59	3481
60	3600
61	3721
62	3844
63	3969
64	4096
65	4225
66	4356
67	4489
68	4624
69	4761
70	4900
71	5041
72	5184
73	5329
74	5476
75	5625
76	5776
77	5929
78	6084
79	6241
80	6400
81	6561
82	6724
83	6889
84	7056
85	7225
86	7396
87	7569
88	7744
89	7921
90	8100
91	8281
92	8464
93	8649
94	8836
95	9025
96	9216
97	9409
98	9604
99	9801
100	10000

978.

PROBLEM V.

Given the obliquity of the ecliptic, and the sun's present declination;
Required the sun's longitude and right ascension.

EXAM. *The obliquity of the ecliptic being $23^{\circ} 29'$, what is the sun's longitude and right ascension when he has $22^{\circ} 43'$ of north declination?*

CONSTRUCTION.

Having described the solstitial colure, drawn the equator EQ, the axis ps, and the ecliptic $\epsilon\psi$, as before; make En, Qn, equal to the given declination, and (3d 968) describe the parallel of declination nn, its intersection with the ecliptic gives $\odot$ the sun's place; thro' P, $\odot$, s, describe the circle of right ascension P $\odot$ s.

COMPUTATION. See art. 516, 517.

In the right angled spheric triangle $\Upsilon\odot B$.

Given sun's greatest decl. $\angle\odot\Upsilon B = 23^{\circ} 29'$ } Required sun's longit. $\Upsilon\odot$.
present decl. $\odot B = 20^{\circ} 43'$ } rt. ascen. ΥB .

To find the sun's longitude.

As f. $\odot$ great decl. $= 23^{\circ} 29'$ 0,39959
To fin. pref. decl. $= 20^{\circ} 43'$ 9,54869
So Radius $= R$ 10,00600

To fin. $\odot$ longit. $= 62^{\circ} 35'$ 9,94828

To find the sun's right ascension.

As Radius $= R$ 10,00000
To cot. gr. decl. $= 23^{\circ} 29'$ 10,36204
So tan. pref. decl. $= 20^{\circ} 43'$ 9,57772

To fin. rt. ascen. $= 60^{\circ} 31'$ 9,93976

Therefore the sun is in $\Pi 2^{\circ} 35'$, or in $\epsilon 27^{\circ} 25'$, according as the time of the year is before or after the summer solstice.

979.

PROBLEM VII.

Given the obliquity of the ecliptic, and the sun's right ascension;
Required the sun's longitude and present declination.

EXAM. *When the sun's right ascension is $60^{\circ} 31'$, what is his longitude and present declination, the greatest declination being $23^{\circ} 29'$?*

CONSTRUCTION.

The solstitial colure, equator, axis and ecliptic being described, as before, make $\Upsilon B =$ given right ascension (4th 968), and describe the circle PBS, cutting the ecliptic in $\odot$ the sun's place.

COMPUTATION.

In the right angled spheric triangle $\Upsilon\odot B$; the leg ΥB and $\angle\odot\Upsilon B$ being known, the hypoth. $\Upsilon\odot$, and other leg $\odot B$, are found as in art. 520, 521.

As Rad : cot. gr. decl. :: cot. rt. ascen.
[: cot. $\odot$ long.

As Rad : cot. $23^{\circ} 29'$:: cot. $60^{\circ} 31'$
[: cot. $62^{\circ} 35'$

As Rad : tan. gr. decl. :: fin. rt. ascen.
[: tan. pr. decl.

As Rad : cot. $23^{\circ} 29'$:: fin. $60^{\circ} 31'$
[: tan. $20^{\circ} 43'$

Three other problems may be formed out of the four things concerned, or greatest declination, present declination, longitude and right ascension: But these being of little more importance than as an exercise for right angled spheric triangles, they are therefore omitted.

980.

PROBLEM VIII.

Given the latitude of the place, and the sun's present declination;
Required the sun's altitude and azimuth at 6 o'clock.

EXAM. *At London, in lat. $51^{\circ} 32' N.$, on the longest day, when the sun's declination is $23^{\circ} 29'$: Required the sun's altitude and azimuth at 6 o'clock in the morning or evening?*

CONSTRUCTION.

Describe the meridian, draw the horizon HR , and prime vertical ZN ; make $RP =$ latitude $51^{\circ} 32' N.$; draw the 6 o'clock hour circle PS , the equator EQ , the $23^{\circ} 29' N.$ parallel of declination nm , cutting the 6 o'clock hour circle PS in $\odot$; and thro' z , $\odot$, N , describe the azimuth circle $z\odot N$, cutting the horizon in A ; then the things given and required fall in either of the triangles $z\odot P$ or $\Upsilon\odot A$, they being supplemental triangles one to the other.

COMPUTATION.

In the spheric triangle $z\odot P$, right angled at P . (522)

Given the co-latit. $zP = 38^{\circ} 28'$ } Required the co-altitude $z\odot$.
the co-decl. $\odot P = 66^{\circ} 31'$ } the azimuth $\angle\odot zP$.

Or in the spheric triangle $\Upsilon A\odot$, right angled at A . (513, 514)

Given the latit. $A\Upsilon\odot = 51^{\circ} 32'$ } Required the altitude $A\odot$.
the decl. $\Upsilon\odot = 23^{\circ} 29'$ } the co-azimuth ΥA .

To find the altitude $A\odot$.

As Radius	= R	10,00000
To sin. decl.	= $23^{\circ} 29'$	9,60041
So sin. lat.	= $51^{\circ} 32'$	9,89375
To sin. alt.	= $18^{\circ} 11'$	9,49416

To find the azimuth AR .

As Radius	= R	10,00000
To cos. lat.	= $51^{\circ} 32'$	9,79383
So tan. decl.	= $23^{\circ} 29'$	9,63796
To cot. azim.	= $74^{\circ} 53'$	9,43179

For the arc AR measures the $\angle RZA$, the azimuth. (389)

981. On the shortest day at London, the parallel of S. declination cuts the 6 o'clock hour circle below the horizon; and as the triangles $\Upsilon A\odot$, $\Upsilon a\odot$, are congruous, the depression below the horizon, on the shortest day, at 6 o'clock, will be equal to the altitude at the same hour on the longest day; and the azimuth will be also equal, if estimated from the south.

So that on the 21st of June, at London, the sun will bear $N. 74^{\circ} 53' E.$ at 6 o'clock in the morning, and $N. 74^{\circ} 53' W.$ at 6 in the evening: But on the 21st of December, at the same hours, it will bear $S. 74^{\circ} 53' E.$, and $S. 74^{\circ} 53' W.$

982. From a due consideration of this problem it is evident, that as the declination increases, the altitude increases and the azimuth lessens; and the contrary happens while the declination is diminishing: So that on the days of the equinoxes, or when the sun has no declination, the altitude at 6 o'clock will be nothing, or the sun will be in the horizon; and the azimuth being then 90 degrees, the sun will be due east in the morning, and west in the evening; that is, on the days of the equinoxes the sun rises and sets at six, in the east and west points of the horizon.

983.

983.

PROBLEM IX.

Given the latitude of the place, and the sun's declination;
Required the altitude and hour when the sun is due east or west.

EXAM. *At London, in lat. $51^{\circ} 32' N.$, what is the sun's altitude, and the hour when he is due east or west, on the longest day, or when the declination is $23^{\circ} 29' N.$?*

CONSTRUCTION.

Describe the primitive circle to represent the meridian of London, draw the horizon HR , and the prime vertical ZN ; make $RP = 51^{\circ} 32'$, the given latitude, draw the 6 o'clock hour circle PS , the equator EQ , the parallel of declination nm (3d 968), cutting the prime vertical in $\odot$, and thro' $P\odot$ describe (163) the hour circle POS , cutting the equator in A .

Here the things concerned in the problem fall in either of the triangles $PZ\odot$ or $\Upsilon A\odot$.

COMPUTATION.

In the spheric triangle $PZ\odot$, right angled at Z .

Given the co-latit. $PZ = 38^{\circ} 28'$ } Required the co-alt. $Z\odot$
the co-decl. $P\odot = 66 \ 31$ } the hour fr. noon $\angle ZP\odot$.

Or in the spherical triangle $\Upsilon A\odot$, right angled at A .

Given the latit. $\angle A\Upsilon\odot = 51^{\circ} 32'$ } Required the altitude $\Upsilon\odot$
the decl. $A\odot = 23 \ 29$ } the hour after 6 ΥA .

To find the altitude $\Upsilon\odot$. (516)

As f. lat. $\angle A\Upsilon\odot = 51^{\circ} 32'$ 0,10625
To fin. decl. $A\odot = 23 \ 29$ 9,60041
So Radius $= R$ 10,00000

To fin. alt. $\Upsilon\odot = 30 \ 36$ 9,70666

To find the hour after 6.

As Radius $= R$ 10,00000
To cot. lat. $A\Upsilon\odot = 51^{\circ} 32'$ 9,90009
So tan. decl. $A\odot = 23 \ 29$ 9,63796

To f. h. fr. 6 $A\Upsilon = 20 \ 12$ 9,53805

Which $20^{\circ} 12'$ converted into time (960), gives 1 h. 20 m. 48 f. for the time after 6 in the morning, and before 6 in the evening; when the sun will appear due east and west; which will be at 7 h. 20 m. 48 f. in the morning, and 4 h. 39 m. 12 f. in the afternoon.

Or, The compl. of $20^{\circ} 12'$, viz. $69^{\circ} 48'$ put into time, which gives 4 h. 39 m. 12 f., shews the time before and after noon when the sun will be due east or west.

984. This problem worked for the shortest day, namely in the $\Delta \Upsilon a\odot$, which is congruous to $\Upsilon A\odot$, would give the sun's depression at the time when he was east or west, which would be before 6 in the morning and after 6 in the evening, by as much as was found above, viz. 1 h. 20 m. 48 f.

985. By this problem it appears, that when the latitude of the place and the sun's declination have the same name, then, the greater the declination and latitude, the greater the altitude and time from 6: And having contrary names, the same things happen; but with this difference, that in the former case the days lengthen on account of the increase of the latitude and declination; whereas, in the latter case, the days shorten on that account.

986.

PROBLEM X.

Given the latitude of a place, and the sun's declination;
Required his amplitude and ascensional difference.

EXAM. At London, whose lat. is $51^{\circ} 32' N.$, in the year 1753, on the 21st of June, being the longest day, when the sun's declination is $23^{\circ} 29' N.$: How far from the north does the sun rise and set, at what time, and what is the length of the day and night?

CONSTRUCTION.

Let the primitive circle represent the meridian of the place, and the diameter HR the horizon; from R , the north point thereof, take $RP = 51^{\circ} 32'$ for the latitude, draw the axis or 6 o'clock hour circle PS , and at right angles to it, draw the equator EQ ; make En , $Qm = 23^{\circ} 29'$ the declination, and (3d 968) describe the parallel of declination nm , cutting the horizon in $\odot$ the place of the sun at its rising and setting; thro' which describe (163) the hour circle $P\odot s$.

COMPUTATION.

Now as the arc $QR =$ co-latitude, measures the $\angle Q\odot R$.

In the spheric triangle $\odot A$, right angled at A .

Given sun's decl. $A\odot = 23^{\circ} 29'$ } Required the amplitude $\odot$.
co-latit. $\angle A\odot = 38^{\circ} 28'$ } the ascen. diff. $\odot A$.

To find the amplitude $\odot$.

As f. co-l. $\angle A\odot = 51^{\circ} 32'$ 0,20617
To sin. decl. $A\odot = 23^{\circ} 29'$ 9,60041
So Radius $= R$ 10,00000

To sin. amp. $\odot = 39^{\circ} 50'$ 9,80658

This $39^{\circ} 50'$ is the amplitude reckoned from the east or west points of the horizon: But its complement $50^{\circ} 10'$ shews how far from the north the sun rises or sets on the longest day at London.

To find the ascensional difference $\odot A$.

As Radius $= R$ 10,00000
To t. lat. $\angle A\odot = 51^{\circ} 31'$ 10,09991
So tan. decl. $A\odot = 23^{\circ} 29'$ 9,63796

To f. af. diff. $\odot A = 33^{\circ} 09'$ 9,73787

Which $33^{\circ} 09'$ converted into time (960) gives 2 h. 12 m. 36 f. for the time which the sun rises before, and sets after, the hour of six on the longest day.

987. Suppose rs to be a parallel of declination as far south, as mn is north; then the hour circle PBS , passing thro' $\odot$ the place of the sun at its rising or setting, will form a triangle $\odot B = \triangle \odot A$, where the amplitude is to the southward of the east and west points.

988. Hence it is evident, that when the latitude and declination have the same name, the sun rises before, and sets after 6: But when of contrary names, the sun rises after, and sets before 6.

989.

989. And as the sun describes the parallel of declination nm in 24 hours, being at n when it is noon, and at m when it is midnight; therefore the time in passing from m to $\odot$, or the time of rising, being doubled, gives the length of the night; and the time of setting being doubled, must give the length of the day.

Then to, and from	6 ^h 0 ^m 0 ^f
Add and subtract the ascen. differ.	2 12 36
Sum, gives $\odot$ setting	8 12 36
Diff. gives $\odot$ rising	3 47 24
Length of day is	16 25 12
Length of night is	7 34 48

990. But when it is the shortest day at London, which is, when the sun has $23^{\circ} 29'$ south declination; then the lengths of the day and night change places; the day being 7 h. 34 m. 48 f. long, and the night 16 h. 25 m. 12 f.

991. When the latitude and declination have the same name, the difference between the right ascension and the ascensional difference, is the oblique ascension; and their sum is the oblique descension.

But when they are of contrary names, their sum is the oblique ascension, and their difference is the oblique descension.

992. When the declination is equal to the co-latitude of any place (which can only happen to places within the polar circles), then the parallel of declination will not cut the horizon, and consequently the sun will not set in those places during the time his declination exceeds the co-latitude: And the same may be said of the stars, every one of which, whose co-declination, or polar distance, is not less than the latitude; or, which is the same thing, whose declination is not less than the co-latitude of a place, those stars will never descend below the horizon of that place. But this is to be only understood, when the sun or stars are in the same hemisphere with the given place; for when the sun or stars are in a contrary hemisphere with any place, whose co-latitude does not exceed the declination of those celestial objects, then will they never rise above the horizon of that place, and consequently are never visible there.

993.

PROBLEM XI.

Given the latitude of a place, the sun's declination and altitude;
Required the hour from noon, and the sun's azimuth.

EXAM. In the latitude of $51^{\circ} 32' N.$ the sun's altitude was observed to be $46^{\circ} 20'$, when his declination was $23^{\circ} 29' N.$: What was the sun's azimuth, and the hour when the observation was made?

CONSTRUCTION.

Let the primitive circle $ZRNH$ represent the meridian of London, HR the horizon, ZN the prime vertical; make $RP = 51^{\circ} 32'$ the height of the pole at London (957), draw the axis PS , and the equator EQ . Lay off the declination En , Qm , $23^{\circ} 29' N.$; the altitude HR , Rs , $46^{\circ} 20'$; and (456) describe the parallel of declination nm , and the parallel of altitude rs , cutting one another in $\odot$ the place of the sun at that time. Thro' z , $\odot$, N , describe an azimuth circle $z\odot N$, and thro' P , $\odot$, s , describe an hour circle $P\odot s$: Then the angles $\odot ZP$, $\odot PZ$, being measured (448), will give the azimuth and hour from noon required.

COMPUTATION.

In the oblique angled spheric triangle $P\odot Z$.

Given the co-lat. $ZP = 38^{\circ} 28'$ } Required the azim. $\angle \odot ZP$.
the co-alt. or zen. dist. $Z\odot = 43^{\circ} 40'$ } and the h. fr. noon $\angle \odot PZ$.
the co-dec. or pol. dist. $\odot P = 66^{\circ} 31'$ } See art. 550.

To find the azimuth $\angle \odot ZP$.

Here $Z\odot = 43^{\circ} 40'$

$ZP = 38^{\circ} 28'$

$\odot P = 66^{\circ} 31'$

$Z\odot - ZP = 5^{\circ} 12' = D.$

$71^{\circ} 43' \quad 35^{\circ} 51\frac{1}{2}'$

$2) \quad 61^{\circ} 19' \quad 30^{\circ} 39\frac{1}{2}'$

$61^{\circ} 19' \quad 30^{\circ} 39\frac{1}{2}'$

Which doubled, gives $113^{\circ} 30'$ for the azimuth sought, reckoning from the north.

To find the hour from noon, $\angle \odot PZ$.

Here $\odot P = 66^{\circ} 31'$

$PZ = 38^{\circ} 28'$

$Z\odot = 43^{\circ} 40'$

$\odot P - PZ = 28^{\circ} 03' = D.$

$71^{\circ} 43' \quad 35^{\circ} 51\frac{1}{2}'$

$2) \quad 15^{\circ} 37' \quad 7^{\circ} 48\frac{1}{2}'$

$15^{\circ} 37' \quad 7^{\circ} 48\frac{1}{2}'$

This doubled, gives $43^{\circ} 52'$ for the measure of the hour from noon, which is 2 h. 55 m. 28 f.

Hence it appears, that the observation was made either at 9 h. 4 m. 32 f. in the morning, or at 2 h. 55 m. 28 f. in the afternoon.

The azimuth being first found, the hour from noon might have been found by the proportion between opposite sides and angles.

Had

Had the declination and latitude been of contrary names, the same kind of operation would have been used to find the things required; only the side $\odot P$ would have been obtuse, by adding the declinat. to 90° , instead of subtracting it as in the case of the lat. and decl. having like names.

994.

PROBLEM XII.

Given the latitude of the place, and the sun's declination;
Required the time when the twilight begins and ends.

EXAM. *At what time does the twilight begin and end at London on the first of May, 1753, the latitude being $51^\circ 32' N.$, and the declination at that time being $15^\circ 12' N.$*

CONSTRUCTION.

Let the circle $ZRNH$ represent the meridian of the place, HR the horizon, ZN the prime vertical, and ts a small circle parallel to the horizon described at 18 degrees below it (456); lay off the latitude RP , draw the axis PS , the equator EQ , and describe the parallel of declination nm (3d 968), and where nm cuts ts in $\odot$, is the sun's place at the time of the beginning or end of the twilight; thro' $\odot$ describe (163) the vertical circle $Z\odot N$, and the hour circle $P\odot S$; then the $\angle ZP\odot$ being measured (448), will give the time before or after noon as required.

COMPUTATION.

In the oblique angled spheric triangle $Z\odot P$.

Given the co-lat. $ZP = 38^\circ 28'$ } Req. the hour fr. noon $\angle = ZP\odot$
the polar dist. $P\odot = 74^\circ 48'$ } The manner of solution is
the zenith dist. $Z\odot = 108^\circ 00'$ } the same as shewn in art. 550.

Here $P\odot = 74^\circ 48'$
 $PZ = 38^\circ 28'$
 $\odot Z = 108^\circ 00'$
 $P\odot - PZ = 36^\circ 20' = D.$

$\begin{array}{r} 144^\circ 20' \\ 2) \quad \hline 71^\circ 40' \end{array} \quad \begin{array}{l} 72^\circ 10' \\ 35^\circ 50' \end{array}$

Then Co-ar. fin. polar dist. $= 74^\circ 48' \quad 0,01547$
Co-ar. fin. co-latit. $= 38^\circ 28' \quad 0,20617$
 $\text{Sine } \frac{1}{2} \text{ sum of zen.d. \& } D = 72^\circ 10' \quad 9,97861$
 $\text{Sine } \frac{1}{2} \text{ diff. of zen.d. \& } D = 35^\circ 50' \quad 9,76747$

The sum of these four logs: $19,96772$

The half sum gives $74^\circ 28'$ $9,98386$

Which doubled gives $148^\circ 56'$ for $\angle ZP\odot$.

And $148^\circ 56'$ reduced to time, gives 9h. 56m' either before or after noon; that is, the twilight begins at 2h. 04m. in the morning, and ends at 9h. 56m. in the evening on the 1st of May at London.

995. When the declination becomes greater than the difference between the co-latitude and 18 degrees, then the parallel of declination nm will not cut the parallel ts 18 degrees below the horizon, and consequently at that time there will be no night in that place, but the twilight will continue from sun-setting to sun-rising; and on this account it is, that from about the 22d of May to about the 21st of July there is no total darkness at London, the sun's declination during that interval being greater than $20^\circ 28'$, which is the difference between 18° and $38^\circ 28'$, the complement of the latitude.

996.

PROBLEM XIII.

Given the time of the year, the latitude of a place, and the altitude of a known fixed star;

Required the hour of the night when the observation was made.

EXAM. *Some time in the night, on the 27th of November, 1752, the star Arcturus was observed at London to be $27^{\circ} 12'$ above the horizon: At what hour was the star observed?*

CONSTRUCTION.

Describe the meridian of the place, draw the horizon HR , whose zenith and nadir are z and N , and describe (10th 968) the parallel of altitude rs at $27^{\circ} 12'$ above the horizon; take P the north pole $51^{\circ} 32'$ above the horizon for the latitude of the place, and s the south pole as much below the horizon; draw the equator EQ , and describe (3d 968) the star's parallel of declination nm , found in the table, art. 1054, to be $20^{\circ} 30' N.$; and where this parallel nm cuts the former rs in $*$, is the position of the star at the time of observation; describe (163) the vertical circle $z*N$, and hour circle $P*s$, and the angle $zP*$ being measured (448), gives the hour from noon.

COMPUTATION.

In the oblique angled spheric triangle $P*z$.

Given the co-lat.

$$PZ = 38^{\circ} 28'$$

the co-altitude

$$z* = 62^{\circ} 48'$$

the polar dist.

$$*P = 69^{\circ} 30'$$

} Required $\angle zP*$, or the hour from noon.

$$\text{Here } P* = 69^{\circ} 30'$$

$$PZ = 38^{\circ} 28'$$

$$z* = 62^{\circ} 48'$$

$$P* - PZ = 31^{\circ} 02' = D.$$

$$\begin{array}{r|l} 93 & 50 \\ 2) & \\ \hline 31 & 40 \end{array} \quad \begin{array}{l} 46^{\circ} 55' \\ 15^{\circ} 53' \end{array}$$

$$\text{Then Co-ar. fin. co-lat.} = 38^{\circ} 28' \quad 0,20617$$

$$\text{Co-ar. fin. pol. dist.} = 69^{\circ} 30' \quad 0,02841$$

$$\text{Sin. } \frac{1}{2} \text{ sum zen. dist. and } D = 46^{\circ} 55' \quad 9,86354$$

$$\text{Sin. } \frac{1}{2} \text{ diff. zen. dist. and } D = 15^{\circ} 53' \quad 9,43724$$

$$\text{The sum of the four logs.} = 19,53536$$

$$\text{The } \frac{1}{2} \text{ sum gives } 35^{\circ} 51' \quad 9,76768$$

Which doubled, gives $71^{\circ} 42' = \angle zP*$.

This $71^{\circ} 42'$ turned into time (960) gives 4 h. 46 m. 48 s. for the time which must elapse before the star comes to the meridian.

Now, at the time of observation, November 27th, 1752,

The sun's right ascension, found in table, art. 1053, is $16^h 13^m$

The star's right ascension, found in table, art. 1054, is $14^h 03^m$

difference of right ascensions

$$2^h 10^m$$

$$12^h 00^m$$

The star souths, or culminates

$$9^h 50^m$$

The time the star has to come to the meridian

$$4^h 47^m$$

The difference gives the hour of the night

$$5^h 03^m \text{ P. M.}$$

Or, the sum gives the hour after midnight

$$14^h 37^m$$

Which is 2 h. 37 m. in the morning.

And whether to subtract or add, will always be known by the star's being in the eastern part of the horizon, or ascending; or by being in the western part of the horizon, or descending.

997.

997,

PROBLEM XIV.

Given the obliquity of the ecliptic, and a star's right ascension and declination;

Required its latitude and longitude.

EXAM. *What is the latitude and longitude of a star, its right ascension being 16 h. 14 m., its declination $25^{\circ} 51' S.$, and the obliquity of the ecliptic $23^{\circ} 29'$?*

CONSTRUCTION.

Let the primitive circle represent the solstitial colure, in which draw the equator EQ , mark its poles P, s , and describe (3d 968) the parallel of the star's declination nm .

The right ascension 16 h. 14 m. = $243^{\circ} 30'$, which being $63^{\circ} 30'$ above 180° , falls in the third quadrant; therefore make (451) $\gamma a = 63^{\circ} 30'$, describe (4th 968) the circle of right ascension, cutting the parallel nm in $*$, the point of the heavens representing the star.

Make $E\varpi = 23^{\circ} 29'$, the obliquity of the ecliptic, draw the ecliptic $\varpi\psi$, find its poles $p q$, and thro' $p * q$ describe a circle of longitude; then the arc $p *$ measured (446) will give the co-latitude, and the $\angle P p *$ will shew the longitude.

COMPUTATION.

In the oblique angled spheric triangle $p P *$.

Given the oblique ecliptic $p P = 23^{\circ} 29'$ } Required the co-lat. $p *$.
the co-declination $P * = 64^{\circ} 09'$ } and the longit. $\angle P p *$.
the right ascen. $\angle p P * = 153^{\circ} 30'$ } See art. 534, 533.

To find the latitude.

As Radius	= R	10,00000	As cos. 4th arc	= $158^{\circ} 45'$	0,03058
To cos. rt. asc.	= $153^{\circ} 30'$	9,95179	To cos. 5th arc	= $94^{\circ} 36'$	8,90417
So tan. ob. ecl.	= $23^{\circ} 29'$	9,63796	So cos. obl. ecl.	= $23^{\circ} 29'$	9,96245
To tan. 4th arc	= $21^{\circ} 15'$	9,58975	To sin. lat.	= $4^{\circ} 32'$	8,89720
Its sup. = 4 th arc	= $158^{\circ} 45'$		Here the latitude of the star is $4^{\circ} 32'$ N. because the declination is north, and the co-lat. reckoned from the N. pole is less than 90° .		
From which take	$64^{\circ} 09' = \text{co-decl.}$				
Leaves a 5th arc	= $94^{\circ} 36'$				

To find the longitude.

As sin. co-lat.	= $85^{\circ} 28'$	0,00136	Here the co-longitude $23^{\circ} 45'$ taken from 270° leaves $246^{\circ} 15'$ for the star's longitude, reckoning from the first point of Aries.		
To sin. co-decl.	= $64^{\circ} 09'$	9,95421			
So sin. rt. ascen.	= $153^{\circ} 30'$	9,64953			
To cos. longit.	= $23^{\circ} 45'$	9,60510			

Because the right ascension being in the third quadrant, the star is there also.

Now in $246^{\circ} 15'$ are 8 signs, 6 deg. 15 min.; that is, the star's place, or longitude, is $6^{\circ} 15'$ in the 9th sign, or $6^{\circ} 15'$ in π .

998. By the precession of the equinoxes, the fixed stars, altho' they always keep the same latitudes, yet they are continually altering their longitude, right ascension and declination; the alteration in longitude is uniformly 50 seconds yearly (848), but that of the right ascension and declination is constantly varying: So that many stars, which once had north decl., come to have south; while others change from S. to N. decl.

999.

999.

PROBLEM XV.

Given the right ascensions and declinations of two fixed stars;
Required their distance.

EXAM. *What is the distance between the fixed stars, Algol in the head of Medusa, and Aldebaran in Taurus; the former having $39^{\circ} 59'$ N. decl., with 2 h. 52 m. right ascension; and the other $15^{\circ} 59'$ N. decl., with 4 h. 22 m. of right ascension.*

CONSTRUCTION.

As Aldebaran precedes Algol in right ascension, let the primitive circle represent the circle of right ascension passing thro' Aldebaran; describe the circle of right ascension PAS, making with PBS an angle of $22^{\circ} 30'$, (451) equal to the difference between the given right ascensions.

Describe the parallels of declination Bm, rs, at the given distances $15^{\circ} 59'$ N., and $39^{\circ} 59'$ N. (3d 968); and the intersections B, of Aldebaran's declination, and A, of Algol's, with their respective circles of right ascension, will be the positions of those stars from one another: Then a great circle BAC, passing thro' B and A, the intercepted arc BA (measured by art. 446) shews the distance of those two stars.

COMPUTATION.

(534)

In the oblique angled spheric triangle PAB.

Given the co-decl. of Ald. PB = $74^{\circ} 01'$
the co-decl. of Algol. PA = $50 01$ } Required their dist. AB.
diff. of right ascen. $\angle APB = 22 30$

As Radius = R 10,00000

To cos. diff. rt. af = $22^{\circ} 30'$ 9,96562

So cot. Aldeb. dec. = $15 59$ 10,54298

To tan. 4th arc = $72 47$ 10,50868

And as cot. 4th arc = $72 47$ 0,52873

To cos. 5th arc = $22 46$ 9,96477

So sin. Aldeb. decl. = $15 59$ 9,43990

To cos. distance = $30 56$ 9,93340

From 4th arc = $72^{\circ} 47'$

Take Algol's co-decl. = $50 01$

Remains 5th arc = $22 46$

The same result would have come out, had the declination of the star Algol been used in the proportions.

1000.

PROBLEM XVI.

Given the latitudes and longitudes of two known fixed stars;
Required their distance.

EXAM. *Aliath, in the Great Bear, Lon. $\cap$ $4^{\circ} 55'$. Lat. $54^{\circ} 18'$ N.
Arcturus, in Bootes, Lon. $\simeq$ $20 15$. Lat. $30 57$ N.*

The construction of this problem is like that of the last; only instead of circles of right ascension, read circles of longitude, and use parallels of latitude instead of parallels of declination.

The computation is also like that of the last, there being given two co-latitudes and the included angle, which is the difference between the given longitudes: Thus Arcturus's long. is $6^{\circ} 20' 15''$, and Aliath's is $5^{\circ} 4^{\circ} 55'$; their difference is $1^{\circ} 15' 20''$, or $45^{\circ} 20'$.

Hence the distance will be found to be $39^{\circ} 42'$.

1661.

1001.

PROBLEM XVII.

Given the latitude and longitude of a fixed star, and also the obliquity of the ecliptic;

Required the right ascension and declination of that star.

EXAM. Suppose the latitude of a star is $7^{\circ} 09' N.$, its longitude $29^{\circ} 01'$: What is the right ascension and declination of that star, the obliquity of the ecliptic being $23^{\circ} 29'$?

The construction of this problem is much like that of Prob. XIV.; only here the intersection of a parallel of latitude cb with a circle of longitude pAq , will give the place of the star.

The computation is also as in Prob. XIV.; for here are given $Pp = 23^{\circ} 29'$, $pA = 82^{\circ} 51'$, and the $\angle PpA = 60^{\circ} 59'$, the co-longitude: To find PA the co-declination, and $\angle pPA$ the right ascension.

The declination of the star will be found to be $17^{\circ} 49' N.$

And the right ascension will be $114^{\circ} 18'$: But as the star's place is in the first quadrant of the ecliptic, its right ascension must be in the first quadrant of the equator; therefore $24^{\circ} 18'$ is the right ascension reckoned from Aries.

1002.

PROBLEM XVIII.

Given the meridional altitude of any celestial object, suppose a comet, its distance from a known star, and the latitude of the place;

Required the declination and right ascension of that comet.

EXAM. Suppose a comet was observed on the meridian at London, when its altitude was $51^{\circ} 55'$, and its distance from the star Arcturus was $59^{\circ} 47'$: What was the declination and right ascension of the comet at that time?

CONSTRUCTION.

In the primitive circle, representing the meridian of the place, draw the horizon HR and prime vertical ZN ; lay off the given latitude $RP = 51^{\circ} 32'$, draw the axis PS , the equator EQ , and (3d 968) nm Arcturus's parallel of declination $= 20^{\circ} 30'$. From the south point of the horizon lay off the given altitude of the comet $= 51^{\circ} 55'$ from H to O : About the point O as a pole, at the given distance between the comet and Arcturus, describe (456) a small circle aa cutting the parallel nm in $*$, the position of Arcturus at that time: Describe the circle of right ascension $P*s$, and a great circle thro' O and $*$.

COMPUTATION.

Since $HE = 38^{\circ} 28'$, the co-lat., and the alt. $HO = 51^{\circ} 55'$, then $EO = (HO - HE =) 13^{\circ} 27'$, is the decl. sought; which is north as the altitude exceeds the co-lat.; consequently the polar distance $OP = 76^{\circ} 33'$.

Then in the triangle $P*O$ are given the three sides to find the $\angle OP*$, the difference between the right ascensions of the comet and Arcturus. And (537) the $\angle OP*$ will be found $= 62^{\circ} 27' = 4$ h. 9 m. 48 s. which is the difference of their right ascensions: Now if Arcturus precedes the comet, the right ascension of the latter is 9 h. 53 m. 12 s.: But if the comet precedes, its right ascension is 18 h. 12 m. 48 s., the former being the diff., and the latter the sum of the right ascensions of Arcturus and the comet.

1003.

PROBLEM XIX.

Given the latitude of a place, the sun's declination and azimuth;
Required his altitude and the time of the observation.

EXAM. In the latitude of $13^{\circ} 30' N.$, and when the sun has $23^{\circ} 29' N.$ declination: What is the sun's altitude and time of the day, when he is seen on the ENE. azimuth circle?

CONSTRUCTION.

Let the primitive circle represent the meridian of the place, wherein HR represents the horizon, and ZN the prime vertical; make RP equal to the latitude, draw the 6 o'clock hour circle PS, the equator EQ, and (3d 968) the parallel of $23^{\circ} 29'$ of declination nm : The tangent of $67^{\circ} 30'$ being laid from the centre a towards H, gives the centre of the vertical circle ZDN, which cuts the parallel nm in the points A and B; and shews that at two distant times in the forenoon the sun will have the azimuth proposed: Thro' the points A and B describe the hour circles PAS, PBS (163); the angles ZPA, ZPB, shewing the times from noon, may be measured by art. 448; and the altitudes DA, DB, by art. 446.

COMPUTATION. See art. 529.

In the spheric triangle PZA, or PZB, there are known

The co-lat. $PZ = 76^{\circ} 30'$; the co-decl. PA, or PB $= 66^{\circ} 31'$; the azim. $\angle PZD = 67^{\circ} 30'$.

To find ZA, or ZB, and the $\angle ZPA$, or $\angle ZPB$.

As Radius : cos. azimuth :: cot. lat. : to tan. of a 4th arc $M = 57^{\circ} 54'$.

And as fin. lat. : fin. decl. :: cos. M : to cos. of a 5th arc $N = 24^{\circ} 54'$.

Then $M + N$, or $57^{\circ} 54' + 24^{\circ} 54' = 82^{\circ} 48' = ZA$, is the comp. of least alt.

And $M - N$, or $57^{\circ} 54' - 24^{\circ} 54' = 33^{\circ} 00' = ZB$, is the comp. of greatest alt.

Therefore, when the sun has $7^{\circ} 12'$, or $57^{\circ} 00'$ of alt. he is on the giv. azim.

Again. As cos. decl. : fin. azim. :: cos. least alt. : fin. hour fr. noon $88^{\circ} 02'$.

And as cos. decl. : fin. azim. :: cos. greater alt. : fin. hour fr. noon $33^{\circ} 16'$.

But $88^{\circ} 02' = 5^h 52^m$; and $33^{\circ} 16' = 2^h 13^m$, the respect. times fr. noon.

Consequently the sun will be seen on the ENE. azimuth at 6 h. 08 m., and again at 9 h. 47 m., both in the morning: Also, in the afternoon he will be on the WNW. azimuth at 2 h. 13 m., and at 5 h. 52 m.

1004. Now to find at what time, and at what altitude, the greatest azimuth will happen at that place on the said day.

As the azimuth circle in this case is to touch the parallel nm , therefore the greatest distance of the azimuth from the equator will be $23^{\circ} 29'$; and as their poles must be at the same distance (413), therefore a small circle rs described about the pole s , at the distance of $23^{\circ} 29'$ (454), its intersection p with the horizon, is the pole of the circle ZCN; then describe an hour circle PCS thro' p .

In the spheric triangle ZPC right angled at c. (414)

Given the co-lat. $PZ = 76^{\circ} 30'$, and the co-decl. $PC = 66^{\circ} 31'$.

Required the greatest azim. $\angle PZC = 70^{\circ} 36'$, the dist. $ZC = 54^{\circ} 08'$, and the hour from noon $= 56^{\circ} 27' = 3^h 46^m$.

1005. So that the azimuth is altered only $3^{\circ} 6'$ in 1 h. 33 m.; and consequently the variation of the compass in the torrid zone may be observed more accurately than elsewhere, could the altitudes when near the zenith be well taken.

SECTION

SECTION VI.

1006. *Of various methods to find the latitude.*

The usual way at sea to find the latitude, is from the sun's meridional altitude and declination; the manner of doing this will be particularly shewn in the next book: But as it frequently happens at sea, that the meridian altitude cannot be taken, therefore the mariner should be furnished with other means to come at this most useful knowledge of his latitude: To help him in this point, and as a farther exercise in the Astronomy of the sphere, the following problems are collected together.

1007. PROBLEM XX.

Given the sun's declination and his amplitude;
Required the latitude of the place.

EXAM. *Being in a place where the compass had no variation, on the first of May, 1753, I observed the sun to rise $62^{\circ} 30'$ from the north towards the east: Required the latitude of that place?*

CONSTRUCTION.

Having described the primitive circle, drawn the horizon HR, and (447) taken $RO = 62^{\circ} 30'$; then about O as a pole describe (454) a small circle, at the distance of $74^{\circ} 48' = \text{co-decl.}$, cutting the primitive in P the place of the north pole: Draw the axis PS, the equator EQ, and the circle POS, cutting the equator in A.

COMPUTATION.

In the spheric triangle γOA right angled at A.

Given the co-amp. $\gamma O = 27^{\circ} 30'$		Then $\text{f. } \gamma O : \text{Rad} :: \text{f. } AO : \text{f. } \angle A \gamma O.$
the declin. $AO = 15 \ 12$		Hence the latitude will be $55^{\circ} 24'$
Required the co-latitude $A \gamma O.$		N.

1008. PROBLEM XXI.

Given the sun's declination, and his ascensional difference;
Required the latitude of the place.

EXAM. *On the 20th of January, 1753, the sun was observed to set at 4 h. 30 m.: Required the latitude of the place?*

As the ascensional difference is the time the sun rises or sets before or after 6 o'clock; therefore $6 \text{ h.} - 4 \text{ h. } 30 \text{ m.} = 1 \text{ h. } 30 \text{ m.} = 22^{\circ} 30' = \text{ascensional difference.}$

CONSTRUCTION. In the primitive circle representing the meridian of the place, draw the equator EQ, the axis PS, the parallel of declination ro , $20^{\circ} 01' \text{ S.}$: Make $\gamma B = 22^{\circ} 30'$, the ascensional difference; describe the circle of right ascension PBS, cutting ro in O; then a diameter HR thro' O will be the horizon, and R p the lat. sought.

COMPUTATION. In the spheric triangle γBO , right angled at B.

Given the asc. diff. $\gamma B = 22^{\circ} 30'$		Then $\text{Rad} : \cot. OB :: \text{f. } \gamma B : \cot. \angle O \gamma B.$
the declin. $OB = 20 \ 01$		Or $\text{Rad} : \cot. \text{decl.} :: \text{fin. rt. asc.} : \tan. \text{lat.}$
Required the co-lat. $\angle O \gamma B.$		Hence the lat. will be $46^{\circ} 25' \text{ N.}$, being contrary to the declin. when the ascen. diff. falls between noon and six.

1009.

PROBLEM XXII.

Given the sun's declination, and altitude at six o'clock;
Required the latitude of the place.

EXAM. *Being at sea on the 20th of May, 1753, at six o'clock in the evening, the sun's altitude was $18^{\circ} 45'$: What was the latitude of the place of observation?*

CONSTRUCTION. Having described the meridian, draw the horizon HR, the prime vertical ZN, and the parallel st of $18^{\circ} 45'$ altitude (10th 968); from the centre γ , with the half tangent of the declination $= 20^{\circ} 04'$, cut the parallel st in o : Thro' o draw the axis PS, and the azimuth circle ZON (163), and the measure of RP will give the latitude sought.

COMPUTATION. In the spheric triangle γAO , right angled at A.

Given the decl. $\gamma O = 20^{\circ} 04' N$,	Then $\sin. \gamma O : \text{Rad} :: \text{f. } AO : \text{f. } \angle O \gamma A$. Or $\sin. \text{decl.} : \text{Rad} :: \sin. \text{alt.} : \sin. \text{latit.}$ Which is $69^{\circ} 32' N$., as the decl. is N.
the altit. $AO = 18^{\circ} 45'$	
Required the lat. $= \angle O \gamma A$.	

1010.

PROBLEM XXIII.

Given the sun's declination, and altitude when he is due east or west;
Required the latitude of the place.

EXAM. *In a place where the compass had no variation, the sun was observed to be due east when his declination was $16^{\circ} 38' N$., and his altitude $20^{\circ} 12'$: What is the latitude of that place?*

CONSTRUCTION. In the meridian HZRN, draw the horizon HR, the prime vertical ZN, and make $\gamma O =$ half tan. of the alt. $20^{\circ} 12'$: About o as a pole, at the distance of $73^{\circ} 22'$, the co-decl., describe (454) a small circle, cutting the meridian in P the elevated pole; draw the axis PS, equator EQ, and thro' P, o, s, describe an hour circle POS; then the measure of PR shews the latitude.

COMPUTATION. In the spheric triangle γAO , right angled at A.

Given the alt. $\gamma O = 20^{\circ} 12'$	Then $\sin. \gamma O : \text{Rad} :: \sin. AO : \sin. \angle A \gamma O$. Or $\sin. \text{alt.} : \text{Rad} :: \sin. \text{decl.} : \sin. \text{lat.}$ Which is $56^{\circ} 00' N$., as the decl. is N.
the decl. $AO = 16^{\circ} 38' N$	
Required the latit. $= \angle A \gamma O$.	

But had the declination been S., the other intersection of the parallel circle and meridian must have been taken for the elevated pole, and the latitude would be south.

$$\sin \phi = \frac{\sin d}{\sin h}$$

1011.

PROBLEM XXIV.

Given the sun's altitude and hour of the day on either equinox;
Required the latitude of the place.

EXAM. *On the day the sun enter'd the vernal equinox, his latitude was found $22^{\circ} 56'$ at 9 o'clock in the morning: In what lat. was that observation made?*

CONSTRUCTION. Describe the meridian, draw the horizon, the prime vertical, and (10th 968) the parallel st of $22^{\circ} 56'$ altitude; from the centre γ , with the half tan. of $45^{\circ} = 3 h$., the ascen. diff., cut st in o , and describe the vertical circle ZON, cutting the horizon in B.

COMPUTATION. In the spheric triangle γBO , right angled at B.

Given the time after 6 $\gamma O = 45^{\circ} 00'$	As $\text{f. } \gamma O : \text{Rad} :: \text{f. } BO : \text{f. } \angle B \gamma O$. Or $\text{f. time } \hat{=} 6 : \text{Rad} :: \text{f. alt.} : \cos. l$. Which is $56^{\circ} 34'$.
the altitude $BO = 22^{\circ} 56'$	
Required the co-latitude $B \gamma O$.	

1012.

1012.

PROBLEM XXV.

Given the sun's altitude, declination and azimuth;
Required the latitude of the place.

EXAM. *June 4th, 1753, being at sea in a place where the compass had no variation, in the afternoon when the sun was $42^{\circ} 30'$ high, his bearing was S. $57^{\circ} 45'$ W.: What is the latitude of that place?*

CONSTRUCTION. Draw the meridian, the horizon HR, the prime vertical ZN, and the parallel st at $42^{\circ} 30'$ above the horizon (456): The tangent of $57^{\circ} 45'$ set from A towards R gives the centre of the azimuth circle ZON, cutting the parallel of altitude st in o: About o as a pole (454), at the distance of $67^{\circ} 30'$, equal to the co-declination, describe a small circle, cutting the meridian in P the place of the pole; then the measure of RP gives the latitude sought.

COMPUTATION. In the oblique angled spheric triangle ZOP.

Given the zenith dist. ZO = $47^{\circ} 30'$
the polar dist. PO = $67^{\circ} 30'$
the azimuth $\angle PZO = 122^{\circ} 15'$ } Required the co-latitude PZ.

As Rad	= R	10,00000	As fin. alt.	= $42^{\circ} 30'$	0,17032
To cos. azim.	= $122^{\circ} 15'$	9,72723	To fin. decl.	= $22^{\circ} 30'$	9,58284
So cot. alt.	= $42^{\circ} 30'$	10,03795	So cos. 4th	= $30^{\circ} 13'$	9,93658
To tan. 4th	= $30^{\circ} 13'$	9,76518	To cos. 5th	= $60^{\circ} 42'$	9,68974

Then the difference between the 5th and 4th arcs, that is $30^{\circ} 13'$ taken from $60^{\circ} 42'$, the remainder $30^{\circ} 29'$ is the co-lat. Therefore $59^{\circ} 31'$ N. is the latitude sought.

1013.

PROBLEM XXVI.

Given the sun's declination, his altitude and hour of the day;
Required the latitude of the place.

EXAM. *Being at sea, the sun's altitude was observed to be $37^{\circ} 20'$ at 9 h. 45 m. in the morning, his declination at that time being $20^{\circ} 30'$ N.: What is the latitude of the place of observation?*

CONSTRUCTION. In the meridian PESQ, draw the equator EQ, axis ps, and parallel of declinat. nm , $22^{\circ} 30'$ dist. from the equator (3d 968): Set off from the centre A towards Q the tang. of $33^{\circ} 45' = 2$ h. 15 m., the distance between the time of observation and noon, which gives the centre of the hour circle POS, cutting the parallel nm in o: About the point o as a pole, describe (454) at the dist. of $52^{\circ} 40'$, the zen. dist., a small circle, cutting the meridian in z the zenith; thro' zo describe an azimuth circle ZON; then the measure of ZE will give the lat. sought.

COMPUTATION. In the oblique angled triangle ZOP.

Given the zenith distance ZO = $52^{\circ} 40'$
the polar distance PO = $67^{\circ} 30'$
the hour from noon $\angle ZPO = 33^{\circ} 45'$ } Required the co-lat. ZP.

As Rad : cos. hour A. M. :: cot. decl. : tan. 4th arc = $63^{\circ} 31'$.

As fin. decl. : fin. alt. :: cos. 4th arc : cos. 5th arc = $45^{\circ} 02'$.

Their difference is the co-latitude $18^{\circ} 29'$. Therefore the lat. is $71^{\circ} 31'$ N.

1014.

$$\begin{aligned} \tan x &= \cos s \cot d \\ \cot y &= \frac{\sin h \cot x}{\sin d} \end{aligned}$$

Given the altitudes of two known fixed stars, having the same azim.;
Required the latitude of the place.

EXAM. Being at sea in an unknown latitude, I observed the star Schedar in Cassiopeia, and Almaach in Andromeda, to have the same Azimuth, when the altitude of Schedar was $37^{\circ} 15'$, and that of Almaach was $17^{\circ} 45'$: What is the latitude of that place?

CONSTRUCTION upon the plane of the equator. Let the primitive circle represent the equator, whose pole is P, and any point γ the place where the right ascension begins, from whence lay off $\gamma a = 27^{\circ} 15'$ for Almaach's right ascension, and $\gamma b = 6^{\circ} 45'$ for Schedar's; draw the circles of right ascension Pa, Pb: Describe (3d 970) Almaach's and Schedar's parallels of declination, cutting Pa, Pb, in A, B; A being Almaach, and B Schedar. A great circle passing thro' A and B, will be the azimuth they are on. About B at the distance of $52^{\circ} 54'$, Schedar's zenith distance, describe (454) a small circle, cutting the azimuth circle thro' A and B in Z, the zenith of the place; draw PZ, which measured on the half tangents gives the co-latitude of the place of observation.

COMPUTATION.

1st. In the oblique angled spheric triangle ABP.

Given Almaach's co-dec. PA = $48^{\circ} 52'$ | Required the angle of position ABP.
Schedar's co-dec. PB = $34^{\circ} 49'$ |
their diff. of alt. AB = $19^{\circ} 30'$ | For the solution see art. 550.

PB = $34^{\circ} 49'$	Co-ar. fin. PB = $34^{\circ} 49'$	0,24340
BA = $19^{\circ} 30'$	Co-ar. fin. BA = $19^{\circ} 30'$	0,47650
PA = $48^{\circ} 52'$	Sin. $\frac{1}{2}$ sum = $32^{\circ} 05\frac{1}{2}'$	9,72532
PB - BA = $15^{\circ} 19'$	Sin. $\frac{1}{2}$ diff. = $16^{\circ} 46\frac{1}{2}'$	9,46031
64 11 $32^{\circ} 05\frac{1}{2}' = \frac{1}{2}$ sum.	Sum of four logs.	19,90553
33 33 $16^{\circ} 46\frac{1}{2}' = \frac{1}{2}$ diff.	$\frac{1}{2}$ sum gives $63^{\circ} 45\frac{1}{2}'$	9,95276
	Which doub. gives $127^{\circ} 31' = \angle ABP$.	

2d. In the oblique spheric triangle PBZ.

Given Schedar's co-dec. PB = $34^{\circ} 49'$ | Required the co-lat. PZ.
Schedar's co-alt. BZ = $52^{\circ} 45'$ | For the solution see art. 548.
angle of position PBZ = $52^{\circ} 29'$ | Here $\angle PBZ = \text{suppl. } \angle PBA$.

As Rad = R	10,00000	As cos. 4th arc = $22^{\circ} 57'$	0,03581
To cos. $\angle$ posit. = $52^{\circ} 29'$	9,78461	To cos. 5th arc = $29^{\circ} 48'$	9,93840
So cot. Sch. decl. = $55^{\circ} 11'$	9,84227	So sin. Sch. decl. = $55^{\circ} 11'$	9,91433
To tan. 4th arc = $22^{\circ} 57'$	9,62688	To sin. latit. = $50^{\circ} 41'$	9,88854
Which taken from $52^{\circ} 45'$	= BZ.		
Leaves 5th arc	$29^{\circ} 48'$		

1015.

PROBLEM XXVIII.

Given the difference of time between the rising of two known stars ;
Required the latitude of the place.

EXAM. *Being at sea in an unknown place, the star Aldebaran was observed to rise 3 h. 15 m. later than the bright star in Aries : Required the latitude of that place?*

Bright star in γ decl. $22^{\circ} 17' N.$; right ascen. 1 h. 53 m. Aldebaran's decl. $15^{\circ} 59' N.$; right ascension 4 h. 22 m.

CONSTRUCTION.

Let the primitive circle represent the equator, describe (3d 970) the parallels of declination of the two stars, that of Aries being $22^{\circ} 17' N.$, and of Aldebaran $15^{\circ} 59' N.$: Draw Pa for the circle of right ascension passing thro' the star in γ , which suppose in A : From a lay off $ab = 37^{\circ} 15' =$ diff. of right ascensions ; and $ac = 48^{\circ} 45' = 3 h. 15 m.$, the diff. of time between their rising ; draw bP, cP , cutting the parallels of declination in B, C : Thro' the points B, C , describe (441) the great circle HOR ; draw PO at right angles to HR , then the measure of PO will give the latitude sought.

COMPUTATION.

In the oblique angled spheric triangle PBC .

Given B 's co-decl. $PB = 74^{\circ} 01'$		Rad : cos. $\angle BPC :: t.PB : t.M = 73^{\circ} 42'$.
A 's co-decl. $PC = 67 \ 43$		From M take PC leaves $N = 5 \ 59$.
$\angle APC - \angle APB = \angle BPC = 11 \ 30$		As fin. $N : f. M :: \tan. \angle CPB : t. \angle C =$
Required the angle PCB .		Hence $\angle PCO = 61^{\circ} 54'$. [$118^{\circ} 06'$.

In the spheric triangle PCO , right angled at O .

Given C 's co-decl. $PC = 67^{\circ} 43'$		As Rad : fin. $PC :: \sin. \angle PCO : \sin. PO$
and the $\angle PCO = 61 \ 54$		[$= 54^{\circ} 43'$.
Required the latitude PO .		Therefore the latitude is $54^{\circ} 43'$.

1016.

PROBLEM XXIX.

Two known fixed stars being observed to have the same altitude ;
Required the latitude of the place of observation.

EXAM. *In the evening, the stars Capella and Procyon were observed at the same time to have each $38^{\circ} 00'$ of altitude : Required the latitude of the place where that observation was taken?*

Capella's decl. $= 45^{\circ} 43' N.$ right ascen. 4 h. 59 m. Procyon's decl. $= 5^{\circ} 49' N.$ right ascen. $= 7 h. 24'$.

CONSTRUCTION.

On the plane of the equator, represented by the primitive circle, describe (3d 970) the parallels of the given declinations ; take $ab = 36^{\circ} 15'$, the diff. of the given right ascensions ; draw Pa, Pb , then A represents Procyon, and B Capella.

About A and B as poles, describe (454) arcs of small circles, at the distance of 52° , the co-altitude, and their intersection gives Z the zenith of the place : Thro' the points $A, B ; Z, A ; Z, B$; describe great circles (441), and draw ZP , whose measure will give the co-latitude of the place of observation.

COMPU-

COMPUTATION.

In the oblique angled spheric triangle APB.

Given A's co-decl. $AP = 84^{\circ} 11'$	Rad: cof. $\angle P :: t. BP : t. M = 38^{\circ} 11'$
B's co-decl. $BP = 44^{\circ} 17'$	And M tak. fr. PA leaves $N = 46^{\circ} 00'$
diff. rt. asc. $\angle ABP = 36^{\circ} 15'$	Sin. $N : \sin. M :: \tan. \angle P : \tan. \angle BAP$
Required the stars distance AB.	$[= 32^{\circ} 13']$
And the angle BAP.	Cof. $M : \cos. N :: \cos. BP : \cos. BA = 50^{\circ} 45'$

In the oblique angled spheric triangle AZB.

Given A's co-alt. $ZA = 52^{\circ} 00'$	The angle BAZ will be found $=$
B's co-alt. $ZB = 52^{\circ} 00'$	$68^{\circ} 15'$
stars distance $AB = 50^{\circ} 45'$	Then $\angle BAZ - \angle BAP = \angle PAZ =$
Required the angle ZAB.	$36^{\circ} 02'$

In the spheric triangle AZP.

Given A's co-alt. $ZA = 52^{\circ} 00'$	Rad: cof. $\angle A :: t. ZA : t. M = 45^{\circ} 59'$
A's co-lat. $AP = 84^{\circ} 11'$	M taken from PA leaves $N = 38^{\circ} 12'$
the angle $ZAP = 36^{\circ} 02'$	Cof. $M : \cos. N :: \cos. ZA : \cos. PZ = 45^{\circ} 52'$
Required the co-lat. ZP.	Therefore the latitude is $44^{\circ} 08' N$.

1017.

PROBLEM XXX.

Given the altitudes of two known fixed stars;

Required the latitude of the place.

EXAM. The altitude of the Hydra's heart was observed to be $40^{\circ} 44'$, and of the Lion's heart $45^{\circ} 00'$: What is the latitude of the place of observation?

Hydra's heart, decl. $= 7^{\circ} 37' S$.; right ascen. $= 9^h. 16^m$. Lion's heart, decl. $= 13^{\circ} 09' N$.; right ascen. $9^h. 53^m$.

CONSTRUCTION.

If this problem is constructed on the plane of the equator, it will be in every respect like the last; only the small circles, described about A and B, are to be unequally distant from their respective poles A, B.

COMPUTATION.

Here, as in the last, there will be three spheric triangles to work in; namely, the triangles APB, ZAB, and ZPB.

In the triangle APB, where $AP = 97^{\circ} 37'$, $BP = 76^{\circ} 51'$, $\angle APB = 9^{\circ} 15'$.

As Rad: cof. $\angle APB :: \tan. BP : \tan. M = 76^{\circ} 41'$. Then $AP - M = N = 20^{\circ} 56'$.

As $\sin. N : \sin. M :: \tan. \angle APB : \tan. \angle BAP = 23^{\circ} 55'$.

And as $\cos. M : \cos. N :: \cos. BP : \cos. BA = 22^{\circ} 42'$.

In the triangle BAZ, where $AZ = 49^{\circ} 16'$, $BZ = 45^{\circ} 00'$, $AB = 22^{\circ} 42'$.

The angle BAZ will be found equal to $68^{\circ} 56'$. (537)

Then $\angle BAZ - \angle BAP = \angle PAZ = 45^{\circ} 01'$.

In the triangle APZ, where $AP = 97^{\circ} 37'$, $AZ = 49^{\circ} 16'$, $\angle PAZ = 45^{\circ} 01'$.

As Rad: cof. $\angle PAZ :: \tan. AZ : \tan. M = 39^{\circ} 23'$. Then $AP - M = N = 58^{\circ} 14'$.

And as $\cos. M : \cos. N :: \cos. AZ : \cos. PZ = 63^{\circ} 23'$.

Hence the latitude sought is $63^{\circ} 23' N$.

1018.

1018.

PROBLEM XXXI.

Given the sun's declination, two altitudes, and the time between the observations;

Required the latitude of the place.

EXAM. On a day when the sun's declination was $20^{\circ} 00' N.$, in the forenoon the sun's altitude was observed to be $18^{\circ} 30'$, and three hours after, his altitude was $44^{\circ} 00'$: What was the latitude of the place?

CONSTRUCTION.

Let the primitive circle represent that hour circle on which the sun was at the first observation, EQ being the equator, then Aa , the parallel of 20° of declination, gives A the sun's place at first; and as CQ is the tangent of 45° , Q will be the centre of the hour circle PBS three hours distant from the former, its intersection B with the parallel of declination, is the sun's place at the second observation: About A as a pole, at the distance of $71^{\circ} 30'$, the first zenith distance, describe (454) a small circle; about B as a pole, at the distance of $46^{\circ} 00'$, the second zenith distance, describe (454) another small circle, cutting the former in Z the zenith: Thro' Z , A ; Z , B ; A , B ; P , Z ; describe (441) great circles; then PZ is the co-latitude required.

COMPUTATION.

Here are three triangles to work in; namely, ABP , ABZ , BPZ .

In the isosceles spheric triangle APB .

Given $AP = 70^{\circ} 00'$	Suppose a perpendicular Pb is drawn.
$BP = 70 00$	
$\angle APB = 45 00$	
Req. $\angle ABP$ and AB .	
	Rad: tan. $Bpb :: \cos. PB : \cot. \angle PBA = 81^{\circ} 56'$.
	Rad: sin. $PB :: \sin. \angle Bpb : \sin. Bb = 21^{\circ} 04\frac{1}{2}'$.
	Then Bb doubled gives $AB = 42^{\circ} 09'$.

In the oblique angled spheric triangle ABZ .

Given $AZ = 71^{\circ} 30'$	Then working by art. 537 for three sides, the
$BZ = 46 00$	
$AB = 42 09$	
Required $\angle ABZ$.	
	angle ABZ will be found $= 114^{\circ} 11'$.
	And $\angle ABZ - \angle PBA = \angle PBZ = 32^{\circ} 15'$.

In the oblique angled spheric triangle PBZ .

Given $PB = 70^{\circ} 00'$	As Rad: $\cos. \angle PBZ :: \tan. BZ : \tan. M = 41^{\circ} 13'$.
$BZ = 46 00$	
$\angle PBZ = 32 15$	
Requir. the co-lat. PZ .	
	And $PB - M = N = 28^{\circ} 47'$.
	As $\cos. M : \cos. N :: \cos. BZ : \cos. PZ = 35^{\circ} 58'$.
	Therefore the latitude is $54^{\circ} 02' N.$

1019. If the sun's altitude can be taken both before and after noon, when he has equal heights, then the time between these two observations being bisected, will give the time when the sun was on the meridian: Now the co-declination, the co-altitude, and the time from noon at either observation being known, the latitude may be readily computed in one oblique angled triangle, wherein are known two sides, and an angle opposite to one of them to find the other side, which is the co-latitude; for which see the problem, art. 1013.

1020.

PROBLEM XXXII.

Given the sun's declination, two altitudes, and the difference of the magnetic azimuths;

Required the latitude of the place.

EXAM. On the 21st of May, the sun's declination being $20^{\circ} 16' N.$, in the morning when the sun was on the ESE. point of the compass, his altitude was $43^{\circ} 30'$; and when he bore S. $20^{\circ} 30' E.$ his altitude was $58^{\circ} 30'$: What is the latitude of the place of observation?

CONSTRUCTION.

Let the primitive circle represent the azimuth circle the sun was on at the greatest altitude $58^{\circ} 30'$, A being the sun's place at that time, HR the horizon, and z the zenith; draw aa a parallel of $43^{\circ} 30'$ of altitude, and (451) describe a vertical circle, making an angle with Az, of $47^{\circ} 00'$, the diff. of the observed azimuths; where this cuts the parallel of altitude aa , gives B the sun's place at the first observation: Then small circles being described about A and B, as poles, at the distances of $69^{\circ} 44'$, the co-declination (454), their intersection will give P the place of the pole: Thro' A, B; P, A; P, B; and z, P, describe great circles (441); then Pz is the co-latitude sought.

COMPUTATION.

1st. In the spheric triangle AZB: Given AZ = A's co-alt. = $31^{\circ} 30'$
 BZ = B's co-alt. = $46 30$
 $\angle AZB$ = diff. of az. = $47 00$

Required $\angle ABZ$ = $45 41$
 AB = $32 17$

2^d. In the spheric triangle APB: Given AP = A's co-decl. = $69 44$
 BP = B's co-decl. = $69 44$
 AB = distance = $32 17$

Required $\angle ABP$ = $83 52$

3^d. In the spheric triangle ZBP: Given BZ = B's co-alt. = $46 30$
 PB = B's co-decl. = $69 44$
 $\angle ZBP$ = $38 11$

Requir. zP = co-lat. = $39 21$

Therefore the latitude of the place is $50^{\circ} 39' N.$

1021.

PROBLEM XXXIII.

Two known stars being observed on the same azimuth, and two other known stars being observed on another azimuth, and the time between the observations being known; to find the latitude of the place.

EXAM. The stars Aldebaran in Taurus, and Regel in Orion, were observed on the same azimuth; and 2 h. 35 m. after, the stars Castor in Gemini and the Hydra's heart were also observed on another azimuth: What was the latitude of the place of observation?

CON-

CONSTRUCTION.

On the plane of the equator put the stars Aldebaran and Regal at A, B, also the stars Castor and Hydra at c, d, by the means of their right ascensions and declinations (2d and 3d of 970): Let the stars c, d, be moved forwards $2^h 35^m$, or $38^\circ 45'$ to C, D; thro' A, B, and D, C, describe (441) great circles intersecting in z, the zenith of the place; draw the great circles AC and PZ; then the measure of PZ gives the co-latitude required.

COMPUTATION.

Aldebaran's declin.	= $15^\circ 59' N.$	right ascen.	= $4^h 22^m = 65^\circ 30'.$
Regal's	= $8^\circ 28' S.$		= $5^h 01^m = 75^\circ 15'.$
Castor's	= $32^\circ 24' N.$		= $7^h 17^m = 109^\circ 15'.$
Hydra's heart	= $7^\circ 37' S.$		= $9^h 16^m = 139^\circ 00'.$

1st. In the triangle PAB, where $PB = 98^\circ 28'$, $PA = 74^\circ 01'$, $\angle APB = 9^\circ 15'.$

Then the $\angle PAB$ will be $157^\circ 46'$, and the $\angle PAZ$, the suppl. = $22^\circ 14'.$

2d. In the triangle PCD, where $PD = 97^\circ 37'$, $PC = 57^\circ 36'$, $\angle CPD = 29^\circ 45'.$

Then the $\angle PCD$ will be $139^\circ 21'$, and the $\angle PCZ$, the suppl. = $40^\circ 39'.$

3d. In the triangle APC, where $AP = 74^\circ 01'$, $CP = 57^\circ 36'$, $\angle APC = 5^\circ 00'$, which is the difference between 2 h. 25 m. and the difference of the right ascensions of A and c.

Then the $\angle PAC$ will be $14^\circ 06'$, $\angle PCA = 163^\circ 23'$, and $AC = 16^\circ 57'.$

Now $\angle PAC + \angle PAZ = \angle CAZ = 36^\circ 20'$; and $\angle PCA - \angle PCZ = \angle ACZ = 122^\circ 44'.$

4th. In the triangle ACZ, where $AC = 16^\circ 57'$; $\angle CAZ = 36^\circ 20'$; $\angle ACZ = 122^\circ 44'.$

Then cz will be found equal to $24^\circ 58'.$

5th. In the triangle CPZ, where $cz = 24^\circ 58'$, $CP = 57^\circ 36'$, $\angle PCZ = 40^\circ 39'.$

Then PZ will be found equal to $40^\circ 52'.$

And the latitude of the place of observation is $49^\circ 08' N.$

There might be given a great variety of other problems to find the latitude from various circumstances; but the trouble of solving them, as well as some of the foregoing ones, is too great to render them of general use: And indeed, some of them were only inserted as trigonometrical exercises for young students; it being generally allowed, that the sciences are most readily learned by working of many examples: And on this account it was judged, that the few following questions might not be unentertaining to those who have a love for these matters.

SECTION VII.

Miscellaneous Problems.

1022.

PROBLEM XXXIV.

In the latitude of $20^{\circ} 00' N.$ stands a horizontal dial, whose gnomon is perpendicular to the plane of the horizon: It is required to know at what hour in the afternoon, on the 21st of June 1753, the shadow of that gnomon shall stand still; and how many degrees shall the shadow run back?

CONSTRUCTION. Let the circle $zRNH$ be the meridian of the place, HR the horizon, whose poles are z, N ; $RP = 20^{\circ}$, the latitude, P and s the north and south poles: About P describe the parallel of decl. aa , cutting the horizon in L ; about s a small circle, being described at the dist. of $23^{\circ} 29'$, its intersection p with the horizon, is the pole of the azimuth circle which will touch the parallel aa in $\odot$, the place of the sun when he has the greatest azimuth that day: Thro' $z, \odot, N$, describe a vertical circle cutting the horizon in K ; and thro' $\odot$ and L describe the hour circles $P\odot S$, PLS .

Then will the $\angle ZP\odot$ be equivalent to the hour when the shadow will stand still; and KL , the difference between the measures of the azimuth and amplitude, will shew how much the shadow will run back.

COMPUTATION. In the right angled spheric triangle PRL .

Given $\angle R = 90^{\circ} 00'$
 lat. $= RP = 20^{\circ} 00'$
 co-dec. $= PL = 66^{\circ} 31'$

Requir. ampl. $= RL = 64^{\circ} 55'$

Then $33^{\circ} 06' = 2$ h. 12 m. 24 f.: And $77^{\circ} 26' - 64^{\circ} 55' = 12^{\circ} 31'$.

So that the shadow will stand still at 2 h. 12 m. 24 f., and will run back $12^{\circ} 31'$.

In rt. angl. spheric triangle $P\odot z$.

Given $\angle \odot = 90^{\circ} 00'$
 co-lat. $= zP = 70^{\circ} 00'$
 co-decl. $= P\odot = 66^{\circ} 31'$

Required hour $= \angle ZP\odot = 33^{\circ} 06'$
 azim. $= \angle \odot zP = 77^{\circ} 26'$

1023.

PROBLEM XXXV.

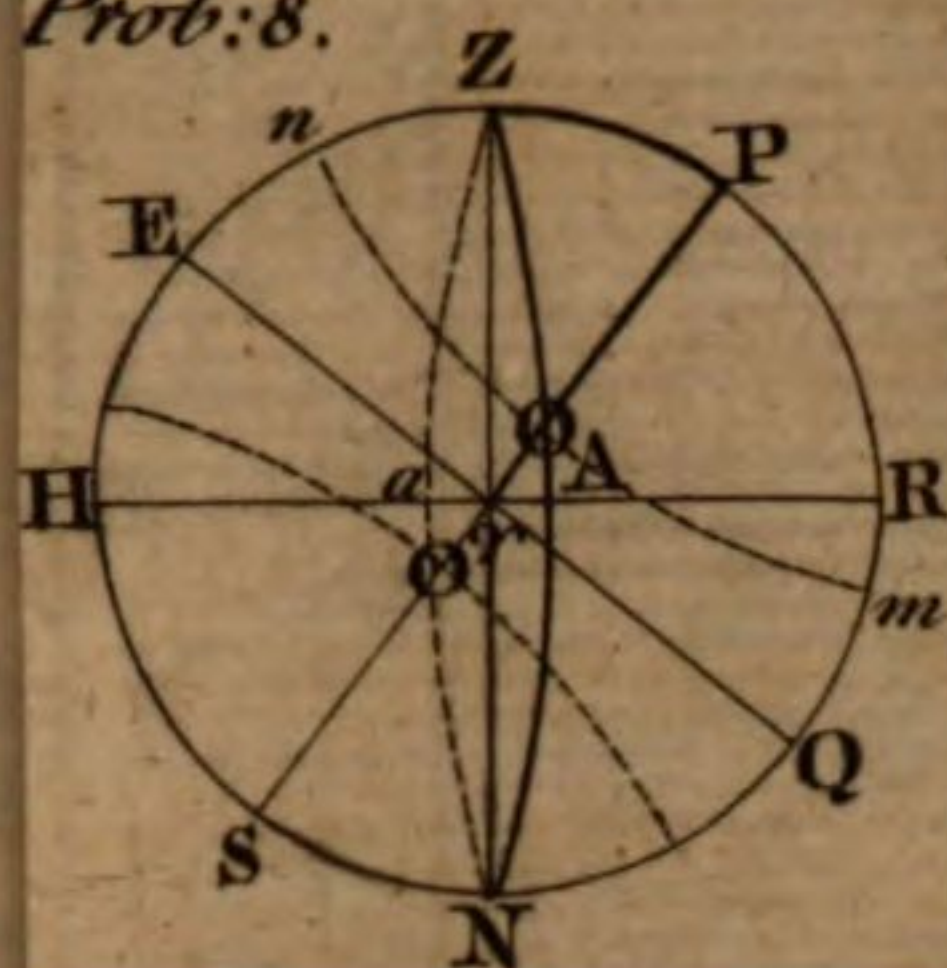
A comet, whose declination was $47^{\circ} 00' N.$, was observed to be distant from a star to the eastward $49^{\circ} 00'$; the star's declination was $36^{\circ} 00' N.$, and its right ascension $45^{\circ} 00'$: What was the latitude and longitude of that comet?

CONSTRUCTION.

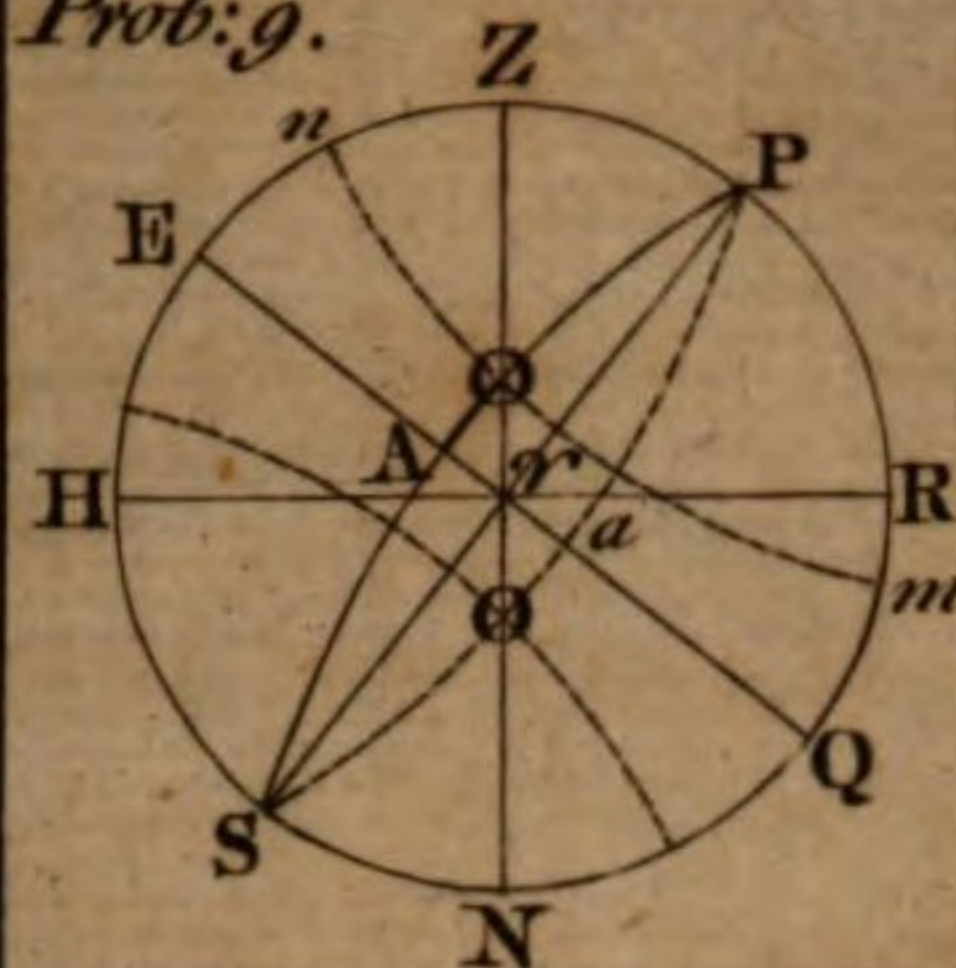
On the plane of the solstitial colure, where P and E are the poles of the equator and ecliptic, put the star at A by its right ascension and declination: About P and A describe small circles, at the distances of the comet from those points, their intersection c gives the place of the comet; describe great circles thro' AC, PC, EC , then EC will be the co-latitude, and the $\angle PEC$ the co-longitude; and their measures may be obtained from articles 446, 448;

COMPU-

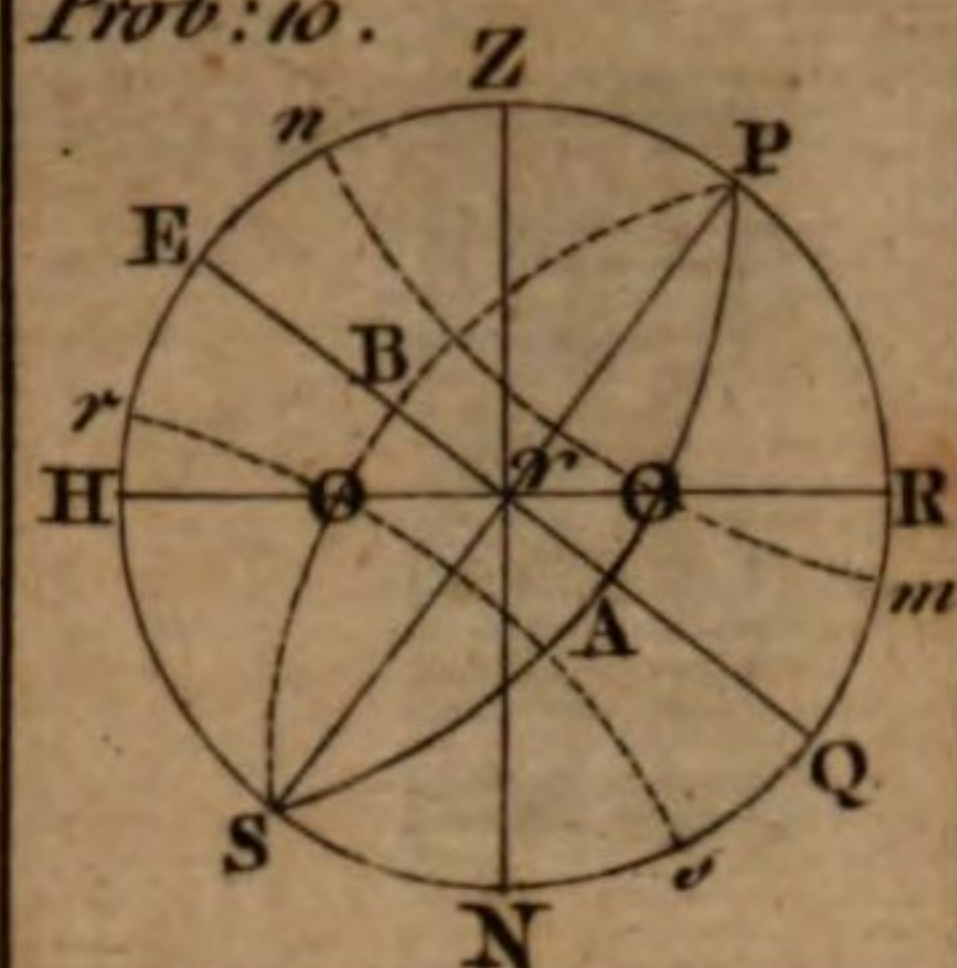
Prob: 8.



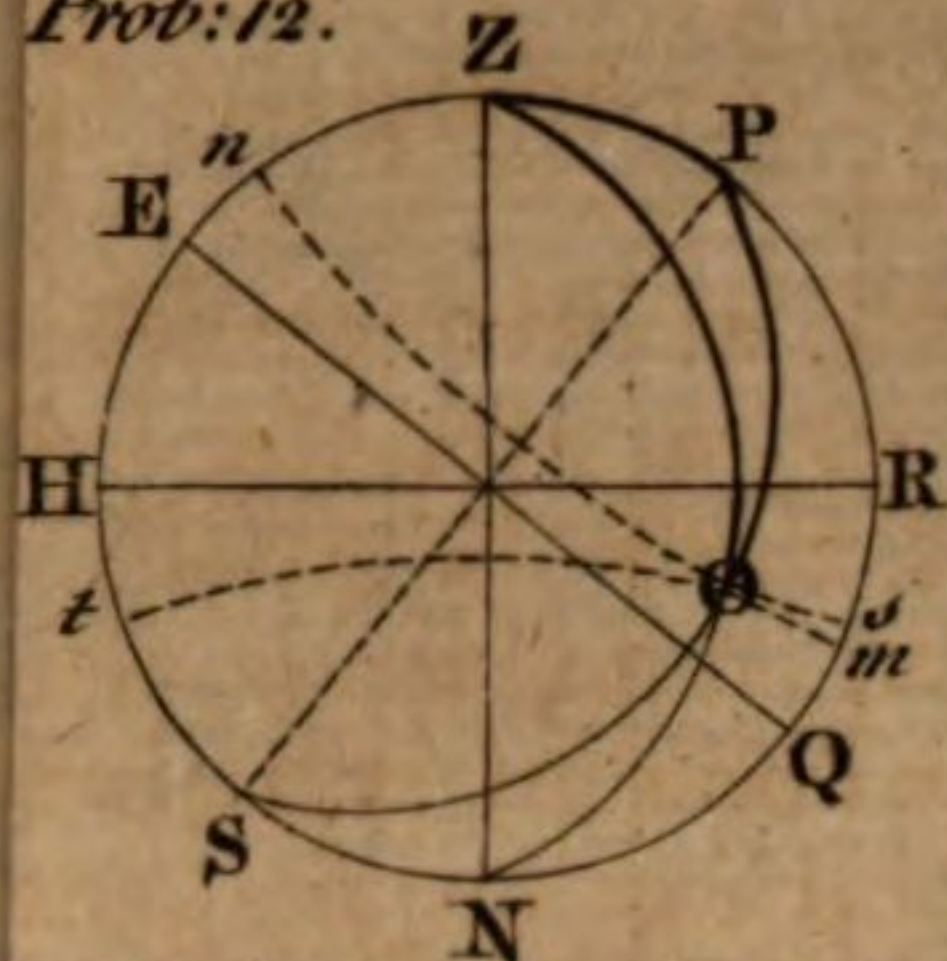
Prob: 9.



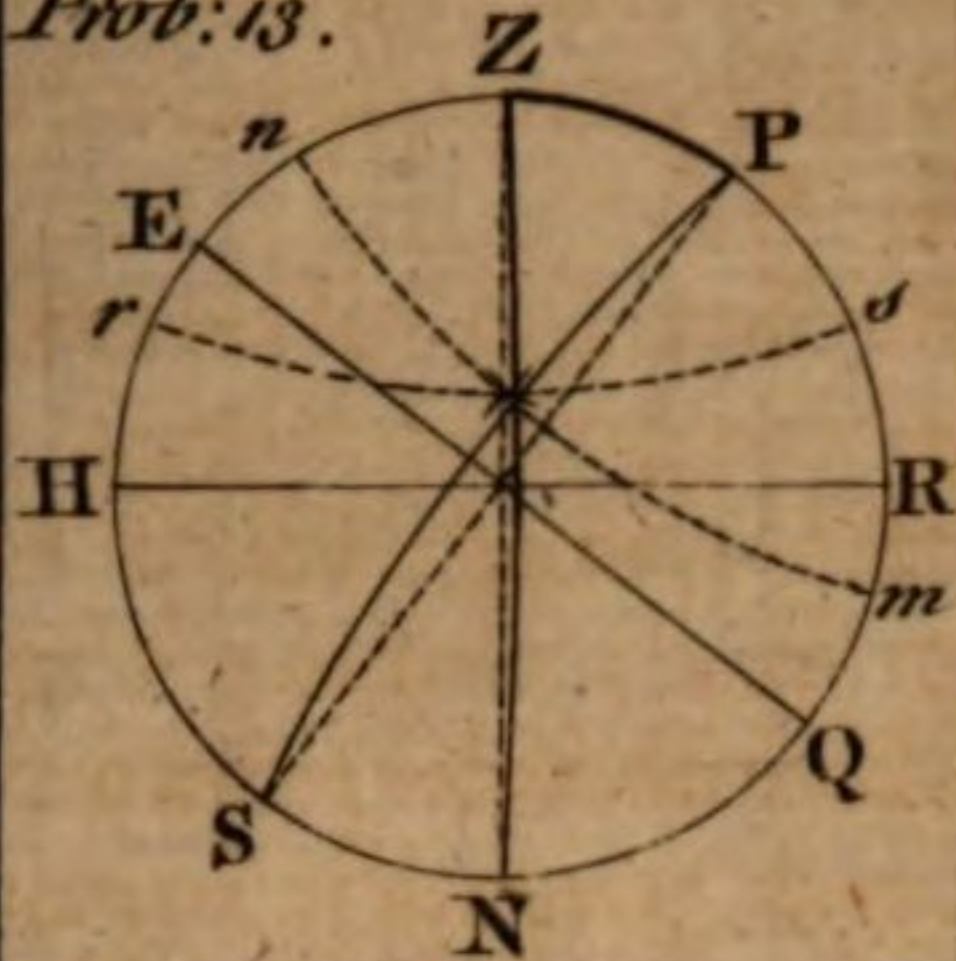
Prob: 10.



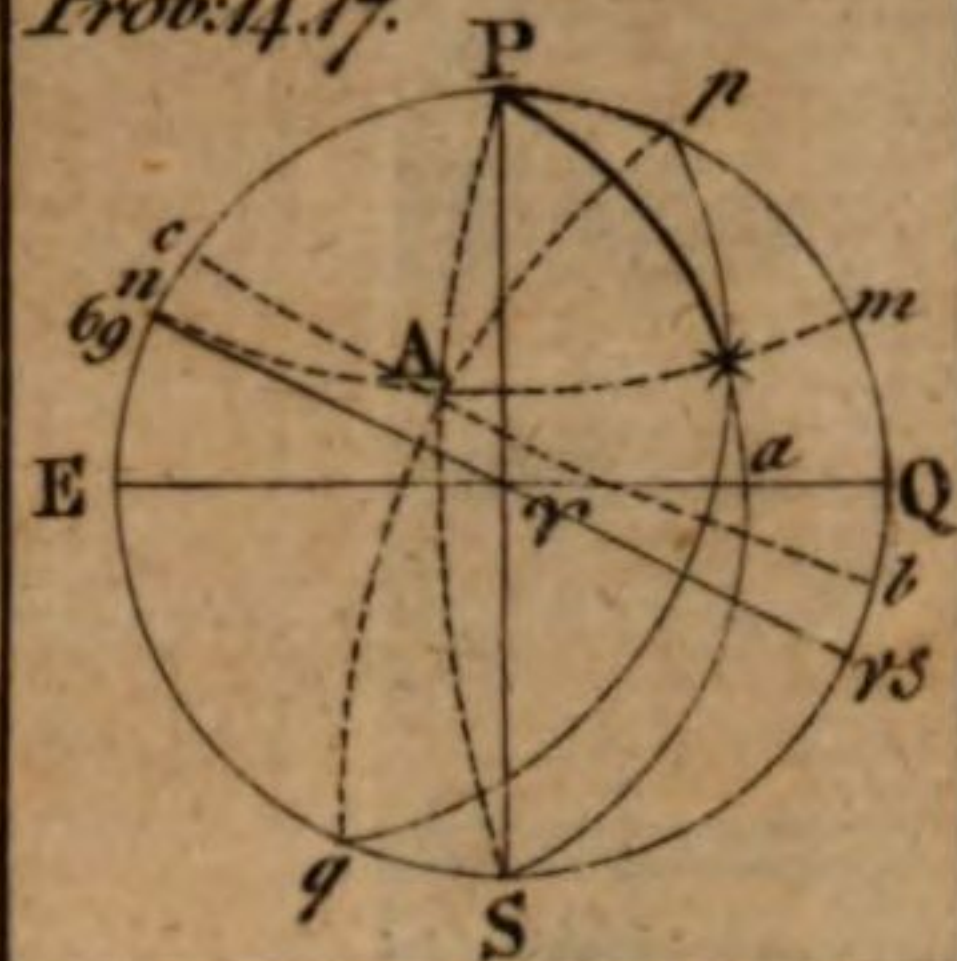
Prob: 12.



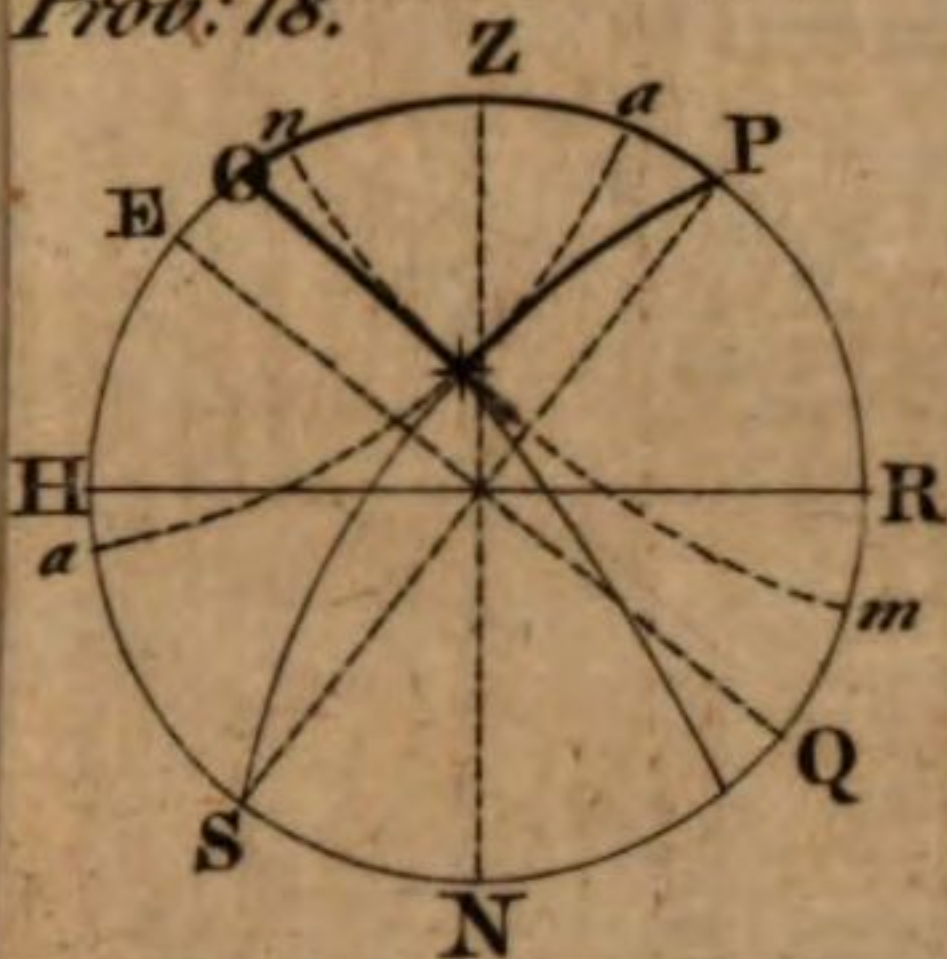
Prob: 13.



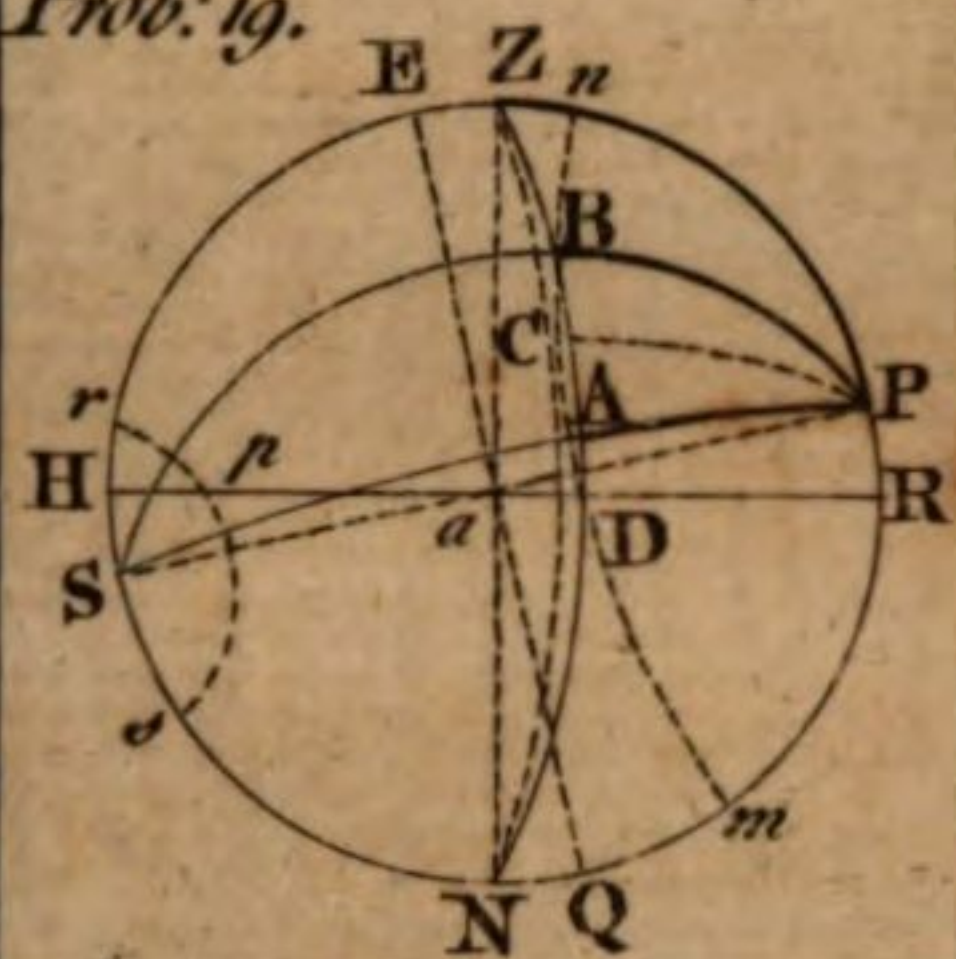
Prob: 14. 17.



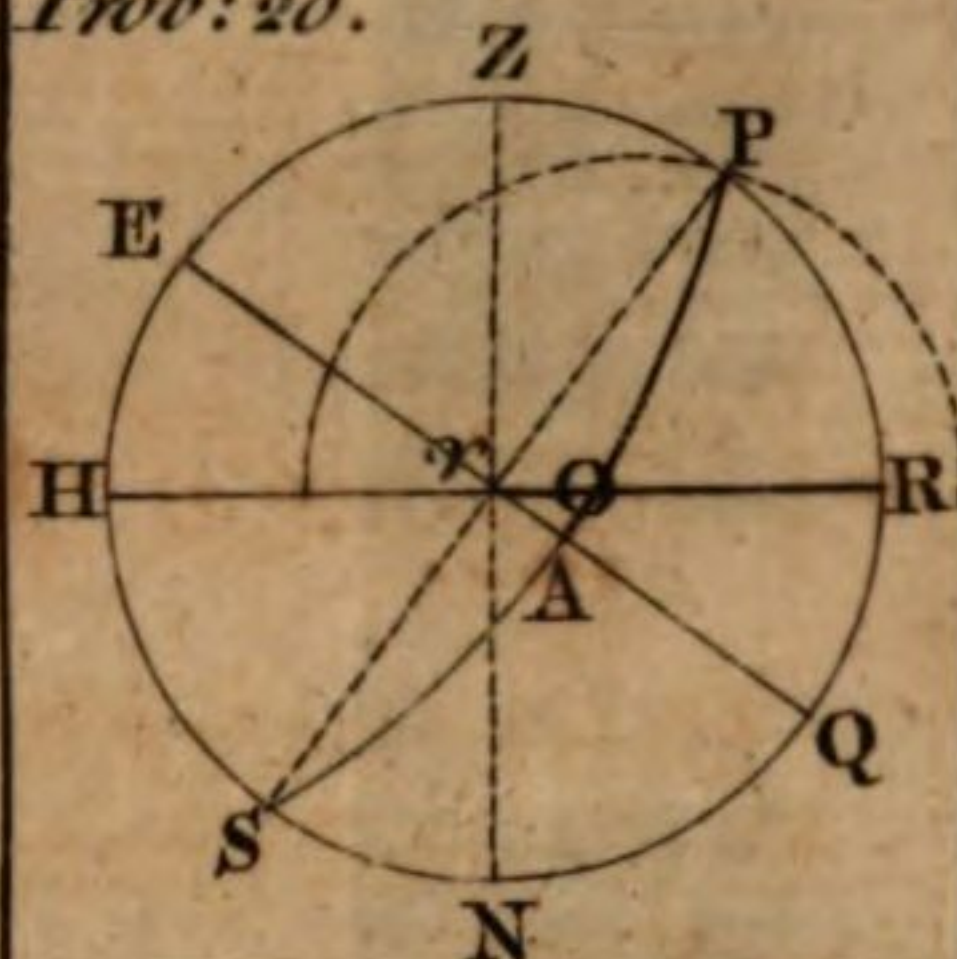
Prob: 18.



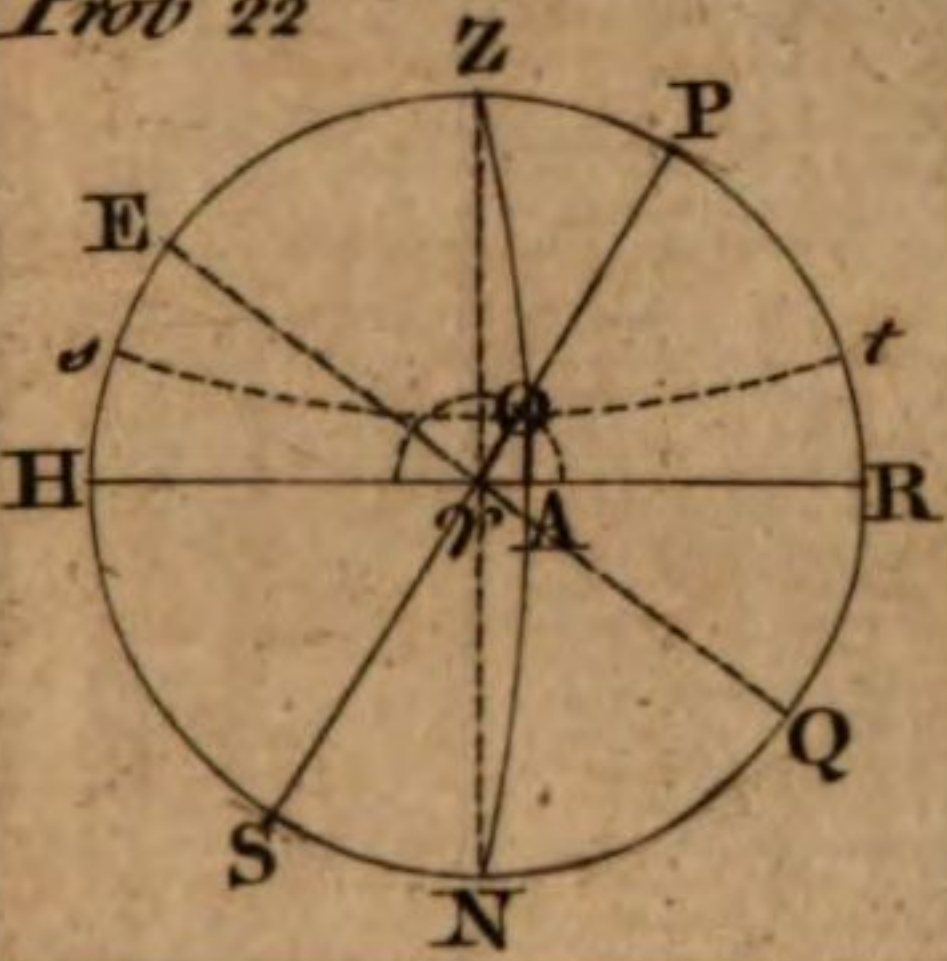
Prob: 19.



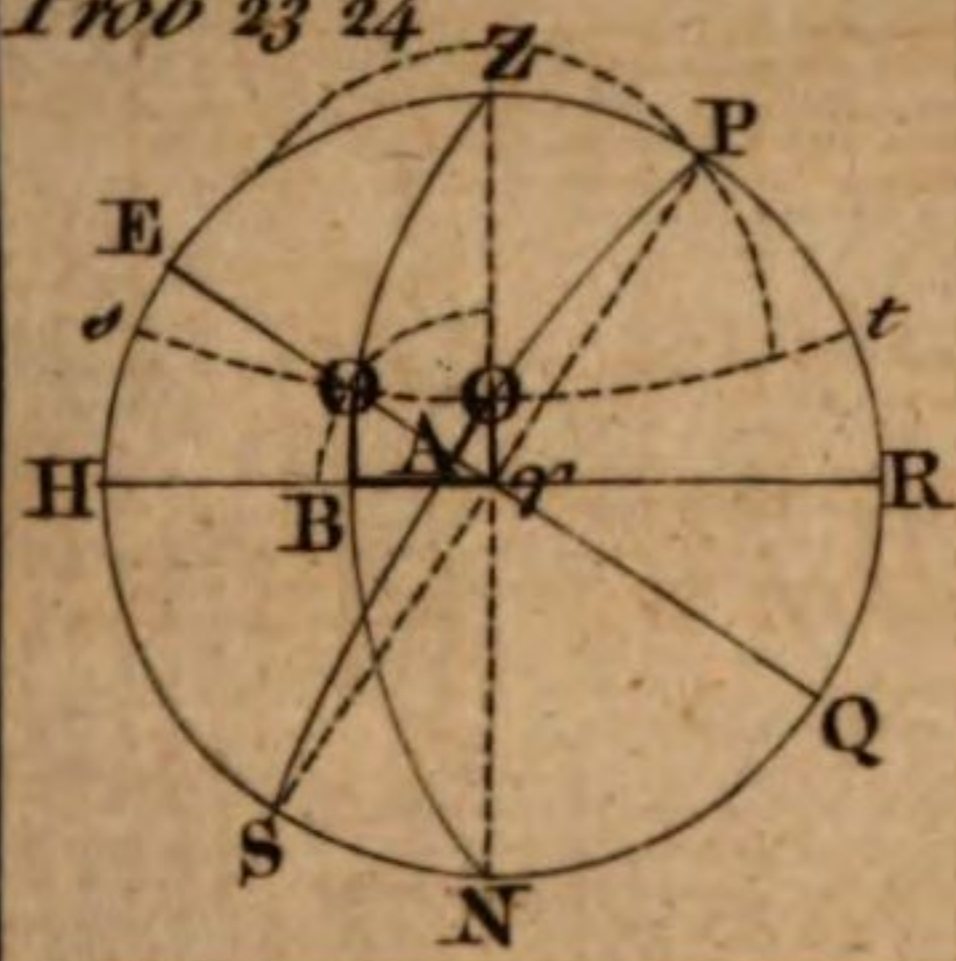
Prob: 20.



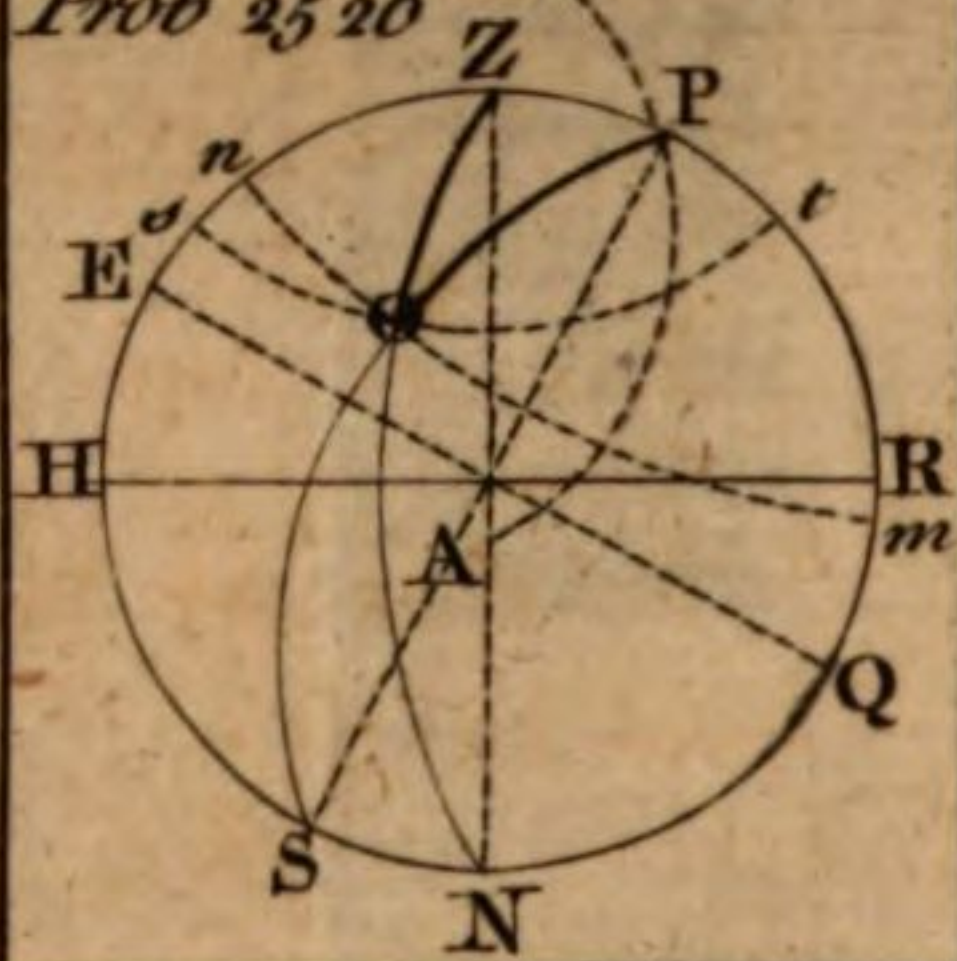
Prob 22



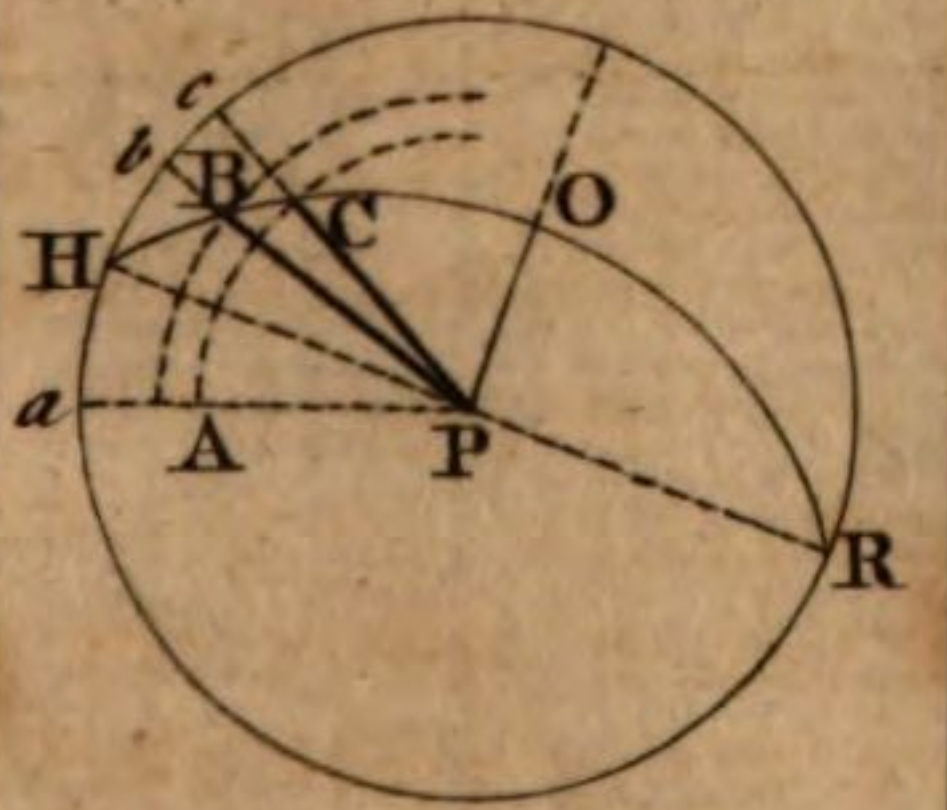
Prob 23 24



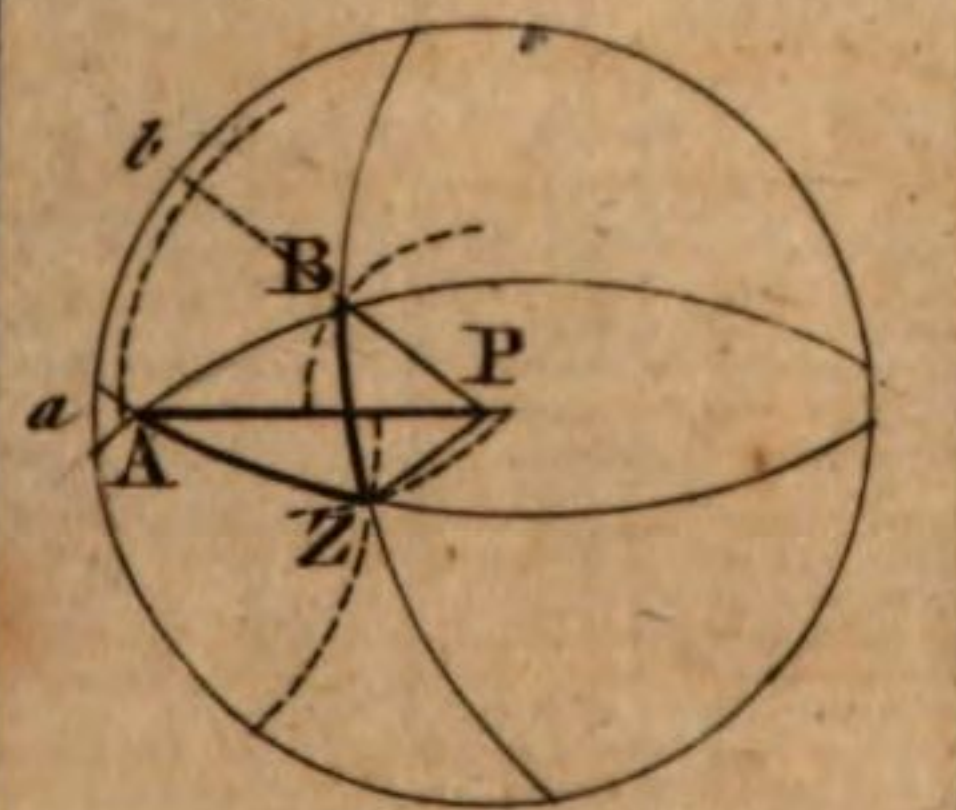
Prob 25 26



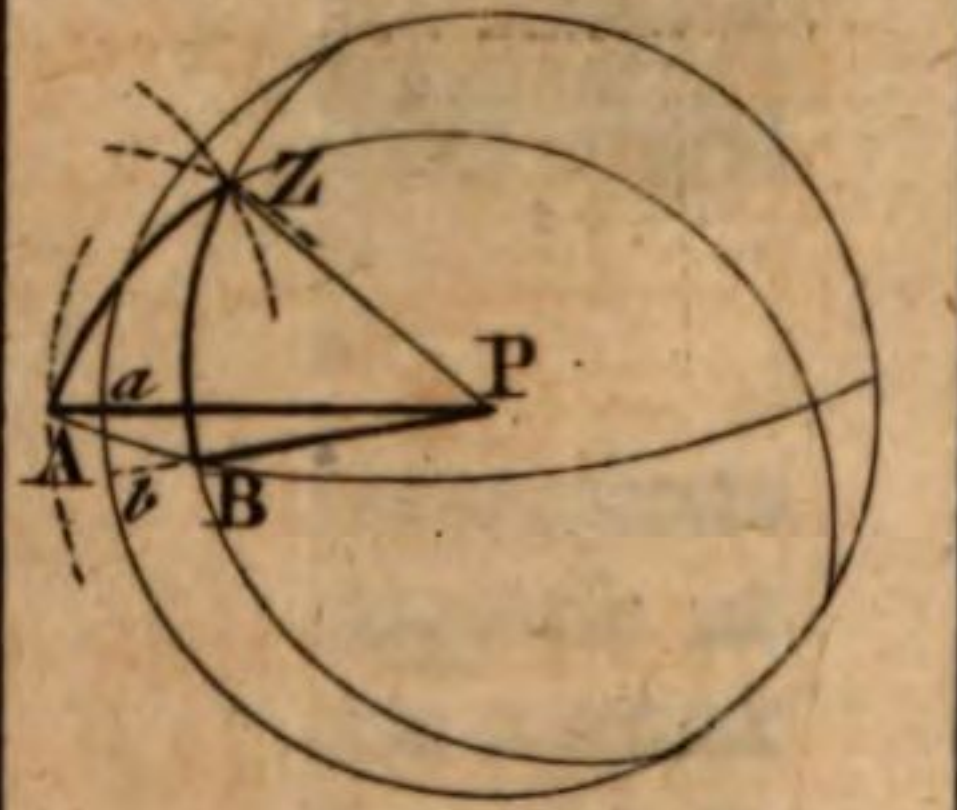
Prob 28



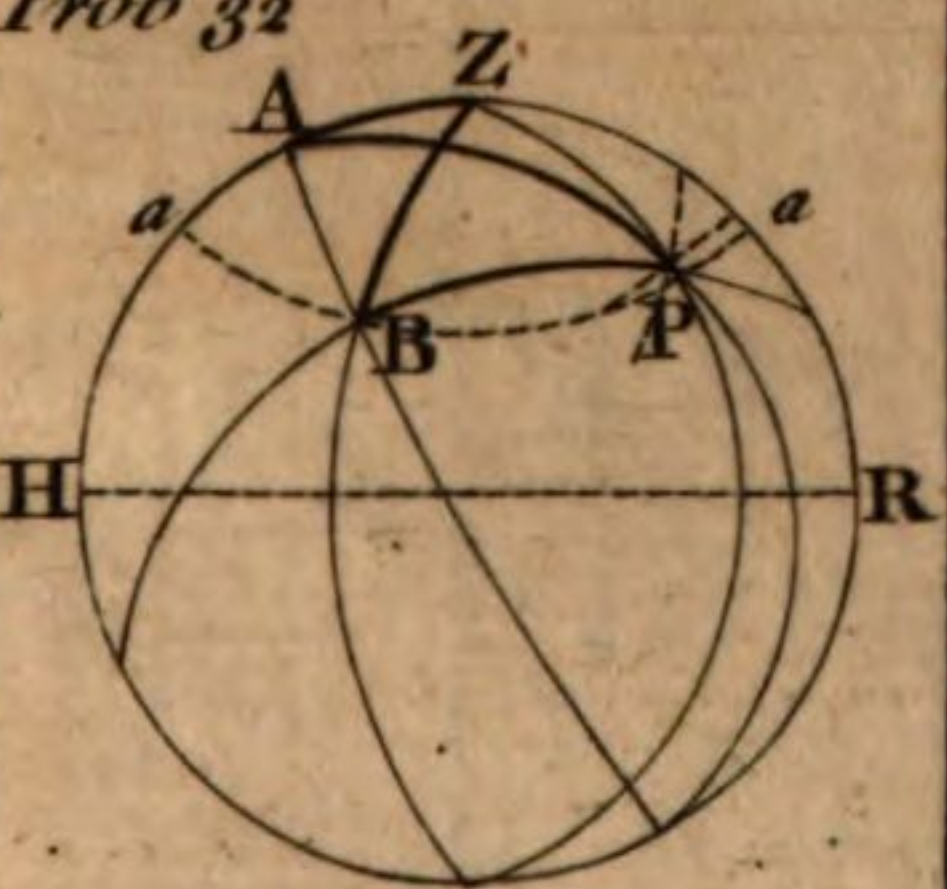
Prob 29



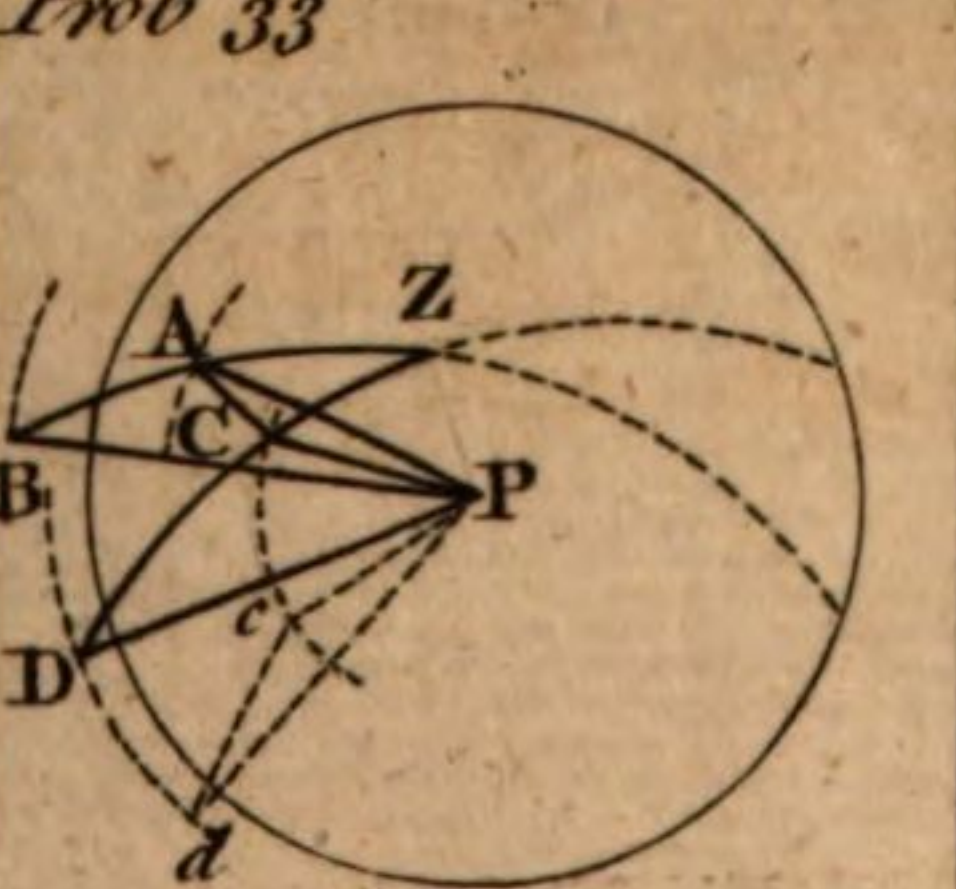
Prob 30



Prob 32



Prob 33



The Problems
belonging to this plate
fall between the
pages 461. 484.
The figures to the last 7
Astron. Problems are in
Plate XI.

7. 11
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65
66
67
68
69
70
71
72
73
74
75
76
77
78
79
80
81
82
83
84
85
86
87
88
89
90
91
92
93
94
95
96
97
98
99
100

COMPUTATION. In the triangle
APC.
Given the *'s co-decl. $PA = 54^{\circ}00'$
comet's co-decl. $PC = 43^{\circ}00'$
their distance $AC = 49^{\circ}00'$
Req. com. rt. asc. fr. * $\angle APC = 65^{\circ}48'$
Then $65^{\circ}48' - 45^{\circ}00' = 20^{\circ}48'$.
And $90^{\circ}00' - 20^{\circ}48' = 69^{\circ}12' = CPE$.

In the triangle EPC.
Given comet's co-decl. $PC = 43^{\circ}00'$
obliq. of ecliptic $PE = 23^{\circ}29'$
comet's rt. asc. $\angle EPC = 69^{\circ}12'$
Req. comet's co-lat. $EC = 39^{\circ}53'$
and longit. $\angle PEC = 83^{\circ}47'$
Which being in the 4th quadrant,
gives $23^{\circ}47'$ in κ .

1024.

PROBLEM XXXVI.

At London, on the 10th of December, 1753, at what time of the night will the stars Aldebaran and Regel be on the same azimuth circle?

Aldeb. decl. $= 15^{\circ}59' N.$; right asc. $= 4 h. 22 m.$ Regel's decl. $= 8^{\circ}28' S.$; right asc. $= 5 h. 01 m.$

Their difference of right ascensions is 39 min., or $9^{\circ}45'$.

CONSTRUCTION. On the plane of the equinoctial put Aldebaran at A, and Regel at B, by their right ascensions and declinations, then a great circle thro' B and A will be the azimuth they are on at the time sought; and the parallel of London's lat. described about P, will cut the azimuth circle BA in Z the zenith, thro' which draw the meridians PZ, Pz.

Here the nearest intersection to the stars is taken for the zenith; for as the stars are both above the horizon, the greatest zenith distance is less than 90 degrees.

COMPUTATION. In the $\triangle APB$.
Given B's co-decl. $PB = 98^{\circ}28'$
A's co-decl. $PA = 74^{\circ}01'$
diff. rt. asc. $\angle APB = 9^{\circ}45'$

In the triangle APZ.
Given A's co-decl. $PA = 74^{\circ}01'$
the co-lat. $PZ = 38^{\circ}28'$
the supl. of BAP or $\angle PAZ = 22^{\circ}14'$

Required the $\angle BAP = 157^{\circ}46'$

Req. $\angle APZ = 144^{\circ}10'$ or $= 23^{\circ}00'$

Now (by 966) the star Aldebaran comes to the meridian at 11 h. 32 m. in the evening; which lessened by 1 h. 32 m. ($= 23^{\circ}00'$) gives 9 h. 40 m. for the time in the evening when those stars will be on the same azimuth.

1025.

PROBLEM XXXVII.

At what time in the evening will the stars Betelgeuse and Pollux have one common altitude above the horizon of London, on the 10th of December, 1753?

Betelgeuse right ascen. $= 5 h. 42 m.$; decl. $= 7^{\circ}20' N.$ Pollux right ascen. $= 7 h. 28 m.$; decl. $= 28^{\circ}36' N.$

Their difference of right ascension is 1 h. 46 m. $= 26^{\circ}30'$.

CONSTRUCTION. On a primitive circle, where any point P represents the pole of the equinoctial, put the stars Pollux at B, and Betelgeuse at A, by their decl. and diff. of right ascensions; thro' A, B, describe a great circle; thro' C, the middle of AB, describe a great circle at right angles to AB, and cutting PA in D; then a small circle described about P, at the distance of $38^{\circ}28'$, the co-lat., will cut the circle CD in Z the zenith.

For the stars A and B having a common altitude, are equally distant from Z.

COMPU-

COMPUTATION. In the triangle APB.

Given A's co-decl.	PA = 82° 40'	} Required the $\angle BAP = 46^\circ 18'$	AB = 32 49
B's co-decl.	PB = 61 24		AC = 16 24
diff. rt. asc. $\angle APB = 26 30$	Its half		

In the triangle ACD.

Given the $\angle C = 90^\circ 00'$	} Required the $\angle D = 46^\circ 05'$	AD = 23 04
$\angle A = 46 18$		
AC = 16 24		Theref. PA — AD = PD = 59 36

In the triangle PDZ.

Given the co-lat. PZ = 38° 28'	} Required $\angle ZPD = 66^\circ 04'$	
PD = 59 36		Or it is = 58 30
$\angle PDZ = 46 05$		

Now (by 966) the star comes to the meridian at 12 h. 32 m. in the morning; from which take 3 h. 54 m. (= 58° 30', as the stars are to the east of the meridian) and it leaves 8 h. 38 m. in the evening for the time when those stars have the same altitude.

1026. PROBLEM XXXVIII.

Wanting to know the latitude and longitude of a comet c, its distance from two known stars A and B were observed, and are as follows;

A's lat. = 49° 12' N. Lon. = 16° 39' π ; distance from c = 45° 05'.

B's lat. = 30 05 N. Lon. = 2 48 π ; distance from c = 45 57.

Hence the place of the comet c is required?

CONSTRUCTION. On the plane of the solstitial colure, where E is the pole of the ecliptic, put the stars A and B by their latitudes and longitudes, and describe a great circle thro' A and B; then small circles described about A and B as poles, at the respective distances of the comet, their intersection c will give its place; describe great circles thro' A, c; B, c; E, c; and EC will be the co-lat., and from $\angle AEC$ will be obtained the longitude.

COMPUTATION. In the $\triangle AEB$.

Given A's co-lat.	AE = 40° 48'
B's co-lat.	BE = 59 55
diff. longit. $\angle AEB = 76 09$	

Required	AB = 59 01
and $\angle EAB = 78 31$	

In the triangle ABC.

Given the distance	AB = 59° 01'
distance	AC = 49 05
distance	BC = 45 57

Required the $\angle BAC = 56 27$
Then $\angle EAB + \angle BAC = \angle EAC = 134^\circ 58'$

In the triangle CEA.

Given A's co-lat.	AE = 40° 48'	} Required the co-lat. EC = 81° 33'	
distance	AC = 49 05		diff. lon. AEC = 32 43
and the $\angle EAC = 134 58$			Hence lat. is 8° 27' N. lon. 19° 22' in γ .

1027. PROBLEM XXXIX.

The distance of the star c being observed from two stars, A, B, whose latitude and distance are known; thence to find the lat. and long. of c.

A's lat. = 5° 30'; dist. from c = 39° 40'; B's lat. = 9° 57'; dist. from c = 10° 7½'; And the distance of AB is 44° 43'.

CON-

CONSTRUCTION. On a circle of longitude, where E is the pole of the ecliptic, put the star B by its lat. ; about the points B and E describe circles at the distances of A from those points, their intersection gives the place of A ; also circles described from A and B, at the distances of C respectively from them, their intersection is the place of C : Then describe the great circles EA, EC ; AC, AB ; BC ; and EC, will be the co-lat., and $\angle BEC$ the longitude from B.

COMPUTATION. In the $\triangle AEB$.		In the triangle ABC.	
Given A's co-lat.	$AE = 84^{\circ} 30'$	Given the distance AB	$= 44^{\circ} 43'$
B's co-lat.	$BE = 80^{\circ} 03'$	distance AC	$= 39^{\circ} 40'$
the distance	$AB = 44^{\circ} 43'$	distance BC	$= 10^{\circ} 07\frac{1}{2}'$
Required the $\angle ABE = 92^{\circ} 14'$		Required the $\angle ABC = 55^{\circ} 22'$	
In the triangle BEC.			
Given B's co-lat.	$BE = 80^{\circ} 03'$	Required C's co-lat.	$CE = 72^{\circ} 01'$
the distance	$BC = 10^{\circ} 07\frac{1}{2}'$	C's lon. fr. B, $\angle BEC$	$= 6^{\circ} 22'$
$(\angle ABE - \angle ABC =) \angle CBE = 36^{\circ} 52'$			

1028.

PROBLEM XL.

From the altitudes of two known fixed stars, and the altitude of a planet when in the same azimuth with one of these stars; to find the place of the planet.

EXAM. Observed the Moon and Cor. Leonis in the same azimuth, when the Moon's zenith distance was $36^{\circ} 37'$.

Cor. Leonis's zenith dist. $= 45^{\circ} 00'$; decl. $13^{\circ} 09' N.$; rt. as. $9^h 53^m$.

Cor. Hydra's zenith dist. $= 49^{\circ} 16'$; decl. $7^{\circ} 37' S.$; rt. as. $9^h 16^m$.

CONSTRUCTION. On the plane of the equinoctial, whose pole is P, draw the colures, and in the solstitial, take E for the pole of the ecliptic; put the given stars at B and A by their declinations and right ascensions: About B and A as poles, with their respective zenith distances, describe circles cutting in Z the zenith; thro' Z and B describe an azimuth circle, and making Z D equal to the D's zenith distance, gives her place: Then describe the great circles ZB, ZA, Z D, E D; and the arc E D will be the co-latitude, and $\angle PED$ the longitude from ϖ .

COMPUTATION. In the $\triangle ABP$.		In the triangle AZB.	
Given A's co-decl.	$PA = 97^{\circ} 37'$	Given A's zen. dist.	$ZA = 49^{\circ} 16'$
B's co-decl.	$PB = 76^{\circ} 51'$	B's zen. dist.	$ZB = 45^{\circ} 00'$
diff. rt. as. $\angle BPA$	$= 9^{\circ} 15'$	the side	$AB = 22^{\circ} 43'$
Required the $\angle ABP$	$= 155^{\circ} 38'$	Required the $\angle ABZ$	$= 89^{\circ} 56'$
the side	$AB = 22^{\circ} 43'$	Then $\angle ABP - \angle ABZ = \angle ZBP$	$= 65^{\circ} 42'$

In the triangle B D P.		In the triangle P D E.	
Given B's co-decl.	$BP = 76^{\circ} 51'$	Given obl. of eclip.	$PE = 23^{\circ} 29'$
$(BZ = ZD =)$ side B D	$= 8^{\circ} 23'$	D's co-decl.	$PD = 73^{\circ} 32'$
$\angle PBD$	$= 65^{\circ} 42'$	D's co-rt. as. $\angle EPD$	$= 129^{\circ} 22'$
Requ. D's co-decl.	$PD = 73^{\circ} 32'$	Requir. D's co-lat.	$DE = 89^{\circ} 00'$
D's rt. as. fr. B, $\angle BPD$	$= 7^{\circ} 57'$	D's lon. fr. ϖ , $\angle PED$	$= 47^{\circ} 52'$
Then $90^{\circ} + \angle BPD + \angle BPD =$			
$\angle EPD = 129^{\circ} 22'$			

SECTION

SECTION VIII.

1029.

Of the Use of the Globes.

By the Globes here are meant two spherical bodies, whose convex surfaces are supposed to give a true representation of the earth and heavens, as visible by observation, and are called the Terrestrial and Celestial globes.

The TERRESTRIAL GLOBE has delineated on its convexity, the whole surface of the *earth* and *sea* in their relative size, form and situation.

The CELESTIAL GLOBE has drawn on its surface, the images of the several constellations and stars: The relative magnitude and position which the stars are observed to have in the heavens, being preserved on this globe.

To the globes are fitted certain appurtenances, whereby a great variety of useful problems are neatly solved.

The BRAZEN MERIDIAN is that ring or hoop in which the globe hangs on its axis; which is represented by two wires passing thro' its poles. This circle is divided into four quarters of 90° each; in one semicircle the divisions begin at each pole, and end at 90° , where they meet: In the other semicircle, the divisions begin at the middle, and proceed thence towards each pole, where they end at 90 degrees. The graduated side of this brazen circle serves as a meridian for any point on the surface of the earth, the globe being turned about till that point comes under the circle.

The HOUR CIRCLE is a small circle of brass which is divided into 24 hours, the quarters and half quarters: It is fixed on the brazen meridian, equally distant from the north end of the axis, to which an index is fitted, that points out the divisions of the hour circle as the globe is turned about.

The HORIZON is represented by the upper surface of the wooden circular frame encompassing the globe about its middle. On this wooden frame is a kind of perpetual calendar, contained in several concentric circles: The inner one is divided into 4 quarters of 90 degrees each; the next circle is divided into the 12 months, with the days in each, according to the new stile; the next contains the 12 equal signs of the zodiac, each being divided into 30 degrees; the next is the 12 months and days according to the old stile; and there is another circle, containing the 32 winds, with their halves and quarters. Altho' these circles are on all horizons, yet their disposition by the workmen are not always the same.

The QUADRANT OF ALTITUDE is a thin slip of brass, one edge of which is graduated into 90 degrees and their quarters, equal to those of the meridian: To one end of this is fixed a brass nut and screw, whereby it is put on and fastened to the meridian; which, if in the zenith or pole of the horizon, then the graduated edge represents a vertical circle passing thro' any point.

Beside these, there are several circles described on the surfaces of both globes, such as the equinoctial, ecliptic, circles of longitude and right ascension, the tropics, polar circles, parallels of lat. and decl., on the celestial globe; and on the terrestrial, the equator, ecliptic, tropics, polar circles, parallels of latitude, hour circles, or meridians to every 15 degrees, and the spiral rhumbs flowing from several centres, called Flies.

1030.

PROBLEM I.

To find the latitude and longitude of any place on the terrestrial globe.

- 1st. Bring the given place under that side of the graduated brazen meridian where the degrees begin at the equator, by turning the globe about.
- 2d. Then the degree of the meridian over it, shews the latitude.
- 3d. And the degree of the equator under the merid., shews the longit.

On some globes, the longitude is reckoned on the equator, from the meridian where it begins, only eastward until it ends at 360° : On such globes, when the longitude of a place exceeds 180° , take it from 360° , and call the remainder the longitude westward.

1031.

PROBLEM II.

To find any place on the globe whose latitude and longitude are given.

- 1st. Bring the given longitude, found on the equator, to the meridian.
- 2d. Then under the given latitude, found on the meridian, is the place sought.

1032.

PROBLEM III.

To find the distance and bearing of any two given places on the globe.

- 1st. Lay the graduated edge of the quadrant of altitude over both places, the beginning, or 0 deg. being on one of them, and the degrees between them shews their distance; these degrees multiplied by 60 gives sea miles, and by 70 gives the distance in land miles nearly.
- 2d. Observe while the quadrant lies in this position, what rhumb of the nearest fly runs mostly parallel to the edge of the quadrant, and that rhumb shews the bearing sought nearly.

1033.

PROBLEM IV.

To find the sun's place and declination on any day.

- 1st. Seek the given day in the circle of months on the horizon, and right against it, in the circle of signs, is the sun's place.

Thus it will be found that the sun enters

The spring signs, Aries, March 20. Taurus, April 20. Gemini, May 21.

The summer signs, Cancer, June 21. Leo July 23. Virgo, August 23.

Autumnal signs, Libra, Sept. 22. Scorpio, Oct. 23. Sagittar. Nov. 22.

The winter signs, Capric. Dec. 21. Aquarius, Jan. 20. Pisces, Feb. 18.

- 2d. Seek the sun's place in the ecliptic on the globe, bring that place to the meridian, and the division it stands under is the sun's declination on the given day.

On the globes, the ecliptic is readily distinguished from the equator, not only by the different colours wherewith they are stained, but also by the ecliptic's approaching towards the poles, after its intersection with the equator. The marks of the signs are also put along the ecliptic, one at the beginning of every successive 30 degrees.

1034.

PROBLEM V.

To rectify the globe for the latitude, zenith, and noon.

1st. Set the globe upon an horizontal plane, with its parts answering to those of the world; move the meridian in its notches, by raising or depressing the pole, until the degrees of latitude cut the horizon; then is the globe rectified for the latitude.

2d. Reckon the latitude from the equator towards the elevated pole, there screw the bevil edge of the nut belonging to the quadrant of altitude, and the rectification is made for the zenith.

3d. Bring the sun's place (found by the last problem) to the meridian, set the index to the XII at noon, or upper XII, and the globe is rectified for the sun's southing, or noon.

1035.

PROBLEM VI.

To find where the sun is vertical at any given time.

1st. Bring the sun's place, found for the given day (1033), to the meridian, note the degree over it, and set the index to the given hour.

2d. Turn the globe till the index comes to 12 at noon, then the place under the said noted degree has the sun in the zenith at that time.

3d. All the places that pass under that degree, by turning the globe round, will have the sun vertical to them on that day.

1036.

PROBLEM VII.

To find what days the sun will be vertical at any given place in the torrid zone.

1st. Note the latitude of the given place on the meridian.

2d. Turn the globe, and note what points of the ecliptic pass under the latitude noted on the meridian.

3d. Seek those points of the ecliptic in the circle of signs on the horizon, and right against them in the circle of months stand the days required.

In this manner it will be found, that the sun will be vertical to the Island of St. Helena on the 6th of November, and on the 4th of February. And at Barbadoes on the 24th of April, and the 18th of August.

1037.

PROBLEM VIII.

At any given hour in a given place, to find what hour it is in any other place?

1st. Bring the place where the time is given to the meridian, and set the index to the given hour.

2d. Bring the other given place to the meridian, and the index shews the hour corresponding to the given time.

1038.

1038.

PROBLEM IX.

At any given time, to find all those places of the earth where the sun is then rising or setting, where mid-day or midnight.

Find the place where the sun is vertical at the given time (1035); rectify the globe for the latitude of that place, and bring it to the meridian.

Then all those places, that are in the west half of the horizon, have the sun rising; and those in the eastern half have it setting.

Those under the meridian above the horizon have the sun culminating, or noon; and those under the meridian below the horizon have midnight:

Those above the horizon have day; those below it have night.

1039.

PROBLEM X.

To find the angle of position of two places; or, the angle made by the meridian of one place, and a great circle passing thro' both places.

Rectify for the latitude of one of the given places, and bring it to the meridian; there fix the quadrant of altitude, and set its graduated edge to the other place: Then will that edge of the quadrant cut the horizon in the degree of position sought.

Thus, the angle of position at the Land's end to Barbadoes is south $71\frac{1}{2}^{\circ}$ westerly: But the angle of position at Barbadoes to the Land's end is north $37\frac{1}{2}$ degrees easterly.

Hence neither of these positions can be the true bearing; for the rhumb passing thro' both places, will be opposite one way to what it is the other.

1040.

PROBLEM XI.

The latitude of any place not within the polar circle being given, to find the time of sun-rising and setting, and the length of the day and night.

Rectify for the latitude and the noon; bring the sun's place to the eastern side of the horizon, and the index shews the time of rising: The sun's place brought to the western side of the horizon, the index gives the setting.

Or, The time of rising taken from 12 hours gives the time of setting.

The time of setting being doubled gives the length of the day.

And the time of rising being doubled gives the length of the night.

Thus, at London, on April 15th, the day is $13\frac{1}{2}$ hours; the night $10\frac{1}{2}$ hours.

1041.

PROBLEM XII.

To find the length of the longest and shortest days in any given place.

Rectify for the latitude; bring the solstitial point of that hemisphere to the eastern part of the horizon, set the index to 12 at noon; turn the globe till the solstitial point comes to the western side of the horizon, the hours past over by the index give the length of the longest day, or night; and its complement to 24 hours gives the length of the shortest night, or day.

1042.

PROBLEM XIII.

A place being given in either frigid or frozen zone, to find the time when the sun begins to appear at, or depart from, that place: Also how many successive days he is present to, or absent from, that place.

Rectify for the latitude, turn the globe, and observe what degrees in the first and second quadrants of the ecliptic are cut by the north point of the horizon, the latitude being supposed north.

Find those degrees in the circle of signs on the horizon, and their corresponding days of the month, and all the time between those days the sun will not set in that place.

Again. Observe what degree in the third and fourth quadrants of the ecliptic will be cut by the south point of the horizon, and the days answering: Then the sun will be quite absent from the given place during the intermediate days; that day in the 3d quadrant, shews when he begins to disappear; and that in the 4th quadrant, shews when he begins to shine in the place proposed.

Thus, at the North cape, in lat. 71° , the sun never sets from May 15 to July 28, which is 74 days; and never rises from November 16 to January 24, which is 69 days.

1043.

PROBLEM XIV.

To find the antæci, periæci, and antipodes of any place.

Bring the given place to the meridian, tell as many degrees of latitude on the contrary side of the equator, and it gives the place of the antæci; that is, of those who have opposite seasons of the year, but the same times of the day.

The given place being under the meridian, set the index to 12 at noon, turn the globe till the index points to 12 at night, and the point under the meridian in the given latitude is the place of the periæci; that is, of those who have the same seasons of the year, but opposite times of the day.

The globe remaining in this position, seek on the contrary side the equator for the degrees of latitude given, and the point under the meridian thus found will be the antipodes to the given place; that is, of such whose seasons of the year and times of the day are directly opposite to those of the given place.

1044.

PROBLEM XV.

To find the beginning and end of the twilight in any place.

Rectify the globe for the latitude, zenith, and noon. (1034)

Seek the point of the ecliptic opposite to the sun's place, turn the globe and quadrant of altitude, till the said opposite point of the ecliptic stands against 18 degrees on the quadrant of altitude; then will the index shew the beginning or end of the twilight: That is, the beginning in the morning, when those points meet in the western hemisphere; or the end in the evening, when the said points meet in the eastern hemisphere.

1045.

1045.

PROBLEM XVI.

The latitude of a place and day of the month being given; to find the sun's declination, meridian altitude, right ascension, amplitude, oblique ascension, ascensional difference; and thence the time of rising, setting, length of the day and night.

Rectify for the latitude and noon. Then

The degree of the meridian over the sun's place is the declination.

The meridian altitude is shewn by the degrees the sun is above the horizon; and is equal to the sum or diff. of the co-lat and declin.

The sun's right ascen. is that degree of the equator under the meridian.

Bring the sun's place to the eastern part of the horizon. Then

The amplitude is that degree of the horizon opposite the sun.

The oblique ascension is that degree of the equator cut by the horizon.

The ascen. diff. is the diff. between the right and oblique ascensions.

The ascen. diff. converted into time, will give the time the sun rises before or after the hour of six, according as his amplitude is to the northward or southward of the east point of the horizon.

1046.

PROBLEM XVII.

Given the latitude of the place and day of the month; to find the sun's altitude and azimuth, either when he is due east or west, at 6 o'clock, or at any other hour while he is above the horizon.

Rectify the globe for the latitude, zenith, and noon.

Set the quadrant of altitude to the east point of the horizon, turn the globe till the sun's place comes to the quadrant's edge, and it shews the altitude, his azimuth being now 90° , and the index shews the hour.

Turn the globe till the index points at 6, there stay it, and move the quadrant till its edge cuts the sun's place; then the degrees at the sun shews its altitude, and the degrees cut by the quadrant in the horizon shews the azimuth, reckoning from the north.

In like manner, the globe being turned till the index is against any other hour, suppose 10 in the forenoon. Then

The graduated edge of the quadrant of altitude being turned to cut the sun's place, will give both the altitude and azimuth at that time.

1047.

PROBLEM XVIII.

Given the latitude, day of the month, and sun's altitude; to find the azimuth and hour of the day.

Rectify the globe for the latitude, zenith, and noon.

Turn the globe and quadrant until the sun's place coincide with the graduated edge of the quadrant.

Then will that edge of the quadrant cut in the horizon the degrees of azimuth, reckoned from the north; and the index will shew the hour of the day.

1048.

1048.

P R O B L E M XIX.

To represent the appearance of the heavens at any time in a given place.

Rectify the celestial globe for the latitude, zenith, and noon, and turn the globe till the index points at the given hour. Then

Those stars in the eastern half of the horizon are rising; those in the western are setting; and those in the meridian are culminating.

The quadrant being set to any given star will shew its altitude, and at the same time its azimuth, reckoned in the horizon.

Now by turning the globe round, it will readily appear what stars never set in that place, and those which never rise; those of perpetual apparition never go below the horizon, and those of perpetual absence never come above it.

1049.

P R O B L E M XX.

To find the latitude and longitude of any star.

Put the centre of the quadrant of altitude on the pole of the ecliptic, and its graduated edge on the given star. Then

The latitude is shewn by the degrees between the ecliptic and star.

The longitude is the degrees cut on the ecliptic by the quadrant.

1050.

P R O B L E M XXI.

To find the declination and right ascension of a star.

Bring the star to the meridian, the degree over it is the declination; and the degree of the equator under the meridian is the right ascension.

1051.

P R O B L E M XXII.

On any day, and in any given place; to find when a proposed star rises, sets, or culminates.

Rectify the globe for the latitude and noon.

Bring the star to the eastern side of the horizon, and the index shews the time of its rising.

Turn the globe till the star comes to the meridian, and the index shews the time of its culminating; and in like manner when it sets, the time will be shewn by the index.

Its meridian altitude, oblique ascension, and ascensional difference, are found in the same manner as for the sun at art: 1045.

Articles 1052, 1053, 1054, 1055, 1056.

ASTRONOMICAL TABLES
OF THE
Declinations and Right Ascensions
OF THE
Sun and Fixed Stars :

Calculated for the year 1753,
And serving for the next twelve years.

A L S O,

A TABLE to reduce the Sun's declination from the
meridian of London to that of any other place.

And a useful TABLE of Refractions.

1052. A TABLE of the Sun's Declination

For the years 1753, 1757, 1761; each being the first after Leap-year.

Days	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Days
	Decl. S.	Decl. S.	Decl. S.	Decl. N.	Decl. N.	Decl. N.	Decl. N.	Decl. N.	Decl. N.	Decl. S.	Decl. S.	Decl. S.	
	° /	° /	° /	° /	° /	° /	° /	° /	° /	° /	° /	° /	
1	22 59	16 58	7 24	4 43	15 12	22 07	23 07	17 58	8 10	3 20	14 36	21 55	1
2	22 53	16 41	7 01	5 06	15 30	22 15	23 03	17 43	7 49	3 44	14 55	22 04	2
3	22 48	16 23	6 38	5 29	15 48	22 23	22 58	17 27	7 27	4 07	15 13	22 12	3
4	22 41	16 05	6 15	5 52	16 05	22 30	22 53	17 11	7 04	4 30	15 32	22 20	4
5	22 35	15 46	5 52	6 14	16 22	22 36	22 48	16 55	6 42	4 53	15 50	22 28	5
6	22 27	15 28	5 29	6 37	16 39	22 43	22 42	16 39	6 20	5 16	16 08	22 35	6
7	22 20	15 09	5 05	6 59	16 56	22 49	22 35	16 22	5 57	5 39	16 26	22 42	7
8	22 12	14 50	4 42	7 22	17 12	22 54	22 28	16 05	5 34	6 02	16 44	22 48	8
9	22 03	14 31	4 19	7 44	17 28	22 59	22 21	15 48	5 12	6 25	17 01	22 54	9
10	21 54	14 11	3 55	8 06	17 44	23 04	22 14	15 30	4 49	6 48	17 18	22 59	10
11	21 45	13 52	3 32	8 28	17 59	23 08	22 06	15 12	4 26	7 11	17 34	23 04	11
12	21 35	13 32	3 08	8 50	18 14	23 12	21 58	14 54	4 03	7 34	17 51	23 09	12
13	21 24	13 11	2 44	9 12	18 29	23 15	21 49	14 36	3 40	7 56	18 07	23 13	13
14	21 14	12 51	2 21	9 34	18 44	23 19	21 40	14 17	3 17	8 18	18 23	23 16	14
15	21 03	12 30	1 57	9 55	18 58	23 21	21 31	13 59	2 54	8 41	18 38	23 20	15
16	20 51	12 10	1 33	10 16	19 12	23 23	21 21	13 40	2 31	9 03	18 53	23 22	16
17	20 39	11 49	1 10	10 37	19 25	23 25	21 11	13 21	2 07	9 25	19 08	23 24	17
18	20 27	11 27	0 46	10 58	19 39	23 27	21 00	13 01	1 44	9 47	19 22	23 26	18
19	20 14	11 06	0 22	11 19	19 52	23 28	20 50	12 42	1 21	10 09	19 36	23 27	19
20	20 01	10 45	N. 01	11 40	20 04	23 29	20 38	12 22	0 57	10 30	19 50	23 28	20
21	19 48	10 23	0 25	12 00	20 16	23 29	20 27	12 02	0 34	10 52	20 03	23 29	21
22	19 34	10 01	0 49	12 20	20 28	23 28	20 15	11 42	0 11	11 13	20 16	23 28	22
23	19 20	9 39	1 12	12 40	20 40	23 27	20 03	11 21	S. 13	11 34	20 28	23 27	23
24	19 06	9 17	1 36	13 00	20 51	23 26	19 50	11 01	0 36	11 55	20 41	23 26	24
25	18 51	8 55	1 59	13 20	21 02	23 25	19 37	10 40	1 00	12 16	20 52	23 25	25
26	18 36	8 32	2 23	13 39	21 12	23 23	19 24	10 19	1 23	12 36	21 04	23 23	26
27	18 20	8 10	2 46	13 58	21 22	23 21	19 10	9 58	1 47	12 57	21 15	23 20	27
28	18 04	7 47	3 10	14 17	21 32	23 18	18 57	9 37	2 10	13 17	21 25	23 17	28
29	17 48		3 33	14 36	21 42	23 15	18 43	9 15	2 34	13 37	21 36	23 14	29
30	17 32		3 56	14 54	21 5	23 11	18 28	8 54	2 57	13 57	21 45	23 10	30
31	17 15		4 20		21 59		18 13	8 32		14 17		23 05	31
Days	S. Decl.	S. Decl.	N. Decl.	N. Decl.	N. Decl.	N. Decl.	N. Decl.	N. Decl.	S. Decl.	S. Decl.	S. Decl.	S. Decl.	Days
	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	

1052. A TABLE of the Sun's Declination

For the years 1754, 1756, 1762; each being the second after Leap-year.

Days	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Days
	Decl. S.	Decl. S.	Decl. S.	Decl. N.	Decl. N.	Decl. N.	Decl. N.	Decl. N.	Decl. N.	Decl. S.	Decl. S.	Decl. S.	
	° /	° /	° /	° /	° /	° /	° /	° /	° /	° /	° /	° /	
1	23 00	17 02	7 30	4 37	15 08	22 05	23 08	18 02	8 16	3 15	14 31	21 53	1
2	22 55	16 45	7 07	5 00	15 26	22 13	23 04	17 47	7 54	3 38	14 50	22 02	2
3	22 49	16 27	6 44	5 23	15 43	22 21	22 59	17 31	7 32	4 01	15 09	22 10	3
4	22 43	16 09	6 21	5 46	16 01	22 28	22 54	17 15	7 10	4 24	15 28	22 18	4
5	22 36	15 51	5 58	6 09	16 18	22 35	22 49	16 59	6 47	4 48	15 46	22 26	5
6	22 29	15 32	5 34	6 31	16 35	22 41	22 43	16 43	6 25	5 11	16 04	22 33	6
7	22 22	15 14	5 11	6 54	16 52	22 47	22 37	16 26	6 03	5 34	16 22	22 40	7
8	22 14	14 55	4 48	7 16	17 08	22 53	22 30	16 09	5 40	5 57	16 40	22 47	8
9	22 05	14 36	4 24	7 39	17 24	22 58	22 23	15 52	5 17	6 20	16 57	22 53	9
10	21 56	14 16	4 01	8 01	17 40	23 01	22 16	15 34	4 55	6 43	17 14	22 58	10
11	21 47	13 56	3 37	8 23	17 56	23 07	22 08	15 17	4 32	7 05	17 31	23 03	11
12	21 37	13 36	3 14	8 45	18 11	23 11	22 00	14 59	4 09	7 28	17 47	23 08	12
13	21 27	13 16	2 50	9 07	18 26	23 15	21 51	14 40	3 46	7 51	18 03	23 12	13
14	21 16	12 56	2 27	9 28	18 40	23 18	21 42	14 22	3 23	8 13	18 19	23 16	14
15	21 05	12 35	2 03	9 50	18 55	23 21	21 33	14 03	2 59	8 35	18 34	23 19	15
16	20 54	12 15	1 39	10 11	19 09	23 23	21 23	13 44	2 36	8 58	18 49	23 22	16
17	20 42	11 54	1 15	10 32	19 22	23 25	21 13	13 25	2 13	9 20	19 04	23 24	17
18	20 30	11 33	0 52	10 53	19 36	23 26	21 03	13 06	1 50	9 42	19 19	23 26	18
19	20 18	11 11	0 28	11 14	19 48	23 27	20 52	12 46	1 26	10 03	19 33	23 27	19
20	20 05	10 50	0 04	11 35	20 01	23 28	20 41	12 27	1 03	10 25	19 46	23 28	20
21	19 51	10 28	N. 19	11 55	20 13	23 29	20 30	12 07	0 40	10 47	20 00	23 29	21
22	19 38	10 06	0 43	12 15	20 25	23 29	20 18	11 47	0 16	11 08	20 13	23 29	22
23	19 24	9 44	1 07	12 35	20 37	23 28	20 06	11 26	S. 07	11 20	20 25	23 28	23
24	19 09	9 22	1 30	12 55	20 48	23 27	19 53	11 06	0 31	11 50	20 38	23 27	24
25	18 54	9 00	1 54	13 15	20 59	23 25	19 40	10 45	0 54	12 11	20 50	23 25	25
26	18 39	8 37	2 17	13 34	21 10	23 24	19 27	10 24	1 18	12 32	21 01	23 23	26
27	18 24	8 15	2 41	13 53	21 20	23 21	19 14	10 03	1 41	12 52	21 12	23 21	27
28	18 08	7 52	3 04	14 12	21 30	23 19	19 00	9 42	2 04	13 12	21 23	23 18	28
29	17 52		3 27	14 31	21 39	23 16	18 46	9 21	2 28	13 32	21 33	23 15	29
30	17 36		3 51	14 50	21 48	23 12	18 32	8 59	2 51	13 52	21 43	23 11	30
31	17 19		4 14		21 57		18 17	8 38		14 12		23 06	31
Days	S. Decl.	S. Decl.	N. Decl.	N. Decl.	N. Decl.	N. Decl.	N. Decl.	N. Decl.	S. Decl.	S. Decl.	S. Decl.	S. Decl.	Days
	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	

1052. A TABLE of the Sun's Declination

For the years 1755, 1759, 1763; each being the third after Leap-year.

Days	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Days
	Decl. S.	Decl. S.	Decl. S.	Decl. N.	Decl. N.	Decl. N.	Decl. N.	Decl. N.	Decl. N.	Decl. S.	Decl. S.	Decl. S.	
	° /	° /	° /	° /	° /	° /	° /	° /	° /	° /	° /	° /	
1	23 02	17 07	7 36	4 31	15 03	22 03	23 09	18 06	8 22	3 08	14 26	21 50	1
2	22 56	16 49	7 13	4 54	15 21	22 11	23 05	17 51	8 00	3 32	14 45	21 59	2
3	22 51	16 32	6 50	5 17	15 39	22 19	23 01	17 35	7 38	3 55	15 04	22 08	3
4	22 45	16 14	6 27	5 40	15 56	22 26	22 56	17 20	7 16	4 18	15 23	22 16	4
5	22 38	15 56	6 04	6 03	16 13	22 33	22 50	17 04	6 54	4 41	15 41	22 24	5
6	22 31	15 37	5 41	6 25	16 30	22 39	22 45	16 47	6 31	5 05	15 59	22 31	6
7	22 24	15 19	5 17	6 48	16 47	22 46	22 39	16 31	6 09	5 28	16 17	22 38	7
8	22 16	15 00	4 54	7 10	17 04	22 51	22 32	16 14	5 46	5 51	16 35	22 45	8
9	22 07	14 41	4 31	7 33	17 20	22 57	22 25	15 56	5 24	6 14	16 52	22 51	9
10	21 59	14 21	4 07	7 55	17 36	23 02	22 18	15 39	5 01	6 36	17 09	22 57	10
11	21 49	14 02	3 44	8 17	17 51	23 06	22 10	15 21	4 38	6 59	17 26	23 02	11
12	21 40	13 42	3 20	8 39	18 07	23 10	22 02	15 03	4 15	7 21	17 42	23 07	12
13	21 30	13 21	2 57	9 01	18 22	23 14	21 54	14 45	3 52	7 44	17 59	23 11	13
14	21 19	13 02	2 33	9 23	18 36	23 17	21 45	14 27	3 29	8 07	18 14	23 15	14
15	21 08	12 41	2 09	9 44	18 51	23 20	21 36	14 08	3 06	8 29	18 30	23 18	15
16	20 57	12 20	1 46	10 05	19 05	23 22	21 26	13 50	2 43	8 52	18 47	23 21	16
17	20 46	11 59	1 22	10 27	19 18	23 24	21 16	13 30	2 19	9 14	19 00	23 22	17
18	20 33	11 38	0 58	10 48	19 32	23 26	21 06	13 11	1 56	9 36	19 11	23 25	18
19	20 21	11 17	0 35	11 08	19 45	23 27	20 55	12 52	1 33	9 57	19 29	23 27	19
20	20 08	10 56	0 11	11 29	19 58	23 28	20 44	12 32	1 09	10 19	19 43	23 28	20
21	19 55	10 34	N. 13	11 50	20 10	23 29	20 33	12 12	0 46	10 41	19 56	23 29	21
22	19 42	10 12	0 36	12 10	20 22	23 29	20 21	11 52	0 23	11 02	20 09	23 29	22
23	19 27	9 50	1 00	12 30	20 34	23 28	20 09	11 32	S. 01	11 23	20 22	23 28	23
24	19 13	9 28	1 24	12 50	20 45	23 27	19 57	11 11	0 24	11 44	20 34	23 27	24
25	18 59	9 06	1 47	13 10	20 56	23 26	19 44	10 51	0 48	12 05	20 46	23 26	25
26	18 43	8 44	2 11	13 29	21 07	23 24	19 31	10 30	1 11	12 26	20 58	23 24	26
27	18 28	8 21	2 34	13 48	21 17	23 22	19 18	10 09	1 35	12 46	21 09	23 21	27
28	18 12	7 59	2 58	14 07	21 27	23 19	19 04	9 48	1 58	13 07	21 19	23 19	28
29	17 56		3 21	14 26	21 37	23 17	18 50	9 26	2 21	13 27	21 30	23 15	29
30	17 40		3 44	14 44	21 46	23 13	18 36	9 00	2 45	13 47	21 40	23 12	30
31	17 23		4 08		21 55		18 21	8 43		14 06		23 08	31
Days	Decl. S.	Decl. S.	Decl. N.	Decl. N.	Decl. N.	Decl. N.	Decl. N.	Decl. N.	Decl. S.	Decl. S.	Decl. S.	Decl. S.	Days
	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	

1052. A TABLE of the Sun's Declination

For the years 1756, 1760, 1764; each being Leap-year.

Days	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Days
	Decl.	Decl.	Decl.	Decl.	Decl.	Decl.	Decl.	Decl.	Decl.	Decl.	Decl.	Decl.	
	S.	S.	S.	N.	N.	N.	N.	N.	N.	S.	S.	S.	
	° /	° /	° /	° /	° /	° /	° /	° /	° /	° /	° /	° /	
1	23 03	17 11	7 19	4 48	15 17	22 09	23 06	17 55	8 05	3 26	14 40	21 57	1
2	22 58	16 53	6 56	5 11	15 34	22 17	23 02	17 39	7 43	3 49	14 59	22 06	2
3	22 52	16 36	6 33	5 34	15 52	22 24	22 57	17 23	7 21	4 13	15 18	22 14	3
4	22 46	16 18	6 10	5 57	16 09	22 31	22 52	17 07	6 59	4 36	15 37	22 22	4
5	22 40	16 00	5 46	6 20	16 26	22 38	22 46	16 51	6 37	4 59	15 55	22 30	5
6	22 33	15 42	5 23	6 42	16 43	22 44	22 40	16 35	6 14	5 22	16 13	22 37	6
7	22 25	15 23	5 00	7 05	17 00	22 50	22 34	16 18	5 52	5 45	16 30	22 43	7
8	22 18	15 05	4 36	7 27	17 16	22 55	22 27	16 00	5 29	6 08	16 48	22 50	8
9	22 10	14 45	4 13	7 50	17 32	23 00	22 20	15 43	5 06	6 31	17 05	22 55	9
10	22 01	14 26	3 49	8 12	17 48	23 05	22 12	15 26	4 43	6 54	17 26	23 01	10
11	21 52	14 07	3 26	8 34	18 03	23 09	22 04	15 08	4 21	7 16	17 39	23 05	11
12	21 42	13 47	3 02	8 56	18 18	23 13	21 56	14 50	3 58	7 39	17 55	23 10	12
13	21 32	13 27	2 39	9 17	18 33	23 16	21 47	14 31	3 35	8 01	18 11	23 14	13
14	21 22	13 06	2 15	9 39	18 47	23 19	21 38	14 13	3 11	8 24	18 26	23 17	14
15	21 11	12 46	1 51	10 00	19 01	23 22	21 28	13 54	2 48	8 46	18 41	23 20	15
16	20 50	12 25	1 28	10 22	19 15	23 24	21 19	13 35	2 25	9 08	18 57	23 23	16
17	20 48	12 05	1 04	10 43	19 29	23 26	21 08	13 16	2 02	9 30	19 11	23 25	17
18	20 36	11 43	0 40	11 03	19 42	23 27	20 58	12 56	1 38	9 52	19 25	23 26	18
19	20 24	11 22	0 17	11 24	19 55	23 28	20 47	12 37	1 15	10 14	19 39	23 28	19
20	20 11	11 01	N. 07	11 45	20 07	23 29	20 36	12 17	0 52	10 36	19 53	23 29	20
21	19 58	10 39	0 31	12 05	20 19	23 29	20 24	11 57	0 28	10 57	20 06	23 29	21
22	19 45	10 18	0 54	12 25	20 31	23 28	20 12	11 37	0 05	11 18	20 19	23 28	22
23	19 31	9 56	1 18	12 45	20 43	23 27	20 00	11 16	S. 19	11 39	20 31	23 27	23
24	19 17	9 34	1 42	13 05	20 54	23 26	19 47	10 56	0 42	12 00	20 43	23 26	24
25	19 02	9 11	2 05	13 24	21 04	23 24	19 34	10 35	1 05	12 21	20 55	23 24	25
26	18 47	8 49	2 29	13 44	21 15	23 22	19 21	10 14	1 29	12 41	21 06	23 22	26
27	18 32	8 27	2 52	14 03	21 25	23 20	19 07	9 53	1 52	13 02	21 17	23 19	27
28	18 16	8 04	3 15	14 21	21 35	23 17	18 53	9 32	2 16	13 22	21 28	23 16	28
29	18 00	7 41	3 39	14 40	21 44	23 14	18 39	9 10	2 39	13 42	21 38	23 13	29
30	17 44		4 02	14 58	21 53	23 10	18 25	8 49	3 03	14 02	21 48	23 09	30
31	17 28		4 25		22 01		18 10	8 27		14 21		23 04	31
Days	S.	S.	N.	N.	N.	N.	N.	N.	S.	S.	S.	S.	Days
	Decl.	Decl.	Decl.	Decl.	Decl.	Decl.	Decl.	Decl.	Decl.	Decl.	Decl.	Decl.	
	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	

1053. A TABLE of the Sun's Right Ascension.

Days	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Days
	Right Asc. H.M.	Right Asc. H.M.	Right Asc. H.M.	Right Asc. H.M.	Right Asc. H.M.	Right Asc. H.M.	Right Asc. H.M.	Right Asc. H.M.	Right Asc. H.M.	Right Asc. H.M.	Right Asc. H.M.	Right Asc. H.M.	
1	18 48	21 01	22 49	0 43	2 34	4 37	6 41	8 46	10 42	12 30	14 26	16 30	1
2	18 53	21 05	22 53	0 47	2 38	4 41	6 45	8 50	10 45	12 34	14 30	16 35	2
3	18 57	21 09	22 57	0 50	2 42	4 45	6 49	8 53	10 49	12 37	14 34	16 39	3
4	19 02	21 13	23 01	0 54	2 46	4 49	6 53	8 57	10 53	12 41	14 38	16 43	4
5	19 06	21 17	23 04	0 7	2 49	4 53	6 57	9 01	10 56	12 45	14 42	16 48	5
6	19 10	21 21	23 08	1 01	2 53	4 57	7 02	9 05	11 00	12 48	14 46	16 52	6
7	19 15	21 25	23 12	1 05	2 57	5 01	7 06	9 09	11 04	12 52	15 50	16 57	7
8	19 19	21 29	23 15	1 08	3 01	5 05	7 10	9 13	11 07	12 56	14 54	17 01	8
9	19 23	21 33	23 19	1 12	3 05	5 10	7 14	9 17	11 11	12 59	14 58	17 05	9
10	19 28	21 37	23 23	1 16	3 09	5 14	7 18	9 20	11 14	13 03	15 02	17 10	10
11	19 32	21 41	23 26	1 19	3 13	5 18	7 22	9 24	11 18	13 07	15 06	17 14	11
12	19 37	21 44	23 30	1 23	3 17	5 22	7 26	9 28	11 22	13 10	15 10	17 19	12
13	19 41	21 48	23 34	1 27	3 20	5 26	7 30	9 32	11 25	13 14	15 15	17 23	13
14	19 45	21 52	23 38	1 30	3 24	5 30	7 34	9 35	11 29	13 18	15 19	17 27	14
15	19 49	21 56	23 41	1 34	3 28	5 34	7 38	9 39	11 32	13 21	15 23	17 32	15
16	19 54	22 00	23 45	1 38	3 32	5 39	7 42	9 43	11 3	13 25	15 27	17 36	16
17	19 58	22 04	23 48	1 41	3 36	5 43	7 46	9 47	11 40	13 29	15 31	17 41	17
18	20 00	22 08	23 52	1 45	3 40	5 47	7 50	9 50	11 43	13 33	15 35	17 45	18
19	20 07	22 12	23 56	1 49	3 44	5 51	7 54	9 54	11 47	13 36	15 39	17 50	19
20	20 11	22 15	23 59	1 53	3 48	5 55	7 58	9 58	11 50	13 40	15 44	17 54	20
21	20 15	22 19	0 03	1 56	3 52	5 59	8 02	10 02	11 54	13 44	15 48	17 58	21
22	20 19	22 23	0 07	2 00	3 56	6 04	8 06	10 05	11 58	13 48	15 52	18 03	22
23	20 23	22 27	0 10	2 04	4 00	6 08	8 10	10 09	12 1	13 52	15 56	18 07	23
24	20 28	22 31	0 14	2 08	4 04	6 12	8 14	10 13	12 05	13 55	16 00	18 12	24
25	20 32	22 34	0 18	2 11	4 08	6 16	8 18	10 16	12 08	13 59	16 05	18 16	25
26	20 36	22 38	0 21	2 15	4 12	6 20	8 22	10 20	12 12	14 03	16 09	18 21	26
27	20 40	22 42	0 25	2 19	4 16	6 24	8 26	10 24	12 16	14 07	16 13	18 25	27
28	20 44	22 46	0 28	2 23	4 20	6 29	8 30	10 27	12 19	14 11	16 17	18 30	28
29	20 48		0 32	2 26	4 24	6 33	8 34	10 31	12 23	14 15	16 22	18 34	29
30	20 52		0 36	2 30	4 29	6 37	8 38	10 35	12 26	14 19	16 26	18 38	30
31	20 56		0 39		4 33		8 42	10 38		14 22		18 43	31
Days	H.M. Right Asc.	H.M. Right Asc.	H.M. Right Asc.	H.M. Right Asc.	H.M. Right Asc.	H.M. Right Asc.	H.M. Right Asc.	H.M. Right Asc.	H.M. Right Asc.	H.M. Right Asc.	H.M. Right Asc.	H.M. Right Asc.	Days
	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	

1054. *A TABLE of the Right Ascension and Declination of some of the principal Fixed Stars; as in the year 1752.*

	Constellation	Place in the Constellat.	Stars names	Mark	Magnit.	Rt. Asc.		Declinat.	
						H.	M.	°	'
CONSTELLATIONS IN THE ZODIAC.	ARIES	Bright star of Aries		α	3	1	53	22	17N.
	TAURUS	The Bull's eye	ALDEBARAN	α	1	4	22	15	59N.
	GEMINI	{ Southern twin	POLLUX	β	2	7	28	28	36N.
		{ Northern twin	CASTOR	α	1	7	17	32	24N.
	CANCER	In Southern claw	ACUBENS	α	4	8	45	13	15N.
	LEO	{ Lion's heart	REGULUS	α	1	9	53	13	09N.
		{ In the Lion's tail	DENER	β	2	11	35	15	55N.
	VIRGO	Virgin's spike		α	1	13	12	9	52 S.
	SCORPIO	Scorpion's heart	ANTARES	α	1	16	14	25	51 S.
	SAGITTARIUS	In the southern part of the bow		ϵ	2	18	06	34	28 S.
NORTHERN CONSTELLATIONS.	ANDROMEDA	{ In the girdle	MIRACH	β	2	0	56	34	18N.
		{ In the foot	ALMAACH	γ	2	1	49	41	08N.
	PERSEUS	{ Medusa's head	ALGOL	β	2	2	52	39	59N.
		{ In Perseus's side	ALGENIB	α	2	3	07	48	58N.
	AURIGA	In the preceding should.	CAPELLA	α	1	4	59	45	43N.
	BOOTES	{ The brightest	ARCTURUS	α	1	14	03	20	30N.
		{ In the following thigh	MIRACH	ϵ	3	14	33	28	06N.
	NORTH CROWN	Brightest in the crown		α	2	15	22	27	33N.
	EAGLE	Eagle's heart	ALTAIR	α	2	19	39	8	14N.
	PEGASUS	{ In Pegasus's wing	MARKAB	α	2	22	53	13	53N.
		{ In Pegasus's leg	SCHEAT	β	2	22	52	26	43N.
	CASSIOPEIA	In the breast	SCHEDAR	α	3	0	27	55	11N.
	GREAT BEAR	{ Upper pointer	DUBHE	α	2	10	46	63	03N.
		{ Lower pointer		β	2	10	45	57	41N.
		{ First in the tail	ALIATH	ϵ	2	12	41	57	18N.
		{ Last in the tail	BENETNASCH	η	3	13	36	50	31N.
	HERCULES	In the head	RAS ALGETHI	α	3	17	03	14	40N.
	LYRA	In the harp	VEGA	α	1	18	29	38	34N.
	SWAN	In the tail	DENER	α	2	20	33	42	25N.
	LITTLE BEAR	Pole star		α	2	0	42	88	00N.
SOUTHERN CONSTELLATIONS.	ORION	{ Bright foot	REGEL	β	1	5	01	8	28 S.
		{ Right shoulder	BETELGEUSE	α	1	5	52	7	20N.
	GREAT DOG	In the mouth	SYRIUS	α	1	6	34	16	23 S.
	LITTLE DOG	In the flank	PROCYON	α	1	7	24	5	49N.
	HYDRA	In the heart		α	2	9	16	7	37 S.
	SOUTH FISH	The brightest	FOMALHAUT	α	1	22	42	30	55 S.
	ARGO	In the waves	CANOPUS		1	6	18	52	33 S.
	ERIDANUS	The last	ACHARNAR	α	1	1	26	58	29 S.
	CENTAURUS	Brightest		α	1	14	13	59	46 S.

1009.

PROBLEM XXII.

Given the sun's declination, and altitude at six o'clock;
Required the latitude of the place.

EXAM. Being at sea on the 20th of May, 1753, at six o'clock in the evening, the sun's altitude was $18^{\circ} 45'$: What was the latitude of the place of observation?

CONSTRUCTION. Having described the meridian, drawn the horizon HR, the prime vertical ZN, and the parallel st of $18^{\circ} 45'$ altitude (10th 968); from the centre γ , with the half tangent of the declination $= 20^{\circ} 04'$, cut the parallel st in o : Thro' o draw the axis PS, and the azimuth circle ZON (163), and the measure of RP will give the latitude sought.

COMPUTATION. In the spheric triangle γAO , right angled at A.
 Given the decl. $\gamma O = 20^{\circ} 04' N$, | Then $\sin. \gamma O : \text{Rad} :: \sin. AO : \sin. \angle O \gamma A$.
 the altit. $AO = 18^{\circ} 45'$ | Or $\sin. \text{decl.} : \text{Rad} :: \sin. \text{alt.} : \sin. \text{latit.}$
 Required the lat. $= \angle O \gamma A$. | Which is $69^{\circ} 32' N$, as the decl. is N.

1010.

PROBLEM XXIII.

Given the sun's declination, and altitude when he is due east or west;
Required the latitude of the place.

EXAM. In a place where the compass had no variation, the sun was observed to be due east when his declination was $16^{\circ} 38' N$, and his altitude $20^{\circ} 12'$: What is the latitude of that place?

CONSTRUCTION. In the meridian HZRN, draw the horizon HR, the prime vertical ZN, and make $\gamma O =$ half tan. of the alt. $20^{\circ} 12'$: About o as a pole, at the distance of $73^{\circ} 22'$, the co-decl., describe (454) a small circle, cutting the meridian in P the elevated pole; draw the axis PS, equator EQ, and thro' P, o , S, describe an hour circle POS; then the measure of PR shews the latitude.

COMPUTATION. In the spheric triangle γAO , right angled at A.
 Given the alt. $\gamma O = 20^{\circ} 12'$ | Then $\sin. \gamma O : \text{Rad} :: \sin. AO : \sin. \angle A \gamma O$.
 the decl. $AO = 16^{\circ} 38' N$ | Or $\sin. \text{alt.} : \text{Rad} :: \sin. \text{decl.} : \sin. \text{lat.}$
 Required the latit. $= \angle A \gamma O$. | Which is $56^{\circ} 00' N$, as the decl. is N.

But had the declination been S., the other intersection of the parallel circle and meridian must have been taken for the elevated pole, and the latitude would be south.

$$\sin \phi = \frac{\sin d}{\sin h.}$$

1011.

PROBLEM XXIV.

Given the sun's altitude and hour of the day on either equinox;
Required the latitude of the place.

EXAM. On the day the sun enter'd the vernal equinox, his latitude was found $22^{\circ} 56'$ at 9 o'clock in the morning: In what lat. was that observation made?

CONSTRUCTION. Describe the meridian, draw the horizon, the prime vertical, and (10th 968) the parallel st of $22^{\circ} 56'$ altitude; from the centre γ , with the half tan. of $45^{\circ} = 3 h.$, the ascen. diff., cut st in o , and describe the vertical circle ZON, cutting the horizon in B.

COMPUTATION. In the spheric triangle γBO , right angled at B.
 Given the time after 6 $\gamma O = 45^{\circ} 00'$ | As $\sin. \gamma O : \text{Rad} :: \sin. BO : \sin. \angle B \gamma O$.
 the altitude $BO = 22^{\circ} 56'$ | Or $\sin. \text{time} \div 6 : \text{Rad} :: \sin. \text{alt.} : \cos. l$.
 Required the co-latitude $B \gamma O$. | Which is $56^{\circ} 34'$.

1012.

1012.

PROBLEM XXV.

Given the sun's altitude, declination and azimuth;
Required the latitude of the place.

EXAM. *June 4th, 1753, being at sea in a place where the compass had no variation, in the afternoon when the sun was $42^{\circ} 30'$ high, his bearing was S. $57^{\circ} 45'$ W.: What is the latitude of that place?*

CONSTRUCTION. Draw the meridian, the horizon HR, the prime vertical ZN, and the parallel st at $42^{\circ} 30'$ above the horizon (456): The tangent of $57^{\circ} 45'$ set from A towards R gives the centre of the azimuth circle ZON, cutting the parallel of altitude st in o: About o as a pole (454), at the distance of $67^{\circ} 30'$, equal to the co-declination, describe a small circle, cutting the meridian in P the place of the pole; then the measure of RP gives the latitude sought.

COMPUTATION. In the oblique angled spheric triangle ZOP.

Given the zenith dist. ZO = $47^{\circ} 30'$
the polar dist. PO = $67^{\circ} 30'$
the azimuth $\angle PZO = 122^{\circ} 15'$ } Required the co-latitude PZ.

As Rad	= R	10,00000	As fin. alt.	= $42^{\circ} 30'$	0,17032
To cos. azim.	= $122^{\circ} 15'$	9,72723	To fin. decl.	= $22^{\circ} 30'$	9,58284
So cot. alt.	= $42^{\circ} 30'$	10,03795	So cos. 4th	= $30^{\circ} 13'$	9,93658
To tan. 4th	= $30^{\circ} 13'$	9,76518	To cos. 5th	= $60^{\circ} 42'$	9,68974

Then the difference between the 5th and 4th arcs, that is $30^{\circ} 13'$ taken from $60^{\circ} 42'$, the remainder $30^{\circ} 29'$ is the co-lat. Therefore $59^{\circ} 31'$ N. is the latitude sought.

1013.

PROBLEM XXVI.

Given the sun's declination, his altitude and hour of the day;
Required the latitude of the place.

EXAM. *Being at sea, the sun's altitude was observed to be $37^{\circ} 20'$ at 9 h. 45 m. in the morning, his declination at that time being $20^{\circ} 30'$ N.: What is the latitude of the place of observation?*

CONSTRUCTION. In the meridian PESQ, draw the equator EQ, axis PS, and parallel of declinat. nm , $22^{\circ} 30'$ dist. from the equator (3d 968): Set off from the centre A towards Q the tang. of $33^{\circ} 45' = 2$ h. 15 m., the distance between the time of observation and noon, which gives the centre of the hour circle POS, cutting the parallel nm in o: About the point o as a pole, describe (454) at the dist. of $52^{\circ} 40'$, the zen. dist., a small circle, cutting the meridian in Z the zenith; thro' ZO describe an azimuth circle ZON; then the measure of ZE will give the lat. sought.

COMPUTATION. In the oblique angled triangle ZOP.

Given the zenith distance ZO = $52^{\circ} 40'$
the polar distance PO = $67^{\circ} 30'$
the hour from noon $\angle ZPO = 33^{\circ} 45'$ } Required the co-lat. ZP.

As Rad : cos. hour A. M. :: cot. decl. : tan. 4th arc = $63^{\circ} 31'$.

As fin. decl. : fin. alt. :: cos. 4th arc : cos. 5th arc = $45^{\circ} 02'$.

Their difference is the co-latitude $18^{\circ} 29'$. Therefore the lat. is $71^{\circ} 31'$ N.

1014.

$$\begin{aligned} \tan x &= \cos s \cot d \\ \cot y &= \frac{\sin h. \cot x}{\sin d.} \end{aligned}$$

Given the altitudes of two known fixed stars, having the same azim.;
Required the latitude of the place.

EXAM. *Being at sea in an unknown latitude, I observed the star Schedar in Cassiopeia, and Almaach in Andromeda, to have the same Azimuth, when the altitude of Schedar was $37^{\circ} 15'$, and that of Almaach was $17^{\circ} 45'$: What is the latitude of that place?*

CONSTRUCTION *upon the plane of the equator.* Let the primitive circle represent the equator, whose pole is P, and any point v the place where the right ascension begins, from whence lay off $va = 27^{\circ} 15'$ for Almaach's right ascension, and $vb = 6^{\circ} 45'$ for Schedar's; draw the circles of right ascension Pa, Pb: Describe (3d 970) Almaach's and Schedar's parallels of declination, cutting Pa, Pb, in A, B; A being Almaach, and B Schedar. A great circle passing thro' A and B, will be the azimuth they are on. About B at the distance of $52^{\circ} 54'$, Schedar's zenith distance, describe (454) a small circle, cutting the azimuth circle thro' A and B in Z, the zenith of the place; draw PZ, which measured on the half tangents gives the co-latitude of the place of observation.

COMPUTATION.

1st. In the oblique angled spheric triangle ABP.

Given Almaach's co-dec. PA = $48^{\circ} 52'$ | Required the angle of position ABP.
Schedar's co-dec. PB = $34^{\circ} 49'$ |
their diff. of alt. AB = $19^{\circ} 30'$ | For the solution see art. 550.

PB = $34^{\circ} 49'$	Co-ar. fin. PB = $34^{\circ} 49'$	0,24340
BA = $19^{\circ} 30'$	Co-ar. fin. BA = $19^{\circ} 30'$	0,47650
PA = $48^{\circ} 52'$	Sin. $\frac{1}{2}$ sum = $32^{\circ} 05\frac{1}{2}'$	9,72532
PB - BA = $15^{\circ} 19'$	Sin. $\frac{1}{2}$ diff. = $16^{\circ} 46\frac{1}{2}'$	9,46031
64 11 $32^{\circ} 05\frac{1}{2}' = \frac{1}{2}$ sum.	Sum of four logs.	19,90553
33 33 $16^{\circ} 46\frac{1}{2}' = \frac{1}{2}$ diff.	$\frac{1}{2}$ sum gives $63^{\circ} 45\frac{1}{2}'$	9,95276
	Which doub. gives $127^{\circ} 31' = \angle ABP.$	

2d. In the oblique spheric triangle PBZ.

Given Schedar's co-dec. PB = $34^{\circ} 49'$ | Required the co-lat. PZ.
Schedar's co-alt. BZ = $52^{\circ} 45'$ | For the solution see art. 548.
angle of position PBZ = $52^{\circ} 29'$ | Here $\angle PBZ = \text{suppl. } \angle PBA.$

As Rad = R	10,00000	As cos. 4th arc = $22^{\circ} 57'$	0,03581
To cos. $\angle$ posit. = $52^{\circ} 29'$	9,78461	To cos. 5th arc = $29^{\circ} 48'$	9,93840
So cot. Sch. decl. = $55^{\circ} 11'$	9,84227	So sin. Sch. decl. = $55^{\circ} 11'$	9,91433
To tan. 4th arc = $22^{\circ} 57'$	9,62688	To sin. latit. = $50^{\circ} 41'$	9,88854
Which taken from $52^{\circ} 45'$	= BZ.		
Leaves 5th arc	$29^{\circ} 48'$		

1015.

PROBLEM XXVIII.

Given the difference of time between the rising of two known stars ;
Required the latitude of the place.

EXAM. *Being at sea in an unknown place, the star Aldebaran was observed to rise 3 h. 15 m. later than the bright star in Aries : Required the latitude of that place?*

Bright star in φ decl. $22^{\circ} 17' N.$; right ascen. 1 h. 53 m. Aldebaran's decl. $15^{\circ} 59' N.$; right ascension 4 h. 22 m.

CONSTRUCTION.

Let the primitive circle represent the equator, describe (3d 970) the parallels of declination of the two stars, that of Aries being $22^{\circ} 17' N.$, and of Aldebaran $15^{\circ} 59' N.$: Draw Pa for the circle of right ascension passing thro' the star in φ , which suppose in A : From a lay off $ab = 37^{\circ} 15' =$ diff. of right ascensions ; and $ac = 48^{\circ} 45' = 3 h. 15 m.$, the diff. of time between their rising ; draw bP, cP , cutting the parallels of declination in B, C : Thro' the points B, C , describe (441) the great circle HOR ; draw PO at right angles to HR , then the measure of PO will give the latitude sought.

COMPUTATION.

In the oblique angled spheric triangle PBC .

Given B 's co-decl. $PB = 74^{\circ} 01'$	Rad : cos. $\angle BPC :: t.PB : t.M = 73^{\circ} 42'$.
A 's co-decl. $PC = 67^{\circ} 43'$	From M take PC leaves $N = 5^{\circ} 59'$.
$\angle APC - \angle APB = \angle BPC = 11^{\circ} 30'$	As sin. $N : f. M :: \tan. \angle CPB : t. \angle C =$
Required the angle PCB .	Hence $\angle PCO = 61^{\circ} 54' [118^{\circ} 06']$.

In the spheric triangle PCO , right angled at O .

Given C 's co-decl. $PC = 67^{\circ} 43'$	As Rad : sin. $PC :: \sin. \angle PCO : \sin. PO$
and the $\angle PCO = 61^{\circ} 54'$	$[= 54^{\circ} 43']$
Required the latitude PO .	Therefore the latitude is $54^{\circ} 43'$.

1016.

PROBLEM XXIX.

Two known fixed stars being observed to have the same altitude ;
Required the latitude of the place of observation.

EXAM. *In the evening, the stars Capella and Procyon were observed at the same time to have each $38^{\circ} 00'$ of altitude : Required the latitude of the place where that observation was taken?*

Capella's decl. $= 45^{\circ} 43' N.$ right ascen. 4 h. 59 m. Procyon's decl. $= 5^{\circ} 49' N.$ right ascen. $= 7 h. 24'$.

CONSTRUCTION.

On the plane of the equator, represented by the primitive circle, describe (3d 970) the parallels of the given declinations ; take $ab = 36^{\circ} 15'$, the diff. of the given right ascensions ; draw Pa, Pb , then A represents Procyon, and B Capella.

About A and B as poles, describe (454) arcs of small circles, at the distance of 52° , the co-altitude, and their intersection gives Z the zenith of the place : Thro' the points $A, B ; Z, A ; Z, B$; describe great circles (441), and draw ZP , whose measure will give the co-latitude of the place of observation.

COMPU-

COMPUTATION.

In the oblique angled spheric triangle APB.

Given A's co-decl. $AP = 84^{\circ} 11'$	Rad: cof. $\angle P :: t. BP : t. M = 38^{\circ} 11'$
B's co-decl. $BP = 44^{\circ} 17'$	And M tak. fr. PA leaves $N = 46^{\circ} 00'$
diff. rt. asc. $\angle ABP = 36^{\circ} 15'$	Sin. N : fin. M :: tan. $\angle P : \tan. \angle BAP$
Required the stars distance AB.	$[= 32^{\circ} 13']$
And the angle BAP.	Cof. M : cof. N :: cof. BP : cof. BA = $50^{\circ} 45'$

In the oblique angled spheric triangle AZB.

Given A's co-alt. $ZA = 52^{\circ} 00'$	The angle BAZ will be found =
B's co-alt. $ZB = 52^{\circ} 00'$	$68^{\circ} 15'$
stars distance AB = $50^{\circ} 45'$	Then $\angle BAZ - \angle BAP = \angle PAZ =$
Required the angle ZAB.	$36^{\circ} 02'$

In the spheric triangle AZP.

Given A's co-alt. $ZA = 52^{\circ} 00'$	Rad: cof. $\angle A :: t. ZA : t. M = 45^{\circ} 59'$
A's co-lat. $AP = 84^{\circ} 11'$	M taken from PA leaves $N = 38^{\circ} 12'$
the angle $ZAP = 36^{\circ} 02'$	Cof. M : cof. N :: cof. ZA : cof. PZ = $45^{\circ} 52'$
Required the co-lat. ZP.	Therefore the latitude is $44^{\circ} 08' N$.

1017.

PROBLEM XXX.

Given the altitudes of two known fixed stars;

Required the latitude of the place.

EXAM. The altitude of the Hydra's heart was observed to be $40^{\circ} 44'$, and of the Lion's heart $45^{\circ} 00'$: What is the latitude of the place of observation?

Hydra's heart, decl. = $7^{\circ} 37' S$.; right ascen. = 9 h. 16 m. Lion's heart, decl. = $13^{\circ} 09' N$.; right ascen. 9 h. 53 m.

CONSTRUCTION.

If this problem is constructed on the plane of the equator, it will be in every respect like the last; only the small circles, described about A and B, are to be unequally distant from their respective poles A, B.

COMPUTATION.

Here, as in the last, there will be three spheric triangles to work in; namely, the triangles APB, ZAB, and ZPB.

In the triangle APB, where $AP = 97^{\circ} 37'$, $BP = 76^{\circ} 51'$, $\angle APB = 9^{\circ} 15'$.

As Rad : cof. $\angle APB :: \tan. BP : \tan. M = 76^{\circ} 41'$. Then $AP - M = N = 20^{\circ} 56'$.

As fin. N : fin. M :: tan. $\angle APB : \tan. \angle BAP = 23^{\circ} 55'$.

And as cof. M : cof. N :: cof. BP : cof. BA = $22^{\circ} 42'$.

In the triangle BAZ, where $AZ = 49^{\circ} 16'$, $BZ = 45^{\circ} 00'$, $AB = 22^{\circ} 42'$.

The angle BAZ will be found equal to $68^{\circ} 56'$. (537)

Then $\angle BAZ - \angle BAP = \angle PAZ = 45^{\circ} 01'$.

In the triangle APZ, where $AP = 97^{\circ} 37'$, $AZ = 49^{\circ} 16'$, $\angle PAZ = 45^{\circ} 01'$.

As Rad : cof. $\angle PAZ :: \tan. AZ : \tan. M = 39^{\circ} 23'$. Then $AP - M = N = 58^{\circ} 14'$.

And as cof. M : cof. N :: cof. AZ : cof. PZ = $63^{\circ} 23'$.

Hence the latitude sought is $63^{\circ} 23' N$.

1018.

PROBLEM XXXI.

Given the sun's declination, two altitudes, and the time between the observations ;

Required the latitude of the place.

EXAM. On a day when the sun's declination was $20^{\circ} 00' N.$, in the forenoon the sun's altitude was observed to be $18^{\circ} 30'$, and three hours after, his altitude was $44^{\circ} 00'$: What was the latitude of the place ?

CONSTRUCTION.

Let the primitive circle represent that hour circle on which the sun was at the first observation, EQ being the equator, then A a, the parallel of 20° of declination, gives A the sun's place at first ; and as CQ is the tangent of 45° , Q will be the centre of the hour circle PBs three hours distant from the former, its intersection B with the parallel of declination, is the sun's place at the second observation : About A as a pole, at the distance of $71^{\circ} 30'$, the first zenith distance, describe (454) a small circle ; about B as a pole, at the distance of $46^{\circ} 00'$, the second zenith distance, describe (454) another small circle, cutting the former in z the zenith : Thro' z, A ; z, B ; A, B ; P, z ; describe (441) great circles ; then Pz is the co-latitude required.

COMPUTATION.

Here are three triangles to work in ; namely, ABP, ABZ, BPZ.

In the isosceles spheric triangle APB.

Given AP = $70^{\circ} 00'$	Suppose a perpendicular Pb is drawn.
BP = 70 00	
$\angle APB = 45^{\circ} 00'$	
Req. $\angle ABP$ and AB.	
	Rad : tan. BPb :: cos. PB : cot. $\angle PBA = 81^{\circ} 56'$.
	Rad : sin. PB :: sin. $\angle BPb$: sin. Pb = $21^{\circ} 04\frac{1}{2}'$.
	Then Bb doubled gives AB = $42^{\circ} 09'$.

In the oblique angled spheric triangle ABZ.

Given AZ = $71^{\circ} 30'$	Then working by art. 537 for three sides, the
BZ = 46 00	
AB = 42 09	
Required $\angle ABZ$.	
	angle ABZ will be found = $114^{\circ} 11'$.
	And $\angle ABZ - \angle PBA = \angle PBZ = 32^{\circ} 15'$.

In the oblique angled spheric triangle PBZ.

Given PB = $70^{\circ} 00'$	As Rad : cos. $\angle PBZ$:: tan. BZ : tan. M = $41^{\circ} 13'$.
BZ = 46 00	
$\angle PBZ = 32^{\circ} 15'$	
Requir. the co-lat. Pz.	
	And PB - M = N = $28^{\circ} 47'$.
	As cos. M : cos. N :: cos. BZ : cos. Pz = $35^{\circ} 58'$.
	Therefore the latitude is $54^{\circ} 02' N.$

1019. If the sun's altitude can be taken both before and after noon, when he has equal heights, then the time between these two observations being bisected, will give the time when the sun was on the meridian : Now the co-declination, the co-altitude, and the time from noon at either observation being known, the latitude may be readily computed in one oblique angled triangle, wherein are known two sides, and an angle opposite to one of them to find the other side, which is the co-latitude ; for which see the problem, art. 1013.

1020.

PROBLEM XXXII.

Given the sun's declination, two altitudes, and the difference of the magnetic azimuths;

Required the latitude of the place.

EXAM. On the 21st of May, the sun's declination being $20^{\circ} 16' N.$, in the morning when the sun was on the ESE. point of the compass, his altitude was $43^{\circ} 30'$; and when he bore S. $20^{\circ} 30' E.$ his altitude was $58^{\circ} 30'$: What is the latitude of the place of observation?

CONSTRUCTION.

Let the primitive circle represent the azimuth circle the sun was on at the greatest altitude $58^{\circ} 30'$, A being the sun's place at that time, HR the horizon, and z the zenith; draw aa a parallel of $43^{\circ} 30'$ of altitude, and (451) describe a vertical circle, making an angle with AZ, of $47^{\circ} 00'$, the diff. of the observed azimuths; where this cuts the parallel of altitude aa , gives B the sun's place at the first observation: Then small circles being described about A and B, as poles, at the distances of $69^{\circ} 44'$, the co-declination (454), their intersection will give P the place of the pole: Thro' A, B; P, A; P, B; and z, P, describe great circles (441); then PZ is the co-latitude sought.

COMPUTATION.

1st. In the spheric triangle AZB: Given AZ = A's co-alt. = $31^{\circ} 30'$
 BZ = B's co-alt. = $46 30$
 $\angle AZB$ = diff. of az. = $47 00$

Required $\angle ABZ$ = $45 41$
 AB = $32 17$

2^d. In the spheric triangle APB: Given AP = A's co-decl. = $69 44$
 BP = B's co-decl. = $69 44$
 AB = distance = $32 17$

Required $\angle ABP$ = $83 52$

3^d. In the spheric triangle ZBP: Given BZ = B's co-alt. = $46 30$
 PB = B's co-decl. = $69 44$
 $\angle ZBP$ = $38 11$

Requir. ZP = co-lat. = $39 21$

Therefore the latitude of the place is $50^{\circ} 39' N.$

1021.

PROBLEM XXXIII.

Two known stars being observed on the same azimuth, and two other known stars being observed on another azimuth, and the time between the observations being known; to find the latitude of the place.

EXAM. The stars Aldebaran in Taurus, and Regel in Orion, were observed on the same azimuth; and 2 h. 35 m. after, the stars Castor in Gemini and the Hydra's heart were also observed on another azimuth: What was the latitude of the place of observation?

CON-

CONSTRUCTION.

On the plane of the equator put the stars Aldebaran and Regal at A, B, also the stars Castor and Hydra at c, d, by the means of their right ascensions and declinations (2d and 3d of 970): Let the stars c, d, be moved forwards $2^h 35^m$, or $38^\circ 45'$ to C, D; thro' A, B, and D, C, describe (441) great circles intersecting in z, the zenith of the place; draw the great circles AC and PZ; then the measure of PZ gives the co-latitude required.

COMPUTATION.

Aldebaran's declin.	$= 15^\circ 59' N.$	right ascen.	$= 4^h 22^m = 65^\circ 30'.$
Regal's	$= 8^\circ 28' S.$		$= 5^h 01^m = 75^\circ 15'.$
Castor's	$= 32^\circ 24' N.$		$= 7^h 17^m = 109^\circ 15'.$
Hydra's heart	$= 7^\circ 37' S.$		$= 9^h 16^m = 139^\circ 00'.$

1st. In the triangle PAB, where $PB = 98^\circ 28'$, $PA = 74^\circ 01'$, $\angle APB = 9^\circ 15'.$

Then the $\angle PAB$ will be $157^\circ 46'$, and the $\angle PAZ$, the suppl. $= 22^\circ 14'.$

2d. In the triangle PCD, where $PD = 97^\circ 37'$, $PC = 57^\circ 36'$, $\angle CPD = 29^\circ 45'.$

Then the $\angle PCD$ will be $139^\circ 21'$, and the $\angle PCZ$, the suppl. $= 40^\circ 39'.$

3d. In the triangle APC, where $AP = 74^\circ 01'$, $CP = 57^\circ 36'$, $\angle APC = 5^\circ 00'$, which is the difference between 2 h. 25 m. and the difference of the right ascensions of A and c.

Then the $\angle PAC$ will be $14^\circ 06'$, $\angle PCA = 163^\circ 23'$, and $AC = 16^\circ 57'.$

Now $\angle PAC + \angle PAZ = \angle CAZ = 36^\circ 20'$; and $\angle PCA - \angle PCZ = \angle ACZ = 122^\circ 44'.$

4th. In the triangle ACZ, where $AC = 16^\circ 57'$; $\angle CAZ = 36^\circ 20'$; $\angle ACZ = 122^\circ 44'.$

Then cz will be found equal to $24^\circ 58'.$

5th. In the triangle CPZ, where $CZ = 24^\circ 58'$, $CP = 57^\circ 36'$, $\angle PCZ = 40^\circ 39'.$

Then PZ will be found equal to $40^\circ 52'.$

And the latitude of the place of observation is $49^\circ 08' N.$

There might be given a great variety of other problems to find the latitude from various circumstances; but the trouble of solving them, as well as some of the foregoing ones, is too great to render them of general use: And indeed, some of them were only inserted as trigonometrical exercises for young students; it being generally allowed, that the sciences are most readily learned by working of many examples: And on this account it was judged, that the few following questions might not be unentertaining to those who have a love for these matters.

SECTION VII.

Miscellaneous Problems.

1022. PROBLEM XXXIV.

In the latitude of $20^{\circ} 00' N.$ stands a horizontal dial, whose gnomon is perpendicular to the plane of the horizon: It is required to know at what hour in the afternoon, on the 21st of June 1753, the shadow of that gnomon shall stand still; and how many degrees shall the shadow run back?

CONSTRUCTION. Let the circle $ZRNH$ be the meridian of the place, HR the horizon, whose poles are Z, N ; $RP = 20^{\circ}$, the latitude, P and s the north and south poles: About P describe the parallel of decl. aa , cutting the horizon in L ; about s a small circle, being described at the dist. of $23^{\circ} 29'$, its intersection p with the horizon, is the pole of the azimuth circle which will touch the parallel aa in $\odot$, the place of the sun when he has the greatest azimuth that day: Thro' $Z, \odot, N$, describe a vertical circle cutting the horizon in K ; and thro' $\odot$ and L describe the hour circles $P\odot s, PLS$.

Then will the $\angle ZP\odot$ be equivalent to the hour when the shadow will stand still; and KL , the difference between the measures of the azimuth and amplitude, will shew how much the shadow will run back.

COMPUTATION. In the right angled spheric triangle PRL .

Given $\angle R = 90^{\circ} 00'$
 lat. $= RP = 20^{\circ} 00'$
 co-dec. $= PL = 66^{\circ} 31'$

Requir. ampl. $= RL = 64^{\circ} 55'$

Then $33^{\circ} 06' = 2 \text{ h. } 12 \text{ m. } 24 \text{ f.}$: And $77^{\circ} 26' - 64^{\circ} 55' = 12^{\circ} 31'$.

So that the shadow will stand still at 2 h. 12 m. 24 f., and will run back $12^{\circ} 31'$.

In rt. angl. spheric triangle $P\odot z$.

Given $\angle \odot = 90^{\circ} 00'$
 co-lat. $= ZP = 70^{\circ} 00'$
 co-decl. $= P\odot = 66^{\circ} 31'$

Required hour $= \angle ZP\odot = 33^{\circ} 06'$
 azim. $= \angle \odot ZP = 77^{\circ} 26'$

1023.

PROBLEM XXXV.

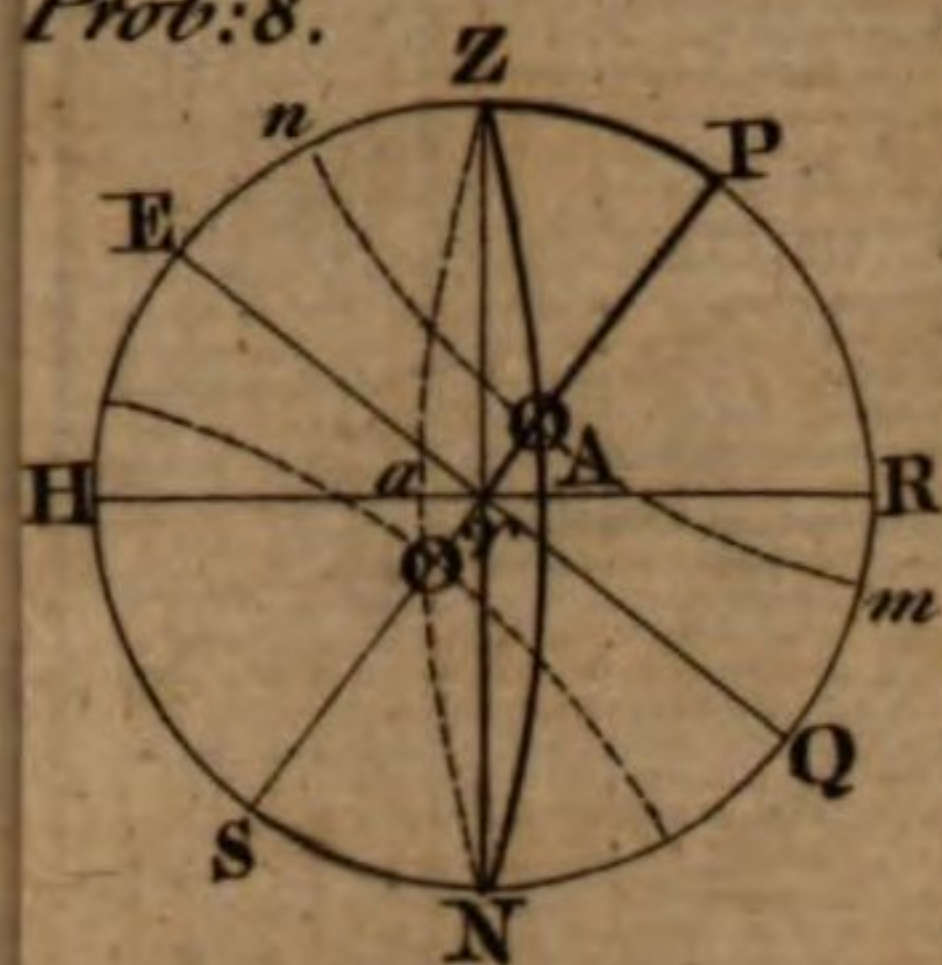
A comet, whose declination was $47^{\circ} 00' N.$, was observed to be distant from a star to the eastward $49^{\circ} 00'$; the star's declination was $36^{\circ} 00' N.$, and its right ascension $45^{\circ} 00'$: What was the latitude and longitude of that comet?

CONSTRUCTION.

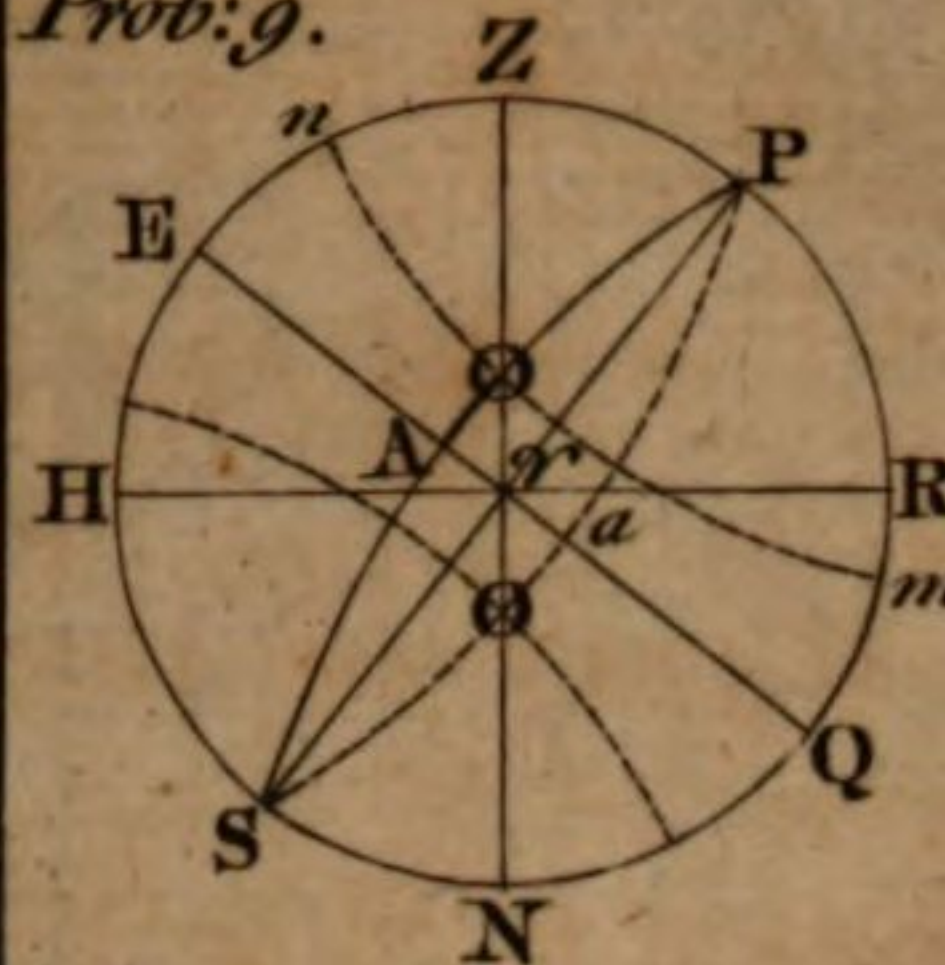
On the plane of the solstitial colure, where P and E are the poles of the equator and ecliptic, put the star at A by its right ascension and declination: About P and A describe small circles, at the distances of the comet from those points, their intersection C gives the place of the comet; describe great circles thro' AC, PC, EC , then EC will be the co-latitude, and the $\angle PEC$ the co-longitude; and their measures may be obtained from articles 446, 448;

COMPU-

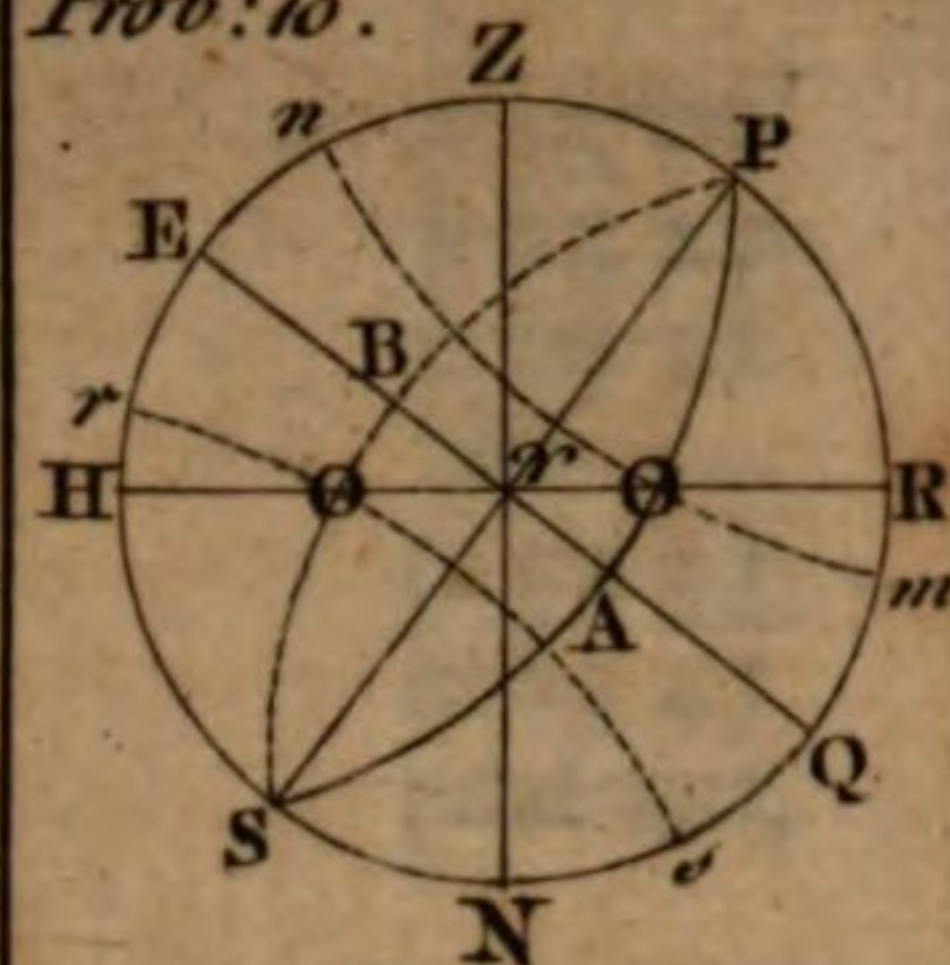
Prob: 8.



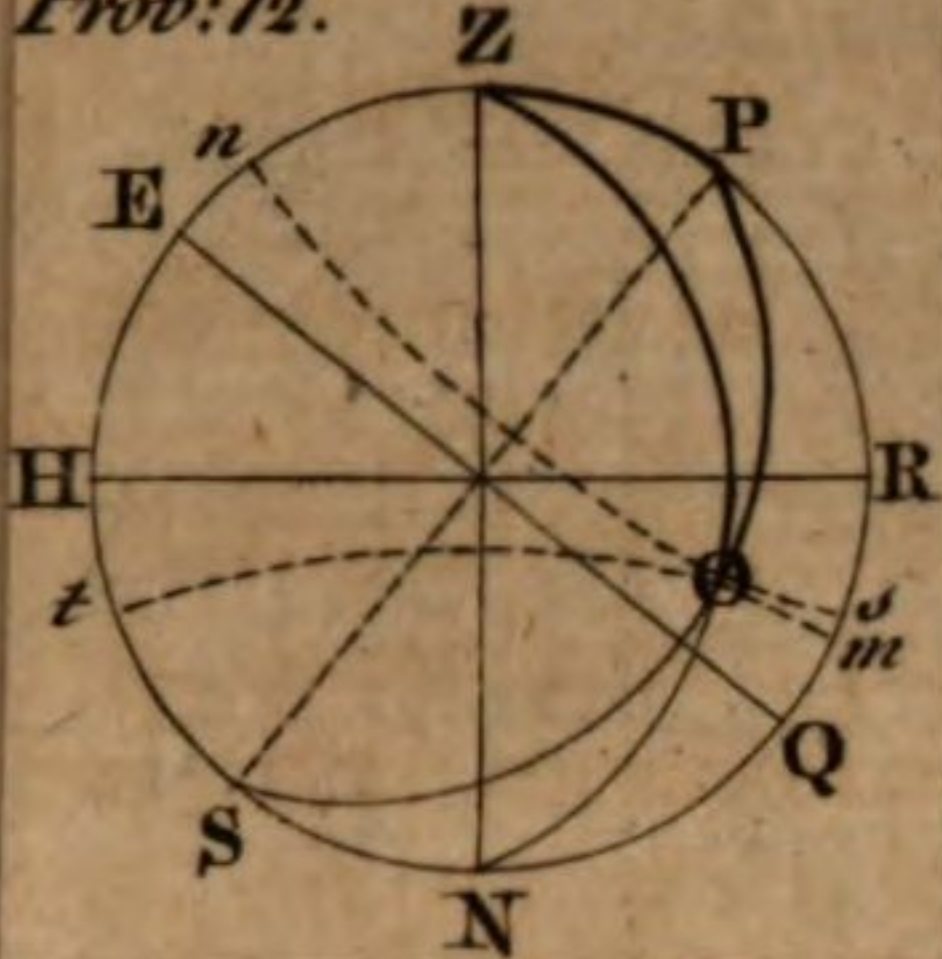
Prob: 9.



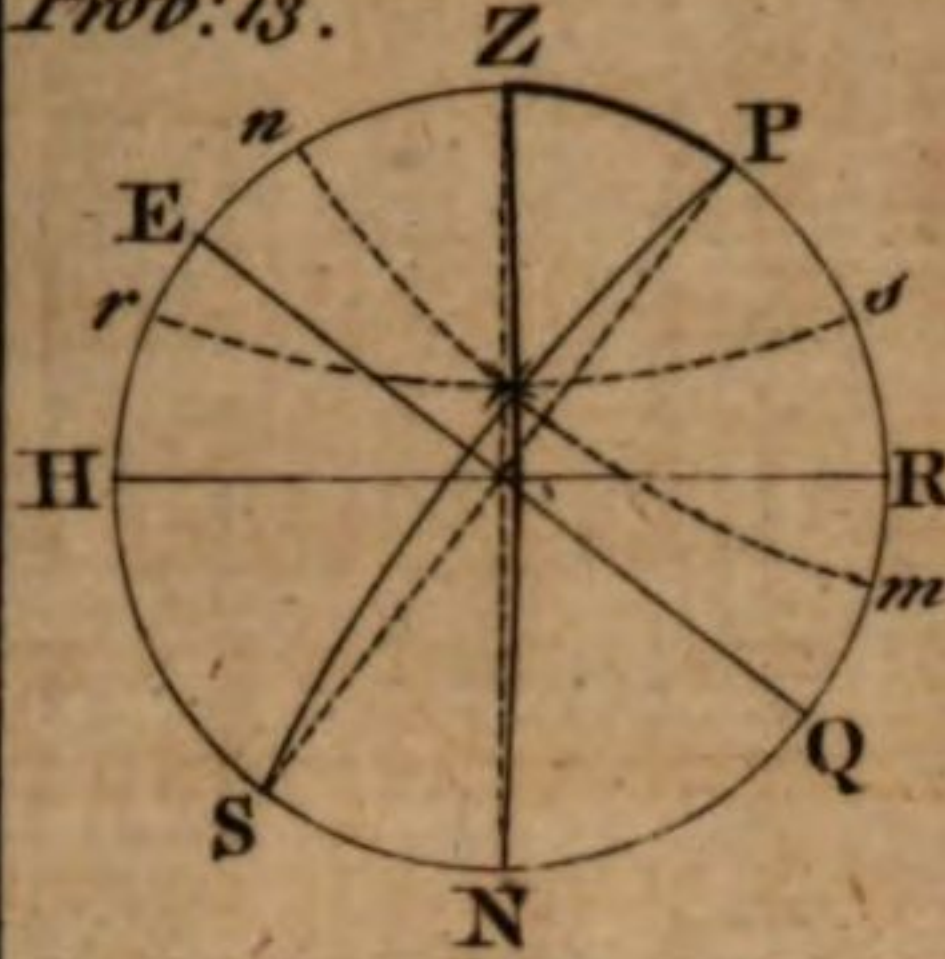
Prob: 10.



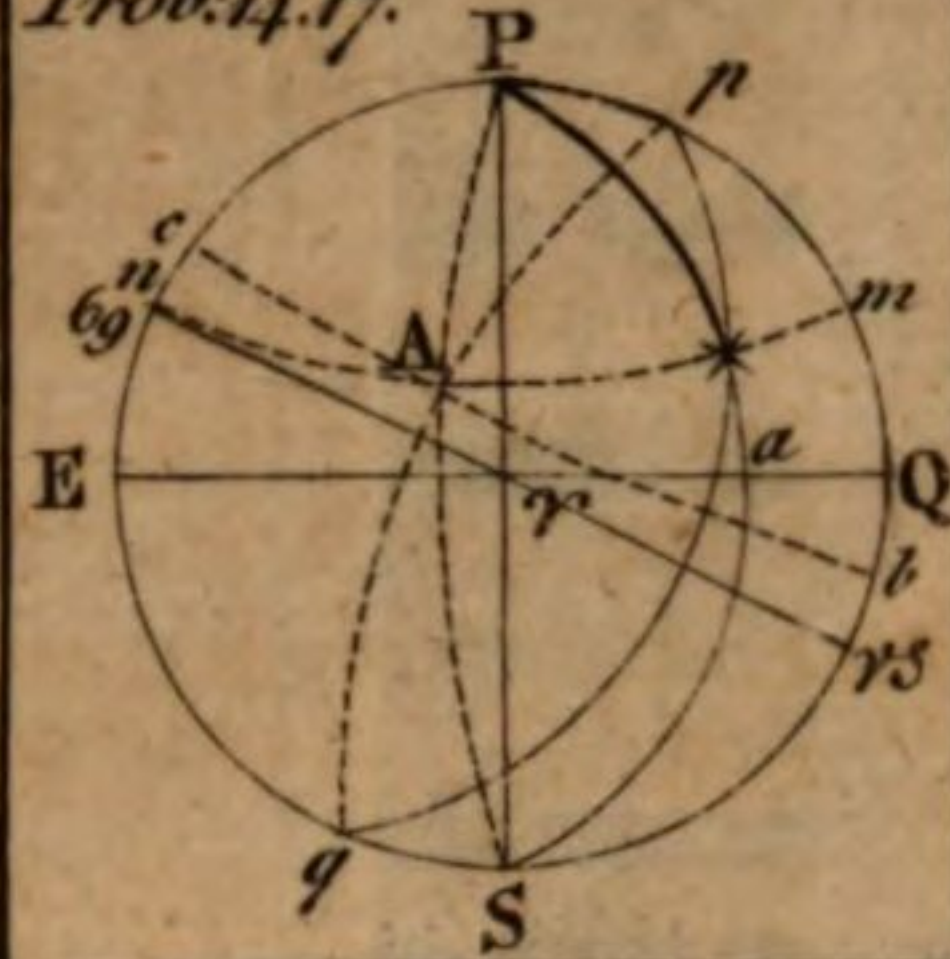
Prob: 12.



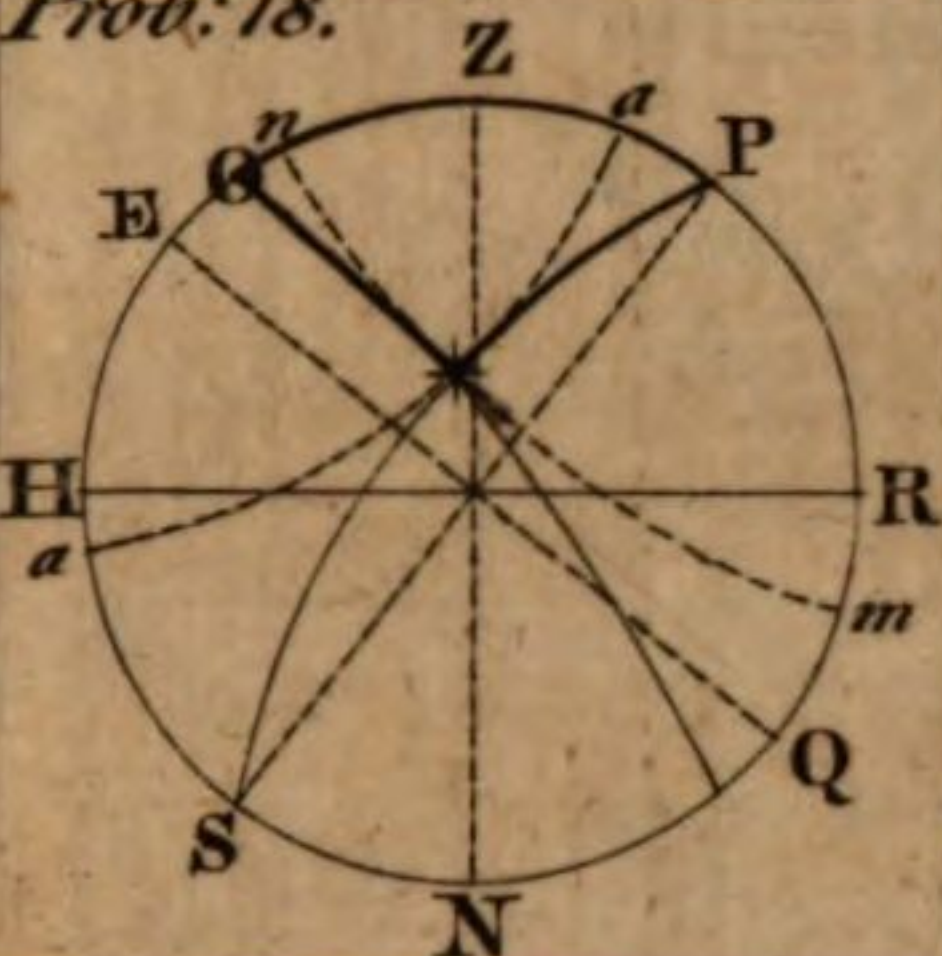
Prob: 13.



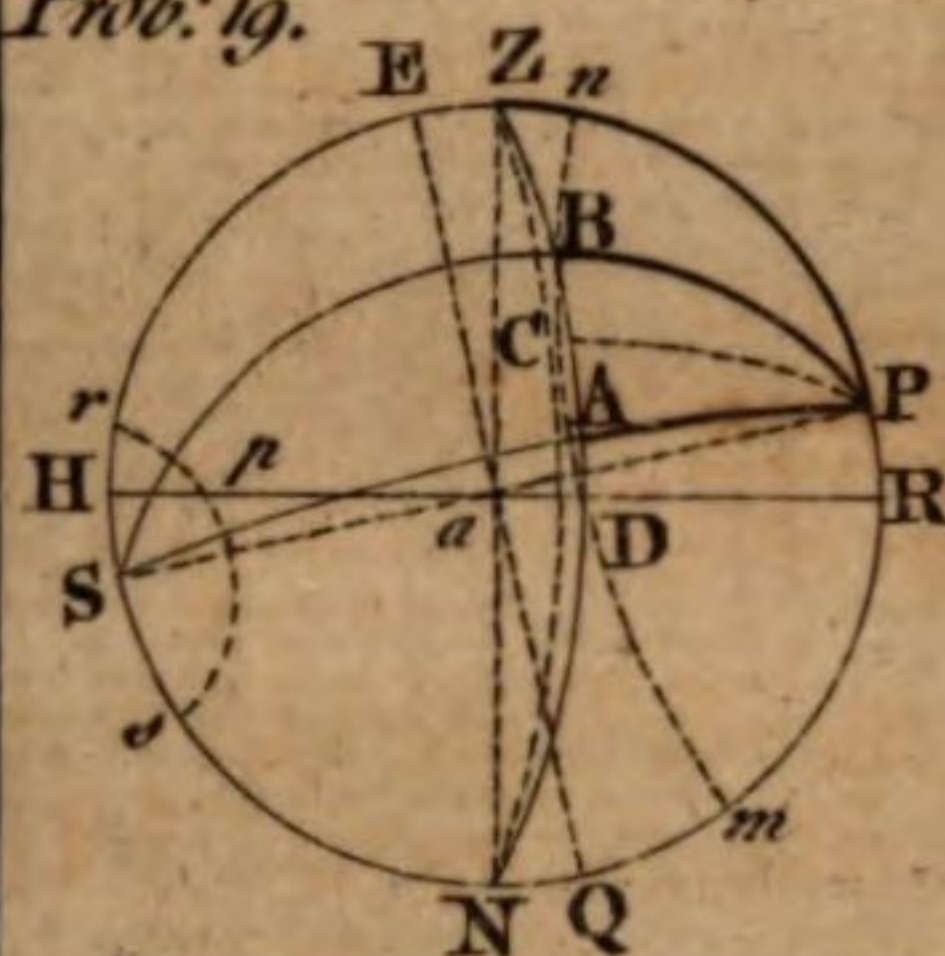
Prob: 14. 17.



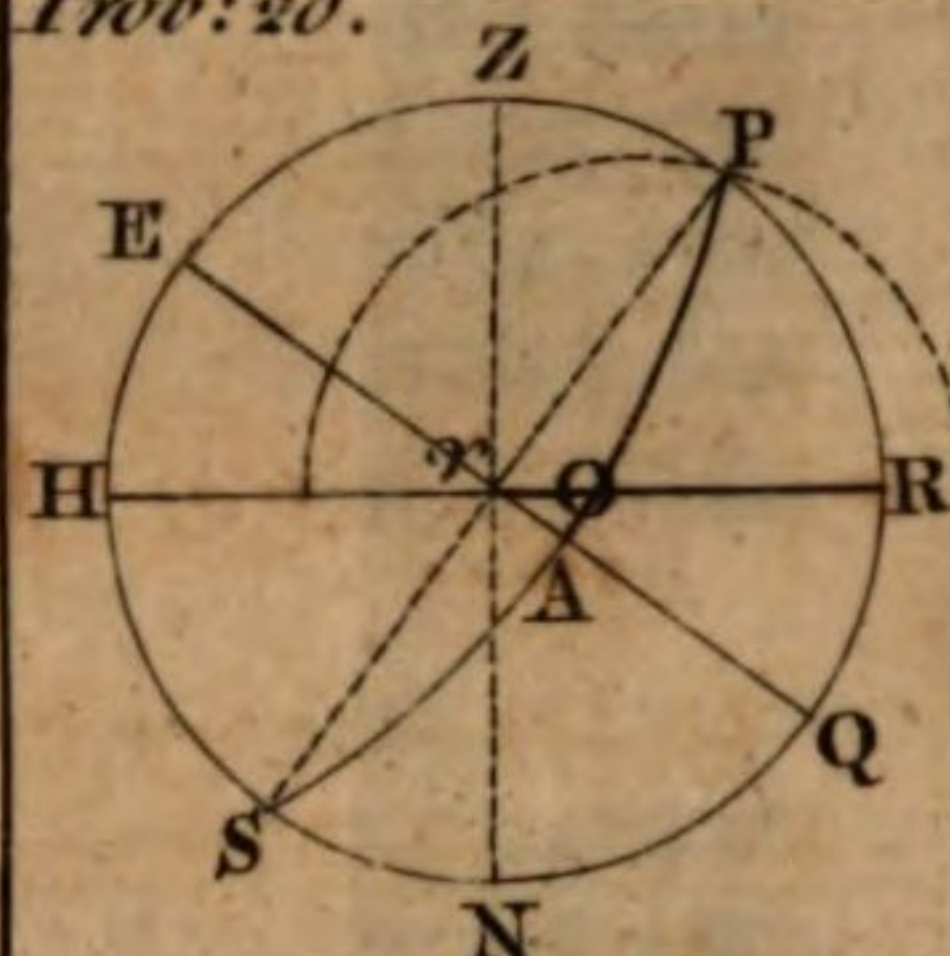
Prob: 18.



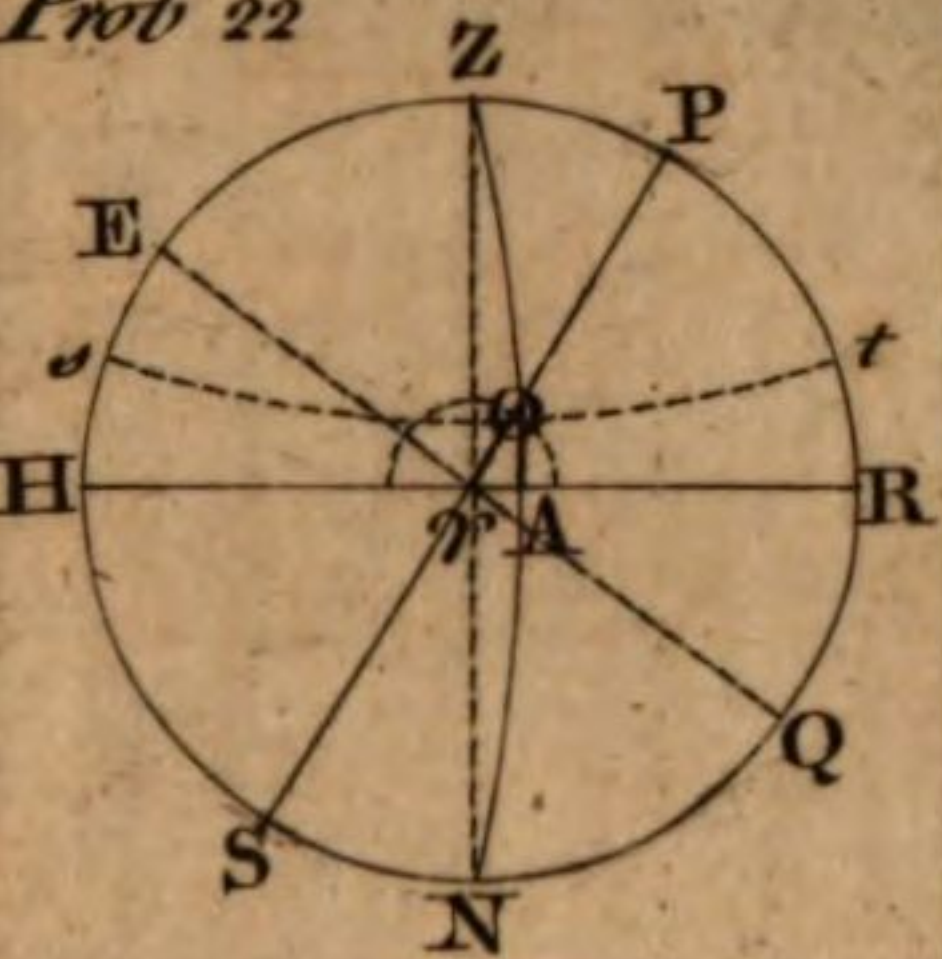
Prob: 19.



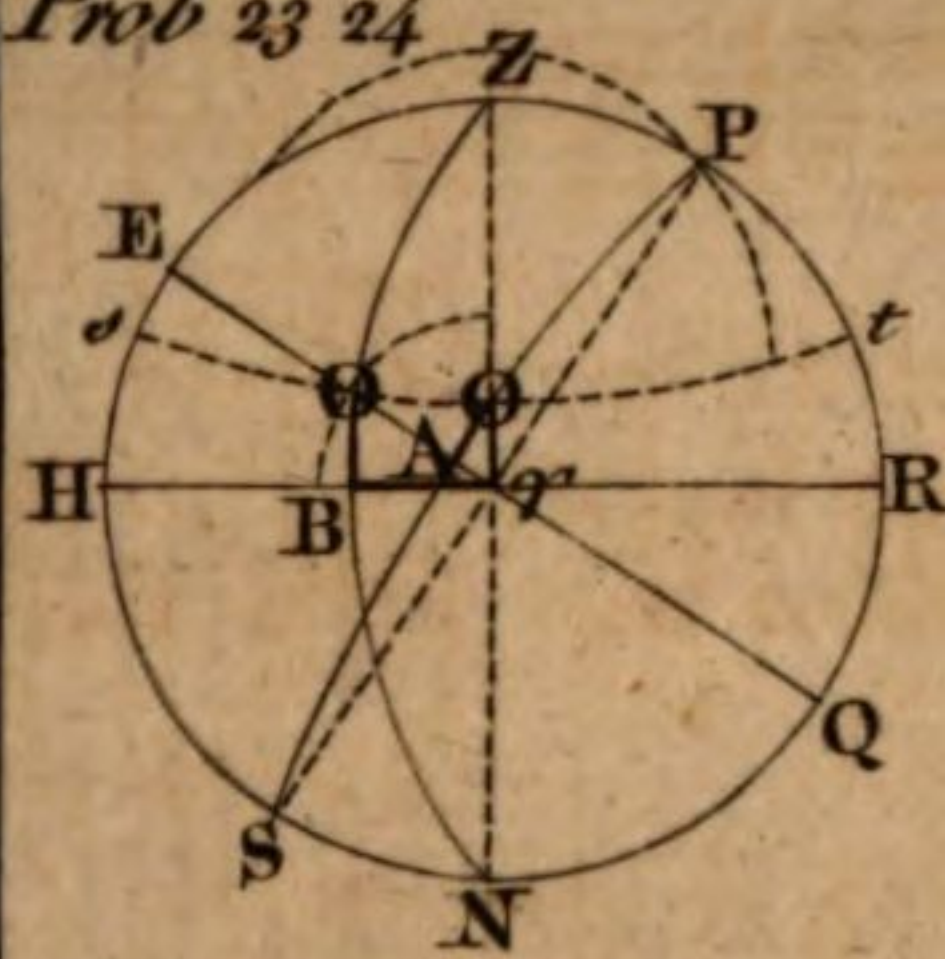
Prob: 20.



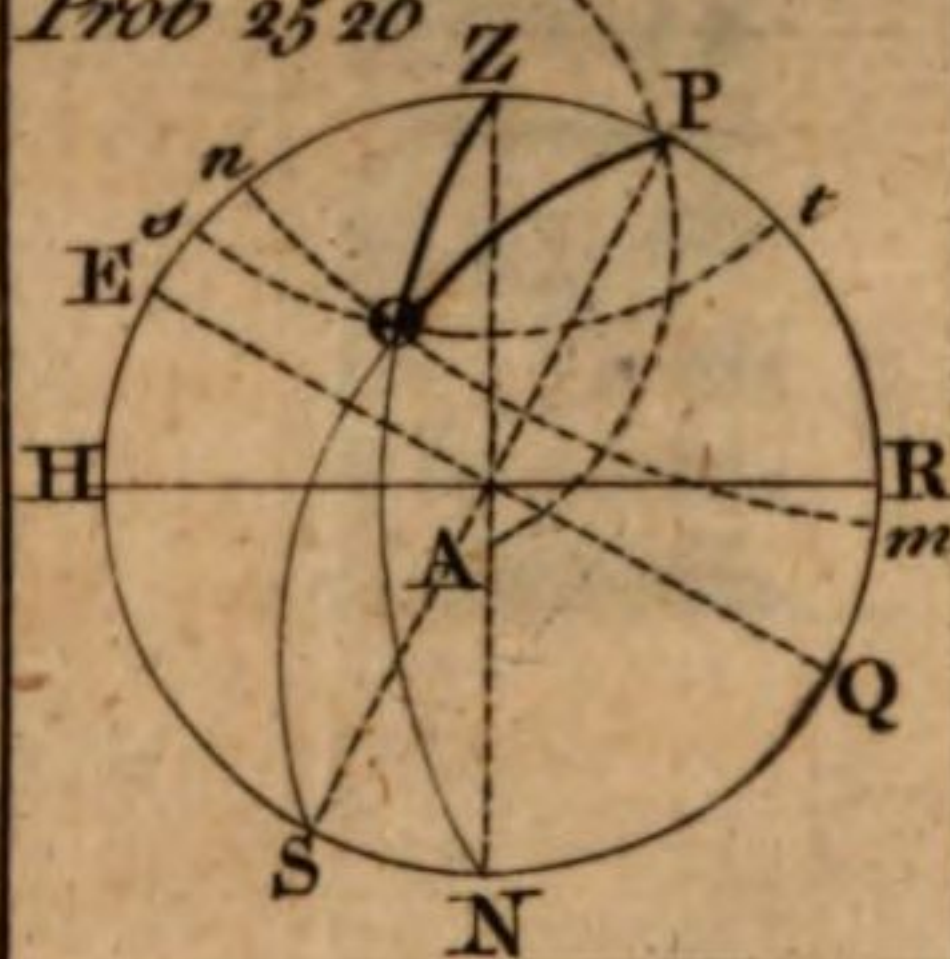
Prob 22



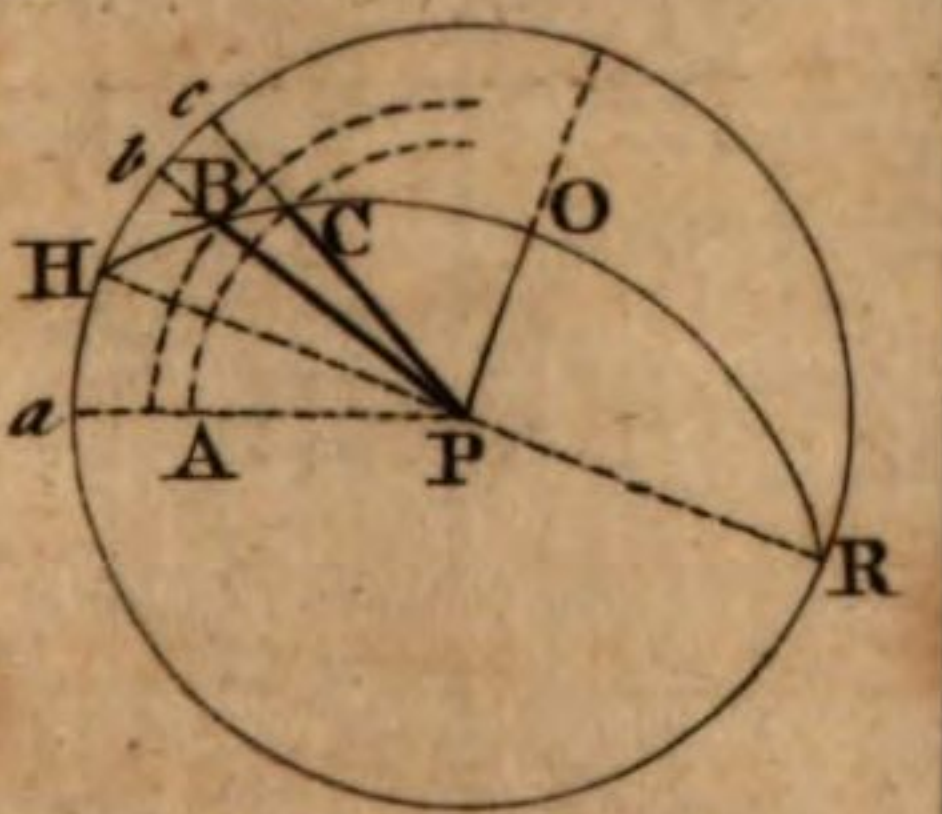
Prob 23 24



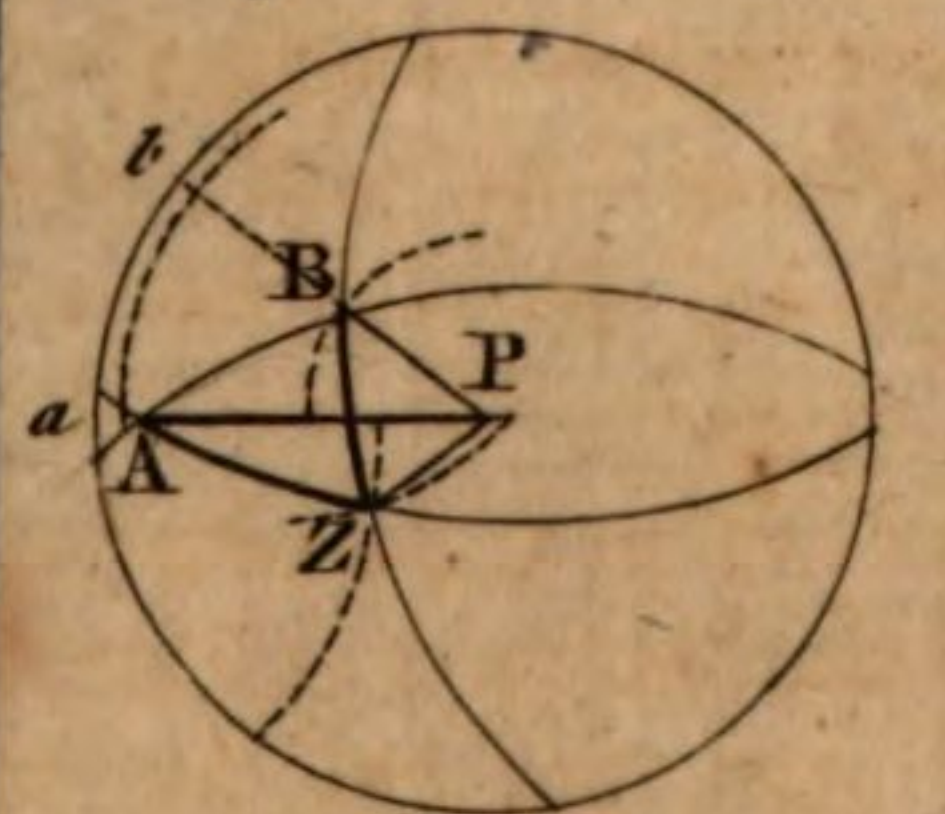
Prob 25 26



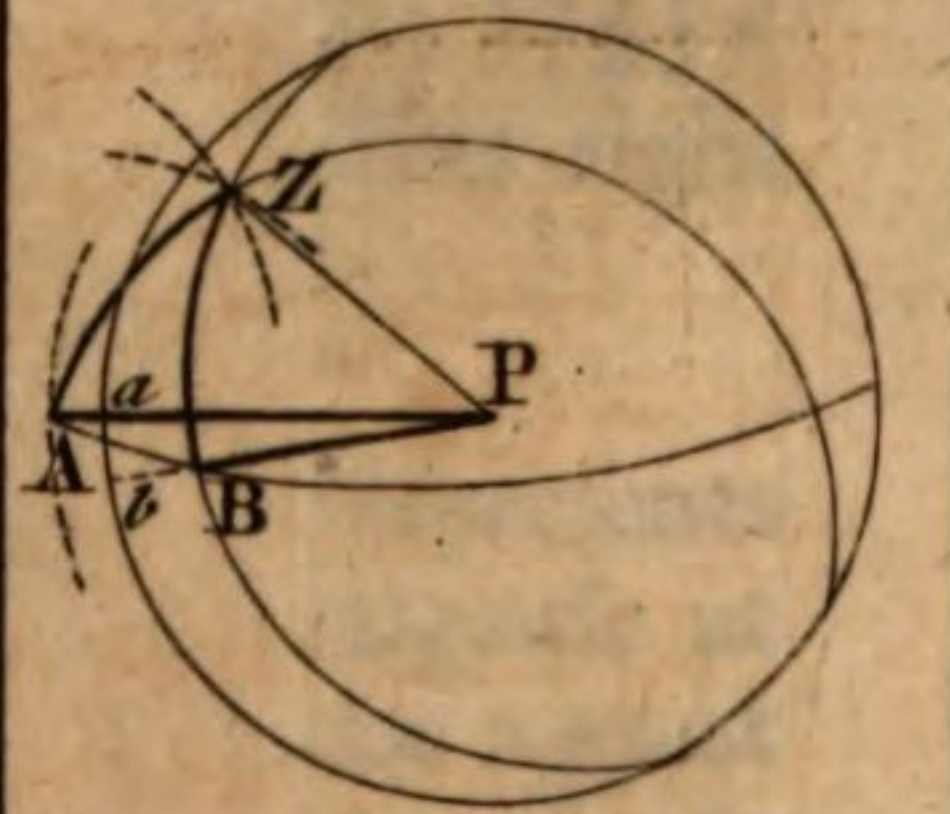
Prob 28



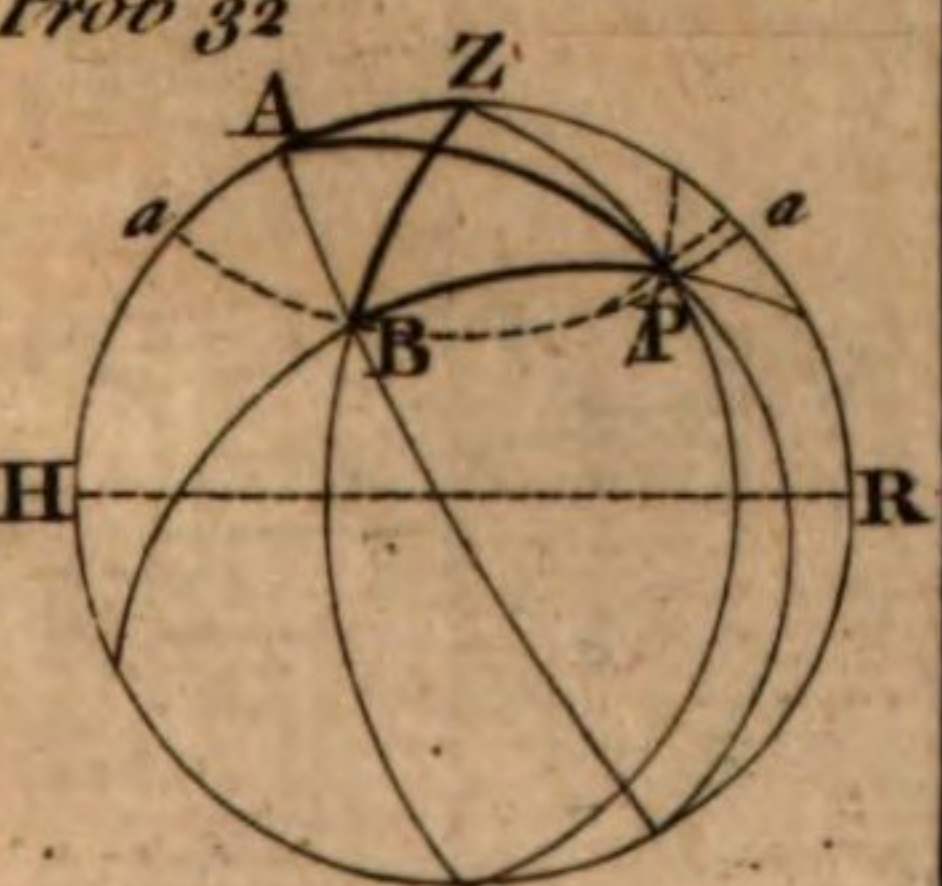
Prob 29



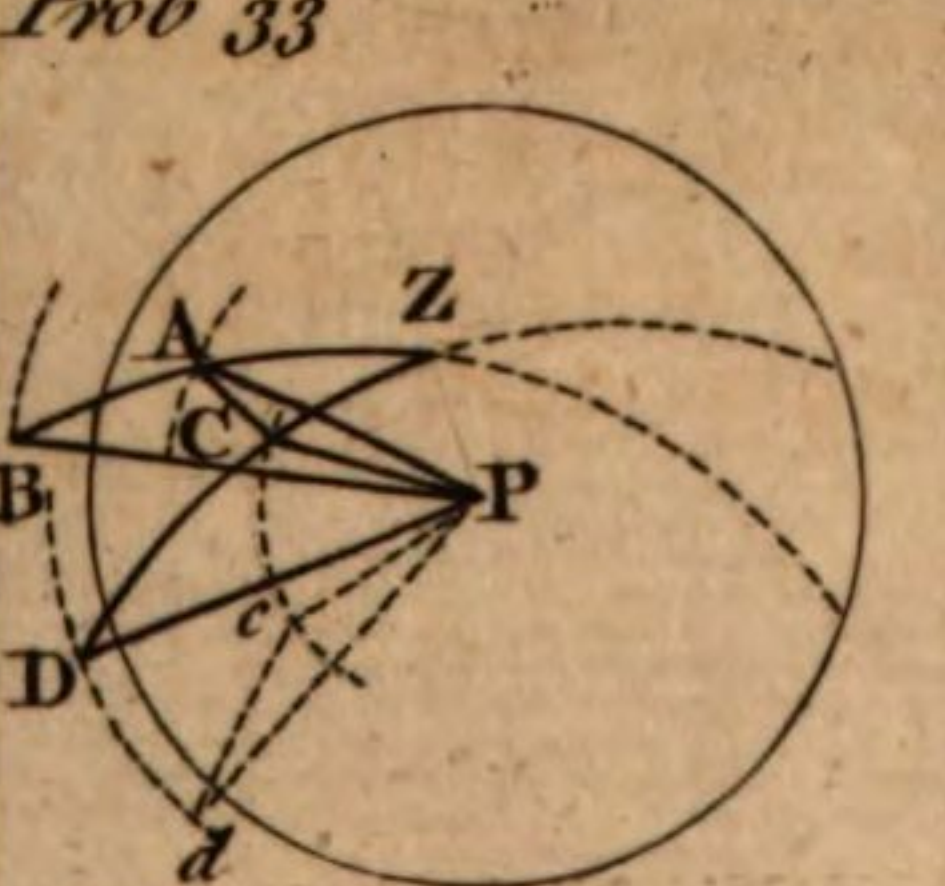
Prob 30



Prob 32



Prob 33



The Problems
belonging to this plate
fall between the
pages 461. 484.
The figures to the last 7
Astron^y Problems are in
Plate XI.

COMPUTATION. In the triangle

APC.

Given the *'s co-decl. $PA = 54^{\circ}00'$
comet's co-decl. $PC = 43^{\circ}00'$
their distance $AC = 49^{\circ}00'$

Req. com. rt. asc. fr. * $\angle APC = 65^{\circ}48'$

Then $65^{\circ}48' - 45^{\circ}00' = 20^{\circ}48'$.

And $90^{\circ}00' - 20^{\circ}48' = 69^{\circ}12' = CPE$.

In the triangle EPC.

Given comet's co-decl. $PC = 43^{\circ}00'$

obliq. of ecliptic $PE = 23^{\circ}29'$

comet's rt. asc. $\angle EPC = 69^{\circ}12'$

Req. comet's co-lat. $EC = 39^{\circ}53'$

and longit. $\angle PEC = 83^{\circ}47'$

Which being in the 4th quadrant,

gives $23^{\circ}47'$ in κ .

1024.

PROBLEM XXXVI.

At London, on the 10th of December, 1753, at what time of the night will the stars Aldebaran and Regal be on the same azimuth circle?

Aldeb. decl. $= 15^{\circ}59' N.$; right asc. $= 4 h. 22 m.$ Regal's decl. $= 8^{\circ}28' S.$; right asc. $= 5 h. 01 m.$

Their difference of right ascensions is 39 min., or $9^{\circ}45'$.

CONSTRUCTION. On the plane of the equinoctial put Aldebaran at A, and Regal at B, by their right ascensions and declinations, then a great circle thro' B and A will be the azimuth they are on at the time sought; and the parallel of London's lat. described about P, will cut the azimuth circle BA in Z the zenith, thro' which draw the meridians PZ, Pz.

Here the nearest intersection to the stars is taken for the zenith; for as the stars are both above the horizon, the greatest zenith distance is less than 90 degrees.

COMPUTATION. In the $\triangle APB$.

Given B's co-decl. $PB = 98^{\circ}28'$

A's co-decl. $PA = 74^{\circ}01'$

diff. rt. asc. $\angle APB = 9^{\circ}45'$

In the triangle APZ.

Given A's co-decl. $PA = 74^{\circ}01'$

the co-lat. $PZ = 38^{\circ}28'$

the supl. of BAP or $\angle PAZ = 22^{\circ}14'$

Required the $\angle BAP = 157^{\circ}46'$

Req. $\angle APZ = 144^{\circ}10'$ or $= 23^{\circ}00'$

Now (by 966) the star Aldebaran comes to the meridian at 11 h. 32 m. in the evening; which lessened by 1 h. 32 m. ($= 23^{\circ}00'$) gives 9 h. 40 m. for the time in the evening when those stars will be on the same azimuth.

1025.

PROBLEM XXXVII.

At what time in the evening will the stars Betelgeuse and Pollux have one common altitude above the horizon of London, on the 10th of December, 1753?

Betelgeuse right ascen. $= 5 h. 42 m.$; decl. $= 7^{\circ}20' N.$ Pollux right ascen. $= 7 h. 28 m.$; decl. $= 28^{\circ}36' N.$

Their difference of right ascension is 1 h. 46 m. $= 26^{\circ}30'$.

CONSTRUCTION. On a primitive circle, where any point P represents the pole of the equinoctial, put the stars Pollux at B, and Betelgeuse at A, by their decl. and diff. of right ascensions; thro' A, B, describe a great circle; thro' C, the middle of AB, describe a great circle at right angles to AB, and cutting PA in D; then a small circle described about P, at the distance of $38^{\circ}28'$, the co-lat., will cut the circle CD in Z the zenith.

For the stars A and B having a common altitude, are equally distant from Z.

COMPU-

COMPUTATION. In the triangle APB.

Given A's co-decl. $PA = 82^{\circ} 40'$ } Required the $\angle BAP = 46^{\circ} 18'$
 B's co-decl. $PB = 61\ 24$ } $AB = 32\ 49$
 diff. rt. asc. $\angle APB = 26\ 30$ } Its half $AC = 16\ 24$

In the triangle ACD.

Given the $\angle C = 90^{\circ} 00'$ } Required the $\angle D = 46^{\circ} 05'$
 $\angle A = 46\ 18$ } $AD = 23\ 04$
 $AC = 16\ 24$ } Theref. $PA - AD = PD = 59\ 36$

In the triangle PDZ.

Given the co-lat. $PZ = 38^{\circ} 28'$ } Required $\angle ZPD = 66^{\circ} 04'$
 $PD = 59\ 36$ } Or it is $= 58\ 30$
 $\angle PDZ = 46\ 05$

Now (by 966) the star comes to the meridian at 12 h. 32 m. in the morning; from which take 3 h. 54 m. ($= 58^{\circ} 30'$, as the stars are to the east of the meridian) and it leaves 8 h. 38 m. in the evening for the time when those stars have the same altitude.

1026. PROBLEM XXXVIII.

Wanting to know the latitude and longitude of a comet c, its distance from two known stars A and B were observed, and are as follows;

A's lat. $= 49^{\circ} 12' N.$ Lon. $= 16^{\circ} 39' \propto$; distance from c $= 45^{\circ} 05'$.
 B's lat. $= 30\ 05 N.$ Lon. $= 2\ 48 \Pi$; distance from c $= 45\ 57$.
 Hence the place of the comet c is required?

CONSTRUCTION. On the plane of the solstitial colure, where E is the pole of the ecliptic, put the stars A and B by their latitudes and longitudes, and describe a great circle thro' A and B; then small circles described about A and B as poles, at the respective distances of the comet, their intersection c will give its place; describe great circles thro' A, c; B, c; E, c; and EC will be the co-lat., and from $\angle AEC$ will be obtained the longitude.

COMPUTATION. In the $\triangle AEB$.

Given A's co-lat. $AE = 40^{\circ} 48'$
 B's co-lat. $BE = 59\ 55$
 diff. longit. $\angle AEB = 76\ 09$

Required $AB = 59\ 01$
 and $\angle EAB = 78\ 31$

In the triangle ABC.

Given the distance $AB = 59^{\circ} 01'$
 distance $AC = 49\ 05$
 distance $BC = 45\ 57$

Required the $\angle BAC = 56\ 27$
 Then $\angle EAB + \angle BAC = \angle EAC = 134^{\circ} 58'$

In the triangle CEA.

Given A's co-lat. $AE = 40^{\circ} 48'$ } Required the co-lat. $EC = 81^{\circ} 33'$
 distance $AC = 49\ 05$ } diff. lon. $AEC = 32\ 43$
 and the $\angle EAC = 134\ 58$ } Hence lat. is $8^{\circ} 27' N.$ lon. $19^{\circ} 22' in \propto$.

1027. PROBLEM XXXIX.

The distance of the star c being observed from two stars, A, B; whose latitude and distance are known; thence to find the lat. and long. of c.

A's lat. $= 5^{\circ} 30'$; dist. from c $= 39^{\circ} 40'$; B's lat. $= 9^{\circ} 57'$; dist. from c $= 10^{\circ} 7\frac{1}{2}'$; And the distance of AB is $44^{\circ} 43'$.

CON-

CONSTRUCTION. On a circle of longitude, where E is the pole of the ecliptic, put the star B by its lat. ; about the points B and E describe circles at the distances of A from those points, their intersection gives the place of A; also circles described from A and B, at the distances of C respectively from them, their intersection is the place of C: Then describe the great circles EA, EC; AC, AB; BC; and EC, will be the co-lat., and $\angle BEC$ the longitude from B.

COMPUTATION. In the $\triangle AEB$.		In the triangle ABC.	
Given A's co-lat.	$AE = 84^{\circ} 30'$	Given the distance	$AB = 44^{\circ} 43'$
B's co-lat.	$BE = 80^{\circ} 03'$	distance AC	$= 39^{\circ} 40'$
the distance	$AB = 44^{\circ} 43'$	distance BC	$= 10^{\circ} 07\frac{1}{2}'$
Required the $\angle ABE = 92^{\circ} 14'$		Required the $\angle ABC = 55^{\circ} 22'$	
In the triangle BEC.			
Given B's co-lat.	$BE = 80^{\circ} 03'$	Required C's co-lat.	$CE = 72^{\circ} 01'$
the distance	$BC = 10^{\circ} 07\frac{1}{2}'$	C's lon. fr. B, $\angle BEC$	$= 6^{\circ} 22'$
$(\angle ABE - \angle ABC =) \angle CBE = 36^{\circ} 52'$			

1028.

PROBLEM XL.

From the altitudes of two known fixed stars, and the altitude of a planet when in the same azimuth with one of these stars; to find the place of the planet.

EXAM. Observed the Moon and Cor. Leonis in the same azimuth, when the Moon's zenith distance was $36^{\circ} 37'$.

Cor. Leonis's zenith dist. $= 45^{\circ} 00'$; decl. $13^{\circ} 09' N.$; rt. as. $9^h 53^m$.

Cor. Hydra's zenith dist. $= 49^{\circ} 16'$; decl. $7^{\circ} 37' S.$; rt. as. $9^h 16^m$.

CONSTRUCTION. On the plane of the equinoctial, whose pole is P, draw the colures, and in the solstitial, take E for the pole of the ecliptic; put the given stars at B and A by their declinations and right ascensions: About B and A as poles, with their respective zenith distances, describe circles cutting in Z the zenith; thro' Z and B describe an azimuth circle, and making Z D equal to the D's zenith distance, gives her place: Then describe the great circles ZB, ZA, Z D, E D; and the arc E D will be the co-latitude, and $\angle PED$ the longitude from ϖ .

COMPUTATION. In the $\triangle ABP$.		In the triangle AZB.	
Given A's co-decl.	$PA = 97^{\circ} 37'$	Given A's zen. dist.	$ZA = 49^{\circ} 16'$
B's co-decl.	$PB = 76^{\circ} 51'$	B's zen. dist.	$ZB = 45^{\circ} 00'$
diff. rt. as. $\angle BPA$	$= 9^{\circ} 15'$	the side	$AB = 22^{\circ} 43'$
Required the $\angle ABP$	$= 155^{\circ} 38'$	Required the $\angle ABZ$	$= 89^{\circ} 56'$
the side	$AB = 22^{\circ} 43'$	Then $\angle ABP - \angle ABZ = \angle ZBP$	$= 65^{\circ} 42'$

In the triangle B D P.		In the triangle P D E.	
Given B's co-decl.	$BP = 76^{\circ} 51'$	Given obl. of eclip.	$PE = 23^{\circ} 29'$
$(BZ = ZD =) \text{side } BD$	$= 8^{\circ} 23'$	D's co-decl.	$PD = 73^{\circ} 32'$
$\angle PBD$	$= 65^{\circ} 42'$	D's co-rt. as. $\angle EPD$	$= 129^{\circ} 22'$
Requ. D's co-decl.	$PD = 73^{\circ} 32'$	Requir. D's co-lat.	$DE = 89^{\circ} 00'$
D's rt. as. fr. B, $\angle BPD$	$= 7^{\circ} 57'$	D's lon. fr. ϖ , $\angle PED$	$= 47^{\circ} 52'$
Then $90^{\circ} + \angle BPD + \angle BPD =$			
$\angle EPD = 129^{\circ} 22'$			

SECTION

SECTION VIII.

1029.

Of the Use of the Globes.

By the Globes here are meant two spherical bodies, whose convex surfaces are supposed to give a true representation of the earth and heavens, as visible by observation, and are called the Terrestrial and Celestial globes.

The TERRESTRIAL GLOBE has delineated on its convexity, the whole surface of the *earth* and *sea* in their relative size, form and situation.

The CELESTIAL GLOBE has drawn on its surface, the images of the several constellations and stars: The relative magnitude and position which the stars are observed to have in the heavens, being preserved on this globe.

To the globes are fitted certain appurtenances, whereby a great variety of useful problems are neatly solved.

The BRAZEN MERIDIAN is that ring or hoop in which the globe hangs on its axis; which is represented by two wires passing thro' its poles. This circle is divided into four quarters of 90° each; in one semicircle the divisions begin at each pole, and end at 90° , where they meet: In the other semicircle, the divisions begin at the middle, and proceed thence towards each pole, where they end at 90 degrees. The graduated side of this brazen circle serves as a meridian for any point on the surface of the earth, the globe being turned about till that point comes under the circle.

The HOUR CIRCLE is a small circle of brass which is divided into 24 hours, the quarters and half quarters: It is fixed on the brazen meridian, equally distant from the north end of the axis, to which an index is fitted, that points out the divisions of the hour circle as the globe is turned about.

The HORIZON is represented by the upper surface of the wooden circular frame encompassing the globe about its middle. On this wooden frame is a kind of perpetual calendar, contained in several concentric circles: The inner one is divided into 4 quarters of 90 degrees each; the next circle is divided into the 12 months, with the days in each, according to the new stile; the next contains the 12 equal signs of the zodiac, each being divided into 30 degrees; the next is the 12 months and days according to the old stile; and there is another circle, containing the 32 winds, with their halves and quarters. Altho' these circles are on all horizons, yet their disposition by the workmen are not always the same.

The QUADRANT OF ALTITUDE is a thin slip of brass, one edge of which is graduated into 90 degrees and their quarters, equal to those of the meridian: To one end of this is fixed a brass nut and screw, whereby it is put on and fastened to the meridian; which, if in the zenith or pole of the horizon, then the graduated edge represents a vertical circle passing thro' any point.

Beside these, there are several circles described on the surfaces of both globes, such as the equinoctial, ecliptic, circles of longitude and right ascension, the tropics, polar circles, parallels of lat. and decl., on the celestial globe; and on the terrestrial, the equator, ecliptic, tropics, polar circles, parallels of latitude, hour circles, or meridians to every 15 degrees, and the spiral rhumbs flowing from several centres, called Flies.

1030.

PROBLEM I.

To find the latitude and longitude of any place on the terrestrial globe.

- 1st. Bring the given place under that side of the graduated brazen meridian where the degrees begin at the equator, by turning the globe about.
- 2d. Then the degree of the meridian over it, shews the latitude.
- 3d. And the degree of the equator under the merid., shews the longit.

On some globes, the longitude is reckoned on the equator, from the meridian where it begins, only eastward until it ends at 360° : On such globes, when the longitude of a place exceeds 180° , take it from 360° , and call the remainder the longitude westward.

1031.

PROBLEM II.

To find any place on the globe whose latitude and longitude are given.

- 1st. Bring the given longitude, found on the equator, to the meridian.
- 2d. Then under the given latitude, found on the meridian, is the place sought.

1032.

PROBLEM III.

To find the distance and bearing of any two given places on the globe.

- 1st. Lay the graduated edge of the quadrant of altitude over both places, the beginning, or 0 deg. being on one of them, and the degrees between them shews their distance; these degrees multiplied by 60 gives sea miles, and by 70 gives the distance in land miles nearly.
- 2d. Observe while the quadrant lies in this position, what rhumb of the nearest fly runs mostly parallel to the edge of the quadrant, and that rhumb shews the bearing sought nearly.

1033.

PROBLEM IV.

To find the sun's place and declination on any day.

- 1st. Seek the given day in the circle of months on the horizon, and right against it, in the circle of signs, is the sun's place.

Thus it will be found that the sun enters

The spring signs, Aries, March 20. Taurus, April 20. Gemini, May 21.

The summer signs, Cancer, June 21. Leo July 23. Virgo, August 23.

Autumnal signs, Libra, Sept. 22. Scorpio, Oct. 23. Sagittar. Nov. 22.

The winter signs, Capric. Dec. 21. Aquarius, Jan. 20. Pisces, Feb. 18.

- 2d. Seek the sun's place in the ecliptic on the globe, bring that place to the meridian, and the division it stands under is the sun's declination on the given day.

On the globes, the ecliptic is readily distinguished from the equator, not only by the different colours wherewith they are stained, but also by the ecliptic's approaching towards the poles, after its intersection with the equator. The marks of the signs are also put along the ecliptic, one at the beginning of every successive 30 degrees.

1034.

PROBLEM V.

To rectify the globe for the latitude, zenith, and noon.

1st. Set the globe upon an horizontal plane, with its parts answering to those of the world; move the meridian in its notches, by raising or depressing the pole, until the degrees of latitude cut the horizon; then is the globe rectified for the latitude.

2d. Reckon the latitude from the equator towards the elevated pole, there screw the bevil edge of the nut belonging to the quadrant of altitude, and the rectification is made for the zenith.

3d. Bring the sun's place (found by the last problem) to the meridian, set the index to the XII at noon, or upper XII, and the globe is rectified for the sun's southing, or noon.

1035.

PROBLEM VI.

To find where the sun is vertical at any given time.

1st. Bring the sun's place, found for the given day (1033), to the meridian, note the degree over it, and set the index to the given hour.

2d. Turn the globe till the index comes to 12 at noon, then the place under the said noted degree has the sun in the zenith at that time.

3d. All the places that pass under that degree, by turning the globe round, will have the sun vertical to them on that day.

1036.

PROBLEM VII.

To find what days the sun will be vertical at any given place in the torrid zone.

1st. Note the latitude of the given place on the meridian.

2d. Turn the globe, and note what points of the ecliptic pass under the latitude noted on the meridian.

3d. Seek those points of the ecliptic in the circle of signs on the horizon, and right against them in the circle of months stand the days required.

In this manner it will be found, that the sun will be vertical to the Island of St. Helena on the 6th of November, and on the 4th of February. And at Barbadoes on the 24th of April, and the 18th of August.

1037.

PROBLEM VIII.

At any given hour in a given place, to find what hour it is in any other place?

1st. Bring the place where the time is given to the meridian, and set the index to the given hour.

2d. Bring the other given place to the meridian, and the index shews the hour corresponding to the given time.

1038.

1038.

PROBLEM IX.

At any given time, to find all those places of the earth where the sun is then rising or setting, where mid-day or midnight.

Find the place where the sun is vertical at the given time (1035); rectify the globe for the latitude of that place, and bring it to the meridian.

Then all those places, that are in the west half of the horizon, have the sun rising; and those in the eastern half have it setting.

Those under the meridian above the horizon have the sun culminating, or noon; and those under the meridian below the horizon have midnight:

Those above the horizon have day; those below it have night.

1039.

PROBLEM X.

To find the angle of position of two places; or, the angle made by the meridian of one place, and a great circle passing thro' both places.

Rectify for the latitude of one of the given places, and bring it to the meridian; there fix the quadrant of altitude, and set its graduated edge to the other place: Then will that edge of the quadrant cut the horizon in the degree of position sought.

Thus, the angle of position at the Land's end to Barbadoes is south $71\frac{1}{2}^{\circ}$ westerly: But the angle of position at Barbadoes to the Land's end is north $37\frac{1}{2}^{\circ}$ easterly.

Hence neither of these positions can be the true bearing; for the rhumb passing thro' both places, will be opposite one way to what it is the other.

1040.

PROBLEM XI.

The latitude of any place not within the polar circle being given, to find the time of sun-rising and setting, and the length of the day and night.

Rectify for the latitude and the noon; bring the sun's place to the eastern side of the horizon, and the index shews the time of rising: The sun's place brought to the western side of the horizon, the index gives the setting.

Or, The time of rising taken from 12 hours gives the time of setting.

The time of setting being doubled gives the length of the day.

And the time of rising being doubled gives the length of the night.

Thus, at London, on April 15th, the day is $13\frac{1}{2}$ hours; the night $10\frac{1}{2}$ hours.

1041.

PROBLEM XII.

To find the length of the longest and shortest days in any given place.

Rectify for the latitude; bring the solstitial point of that hemisphere to the eastern part of the horizon, set the index to 12 at noon; turn the globe till the solstitial point comes to the western side of the horizon, the hours past over by the index give the length of the longest day, or night; and its complement to 24 hours gives the length of the shortest night, or day.

1042.

PROBLEM XIII.

A place being given in either frigid or frozen zone, to find the time when the sun begins to appear at, or depart from, that place: Also how many successive days he is present to, or absent from, that place.

Rectify for the latitude, turn the globe, and observe what degrees in the first and second quadrants of the ecliptic are cut by the north point of the horizon, the latitude being supposed north.

Find those degrees in the circle of signs on the horizon, and their corresponding days of the month, and all the time between those days the sun will not set in that place.

Again. Observe what degree in the third and fourth quadrants of the ecliptic will be cut by the south point of the horizon, and the days answering: Then the sun will be quite absent from the given place during the intermediate days; that day in the 3d quadrant, shews when he begins to disappear; and that in the 4th quadrant, shews when he begins to shine in the place proposed.

Thus, at the North cape, in lat. 71° , the sun never sets from May 15 to July 28, which is 74 days; and never rises from November 16 to January 24, which is 69 days.

1043.

PROBLEM XIV.

To find the antæci, periæci, and antipodes of any place.

Bring the given place to the meridian, tell as many degrees of latitude on the contrary side of the equator, and it gives the place of the antæci; that is, of those who have opposite seasons of the year, but the same times of the day.

The given place being under the meridian, set the index to 12 at noon, turn the globe till the index points to 12 at night, and the point under the meridian in the given latitude is the place of the periæci; that is, of those who have the same seasons of the year, but opposite times of the day.

The globe remaining in this position, seek on the contrary side the equator for the degrees of latitude given, and the point under the meridian thus found will be the antipodes to the given place; that is, of such whose seasons of the year and times of the day are directly opposite to those of the given place.

1044.

PROBLEM XV.

To find the beginning and end of the twilight in any place.

Rectify the globe for the latitude, zenith, and noon. (1034)

Seek the point of the ecliptic opposite to the sun's place, turn the globe and quadrant of altitude, till the said opposite point of the ecliptic stands against 18 degrees on the quadrant of altitude; then will the index shew the beginning or end of the twilight: That is, the beginning in the morning, when those points meet in the western hemisphere; or the end in the evening, when the said points meet in the eastern hemisphere.

1045.

1045.

PROBLEM XVI.

The latitude of a place and day of the month being given; to find the sun's declination, meridian altitude, right ascension, amplitude, oblique ascension, ascensional difference; and thence the time of rising, setting, length of the day and night.

Rectify for the latitude and noon. Then

The degree of the meridian over the sun's place is the declination.

The meridian altitude is shewn by the degrees the sun is above the horizon; and is equal to the sum or diff. of the co-lat and declin.

The sun's right ascen. is that degree of the equator under the meridian.

Bring the sun's place to the eastern part of the horizon. Then

The amplitude is that degree of the horizon opposite the sun.

The oblique ascension is that degree of the equator cut by the horizon.

The ascen. diff. is the diff. between the right and oblique ascensions.

The ascen. diff. converted into time, will give the time the sun rises before or after the hour of six, according as his amplitude is to the northward or southward of the east point of the horizon.

1046.

PROBLEM XVII.

Given the latitude of the place and day of the month; to find the sun's altitude and azimuth, either when he is due east or west, at 6 o'clock, or at any other hour while he is above the horizon.

Rectify the globe for the latitude, zenith, and noon.

Set the quadrant of altitude to the east point of the horizon, turn the globe till the sun's place comes to the quadrant's edge, and it shews the altitude, his azimuth being now 90° , and the index shews the hour.

Turn the globe till the index points at 6, there stay it, and move the quadrant till its edge cuts the sun's place; then the degrees at the sun shews its altitude, and the degrees cut by the quadrant in the horizon shews the azimuth, reckoning from the north.

In like manner, the globe being turned till the index is against any other hour, suppose 10 in the forenoon. Then

The graduated edge of the quadrant of altitude being turned to cut the sun's place, will give both the altitude and azimuth at that time.

1047.

PROBLEM XVIII.

Given the latitude, day of the month, and sun's altitude; to find the azimuth and hour of the day.

Rectify the globe for the latitude, zenith, and noon.

Turn the globe and quadrant until the sun's place coincide with the graduated edge of the quadrant.

Then will that edge of the quadrant cut in the horizon the degrees of azimuth, reckoned from the north; and the index will shew the hour of the day.

1048.

1048.

P R O B L E M XIX.

To represent the appearance of the heavens at any time in a given place.

Rectify the celestial globe for the latitude, zenith, and noon, and turn the globe till the index points at the given hour. Then

Those stars in the eastern half of the horizon are rising; those in the western are setting; and those in the meridian are culminating.

The quadrant being set to any given star will shew its altitude, and at the same time its azimuth, reckoned in the horizon.

Now by turning the globe round, it will readily appear what stars never set in that place, and those which never rise; those of perpetual apparition never go below the horizon, and those of perpetual absence never come above it.

1049.

P R O B L E M XX.

To find the latitude and longitude of any star.

Put the centre of the quadrant of altitude on the pole of the ecliptic, and its graduated edge on the given star. Then

The latitude is shewn by the degrees between the ecliptic and star.

The longitude is the degrees cut on the ecliptic by the quadrant.

1050.

P R O B L E M XXI.

To find the declination and right ascension of a star.

Bring the star to the meridian, the degree over it is the declination; and the degree of the equator under the meridian is the right ascension.

1051.

P R O B L E M XXII.

On any day, and in any given place; to find when a proposed star rises, sets, or culminates.

Rectify the globe for the latitude and noon.

Bring the star to the eastern side of the horizon, and the index shews the time of its rising.

Turn the globe till the star comes to the meridian, and the index shews the time of its culminating; and in like manner when it sets, the time will be shewn by the index.

Its meridian altitude, oblique ascension, and ascensional difference, are found in the same manner as for the sun at art. 1045.

Articles 1052, 1053, 1054, 1055, 1056.

ASTRONOMICAL TABLES
OF THE
Declinations and Right Ascensions
OF THE
Sun and Fixed Stars :

Calculated for the year 1753,
And serving for the next twelve years.

A L S O,

A TABLE to reduce the Sun's declination from the
meridian of London to that of any other place.

And a useful TABLE of Refractions.

1052. A TABLE of the Sun's Declination

For the years 1753, 1757, 1761; each being the first after Leap-year.

Days	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Days
	Decl. S.	Decl. S.	Decl. S.	Decl. N.	Decl. N.	Decl. N.	Decl. N.	Decl. N.	Decl. N.	Decl. S.	Decl. S.	Decl. S.	
	° /	° /	° /	° /	° /	° /	° /	° /	° /	° /	° /	° /	
1	22 59	16 58	7 24	4 43	15 12	22 07	23 07	17 58	8 10	3 20	14 36	21 55	1
2	22 53	16 41	7 01	5 06	15 30	22 15	23 03	17 43	7 49	3 44	14 55	22 04	2
3	22 48	16 23	6 38	5 29	15 48	22 23	22 58	17 27	7 27	4 07	15 13	22 12	3
4	22 41	16 05	6 15	5 52	16 05	22 30	22 53	17 11	7 04	4 30	15 32	22 20	4
5	22 35	15 46	5 52	6 14	16 22	22 36	22 48	16 55	6 42	4 53	15 50	22 28	5
6	22 27	15 28	5 29	6 37	16 39	22 43	22 42	16 39	6 20	5 16	16 08	22 35	6
7	22 20	15 09	5 05	6 59	16 56	22 49	22 35	16 22	5 57	5 39	16 26	22 42	7
8	22 12	14 50	4 42	7 22	17 12	22 54	22 28	16 05	5 34	6 02	16 44	22 48	8
9	22 03	14 31	4 19	7 44	17 28	22 59	22 21	15 48	5 12	6 25	17 01	22 54	9
10	21 54	14 11	3 55	8 06	17 44	23 04	22 14	15 30	4 49	6 48	17 18	22 59	10
11	21 45	13 52	3 32	8 28	17 59	23 08	22 06	15 12	4 26	7 11	17 34	23 04	11
12	21 35	13 32	3 08	8 50	18 14	23 12	21 58	14 54	4 03	7 34	17 51	23 09	12
13	21 24	13 11	2 44	9 12	18 29	23 15	21 49	14 36	3 40	7 56	18 07	23 13	13
14	21 14	12 51	2 21	9 34	18 44	23 19	21 40	14 17	3 17	8 18	18 23	23 16	14
15	21 03	12 30	1 57	9 55	18 58	23 21	21 31	13 59	2 54	8 41	18 38	23 20	15
16	20 51	12 10	1 33	10 16	19 12	23 23	21 21	13 40	2 31	9 03	18 53	23 22	16
17	20 39	11 49	1 10	10 37	19 25	23 25	21 11	13 21	2 07	9 25	19 08	23 24	17
18	20 27	11 27	0 46	10 58	19 39	23 27	21 00	13 01	1 44	9 47	19 22	23 26	18
19	20 14	11 06	0 22	11 19	19 52	23 28	20 50	12 42	1 21	10 09	19 36	23 27	19
20	20 01	10 45	N. 01	11 40	20 04	23 29	20 38	12 22	0 57	10 30	19 50	23 28	20
21	19 48	10 23	0 25	12 00	20 16	23 29	20 27	12 02	0 34	10 52	20 03	23 29	21
22	19 34	10 01	0 49	12 20	20 28	23 28	20 15	11 42	0 11	11 13	20 16	23 28	22
23	19 20	9 39	1 12	12 40	20 40	23 27	20 03	11 21	S. 13	11 34	20 28	23 27	23
24	19 06	9 17	1 36	13 00	20 51	23 26	19 50	11 01	0 36	11 55	20 41	23 26	24
25	18 51	8 55	1 59	13 20	21 02	23 25	19 37	10 40	1 00	12 16	20 52	23 25	25
26	18 36	8 32	2 23	13 39	21 12	23 23	19 24	10 19	1 23	12 36	21 04	23 23	26
27	18 20	8 10	2 46	13 58	21 22	23 21	19 10	9 58	1 47	12 57	21 15	23 20	27
28	18 04	7 47	3 10	14 17	21 32	23 18	18 57	9 37	2 10	13 17	21 25	23 17	28
29	17 48		3 33	14 36	21 42	23 15	18 43	9 15	2 34	13 37	21 36	23 14	29
30	17 32		3 56	14 54	21 5	23 11	18 28	8 54	2 57	13 57	21 45	23 10	30
31	17 15		4 20		21 59		18 13	8 32		14 17		23 05	31
Days	S. Decl.	S. Decl.	N. Decl.	N. Decl.	N. Decl.	N. Decl.	N. Decl.	N. Decl.	S. Decl.	S. Decl.	S. Decl.	S. Decl.	Days
	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	

1052. A TABLE of the Sun's Declination

For the years 1754, 1756, 1762; each being the second after Leap-year.

Days	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Days
	Decl.	Decl.	Decl.	Decl.	Decl.	Decl.	Decl.	Decl.	Decl.	Decl.	Decl.	Decl.	
	S.	S.	S.	N.	N.	N.	N.	N.	N.	S.	S.	S.	
	o /	o /	o /	o /	o /	o /	o /	o /	o /	o /	o /	o /	
1	23 00	17 02	7 30	4 37	15 08	22 05	23 08	18 02	8 16	3 15	14 31	21 53	1
2	22 55	16 45	7 07	5 00	15 26	22 13	23 04	17 47	7 54	3 38	14 50	22 02	2
3	22 49	16 27	6 44	5 23	15 43	22 21	22 59	17 31	7 32	4 01	15 09	22 10	3
4	22 43	16 09	6 21	5 46	16 01	22 28	22 54	17 15	7 10	4 24	15 28	22 18	4
5	22 36	15 51	5 58	6 09	16 18	22 35	22 49	16 59	6 47	4 48	15 46	22 26	5
6	22 29	15 32	5 34	6 31	16 35	22 41	22 43	16 43	6 25	5 11	16 04	22 33	6
7	22 22	15 14	5 11	6 54	16 52	22 47	22 37	16 26	6 03	5 34	16 22	22 40	7
8	22 14	14 55	4 48	7 16	17 08	22 53	22 30	16 09	5 40	5 57	16 40	22 47	8
9	22 05	14 36	4 24	7 39	17 24	22 58	22 23	15 52	5 17	6 20	16 57	22 53	9
10	21 56	14 16	4 01	8 01	17 40	23 01	22 16	15 34	4 55	6 43	17 14	22 58	10
11	21 47	13 56	3 37	8 23	17 56	23 07	22 08	15 17	4 32	7 05	17 31	23 03	11
12	21 37	13 36	3 14	8 45	18 11	23 11	22 00	14 59	4 09	7 28	17 47	23 08	12
13	21 27	13 16	2 50	9 07	18 26	23 15	21 51	14 40	3 46	7 51	18 03	23 12	13
14	21 16	12 56	2 27	9 28	18 40	23 18	21 42	14 22	3 23	8 13	18 19	23 16	14
15	21 05	12 35	2 03	9 50	18 55	23 21	21 33	14 03	2 59	8 35	18 34	23 19	15
16	20 54	12 15	1 39	10 11	19 09	23 23	21 23	13 44	2 36	8 58	18 49	23 22	16
17	20 42	11 54	1 15	10 32	19 22	23 25	21 13	13 25	2 13	9 20	19 04	23 24	17
18	20 30	11 33	0 52	10 53	19 36	23 26	21 03	13 06	1 50	9 42	19 19	23 26	18
19	20 18	11 11	0 28	11 14	19 48	23 27	20 52	12 46	1 26	10 03	19 33	23 27	19
20	20 05	10 50	0 04	11 35	20 01	23 28	20 41	12 27	1 03	10 25	19 46	23 28	20
21	19 51	10 28	N. 19	11 55	20 13	23 29	20 30	12 07	0 40	10 47	20 00	23 29	21
22	19 38	10 06	0 43	12 15	20 25	23 29	20 18	11 47	0 16	11 08	20 13	23 29	22
23	19 24	9 44	1 07	12 35	20 37	23 28	20 06	11 26	S. 07	11 20	20 25	23 28	23
24	19 09	9 22	1 30	12 55	20 48	23 27	19 53	11 06	0 31	11 50	20 38	23 27	24
25	18 54	9 00	1 54	13 15	20 59	23 25	19 40	10 45	0 54	12 11	20 50	23 25	25
26	18 39	8 37	2 17	13 34	21 10	23 24	19 27	10 24	1 18	12 32	21 01	23 23	26
27	18 24	8 15	2 41	13 53	21 20	23 21	19 14	10 03	1 41	12 52	21 12	23 21	27
28	18 08	7 52	3 04	14 12	21 30	23 19	19 00	9 42	2 04	13 12	21 23	23 18	28
29	17 52		3 27	14 31	21 39	23 16	18 46	9 21	2 28	13 32	21 33	23 15	29
30	17 36		3 51	14 50	21 48	23 12	18 32	8 59	2 51	13 52	21 43	23 11	30
31	17 19		4 14		21 57		18 17	8 38		14 12		23 06	31
Days	S.	S.	N.	N.	N.	N.	N.	N.	S.	S.	S.	S.	Days
	Decl.	Decl.	Decl.	Decl.	Decl.	Decl.	Decl.	Decl.	Decl.	Decl.	Decl.	Decl.	
	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	

1052. A TABLE of the Sun's Declination

For the years 1755, 1759, 1763; each being the third after Leap-year.

Days	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Days
	Decl. S.	Decl. S.	Decl. S.	Decl. N.	Decl. N.	Decl. N.	Decl. N.	Decl. N.	Decl. N.	Decl. S.	Decl. S.	Decl. S.	
	° /	° /	° /	° /	° /	° /	° /	° /	° /	° /	° /	° /	
1	23 02	17 07	7 36	4 31	15 03	22 03	23 09	18 06	8 22	3 08	14 26	21 50	1
2	22 56	16 49	7 15	4 54	15 21	22 11	23 05	17 51	8 00	3 32	14 45	21 59	2
3	22 51	16 31	6 50	5 17	15 39	22 19	23 01	17 35	7 38	3 55	15 04	22 08	3
4	22 45	16 14	6 27	5 40	15 56	22 26	22 56	17 20	7 16	4 18	15 23	22 16	4
5	22 38	15 56	6 04	6 03	16 13	22 33	22 5	17 04	6 54	4 41	15 41	22 24	5
6	22 31	15 37	5 41	6 25	16 30	22 39	22 45	16 47	6 31	5 05	15 59	22 31	6
7	22 24	15 19	5 17	6 48	16 47	22 46	22 39	16 31	6 09	5 28	16 17	22 38	7
8	22 16	15 00	4 54	7 10	17 04	22 51	22 32	16 14	5 46	5 51	16 35	22 45	8
9	22 07	14 41	4 31	7 33	17 20	22 57	22 25	15 56	5 24	6 14	16 52	22 51	9
10	21 59	14 21	4 07	7 55	17 36	23 02	22 18	15 39	5 01	6 36	17 09	22 57	10
11	21 49	14 02	3 44	8 17	17 51	23 06	22 10	15 21	4 38	6 59	17 26	23 02	11
12	21 40	13 42	3 20	8 39	18 07	23 10	22 02	15 03	4 15	7 21	17 42	23 07	12
13	21 30	13 21	2 57	9 01	18 22	23 14	21 54	14 45	3 52	7 44	17 59	23 11	13
14	21 19	13 02	2 33	9 23	18 36	23 17	21 45	14 27	3 29	8 07	18 14	23 15	14
15	21 08	12 41	2 09	9 44	18 51	23 20	21 36	14 08	3 06	8 29	18 30	23 18	15
16	20 57	12 20	1 46	10 05	19 05	23 22	21 26	13 50	2 43	8 52	18 4	23 21	16
17	20 46	11 59	1 22	10 27	19 18	23 24	21 16	13 30	2 19	9 14	19 00	23 22	17
18	20 33	11 38	0 58	10 48	19 32	23 26	21 06	13 11	1 56	9 36	19 1	23 25	18
19	20 21	11 17	0 35	11 08	19 45	23 27	20 55	12 52	1 33	9 57	19 29	23 27	19
20	20 08	10 56	0 11	11 29	19 58	23 28	20 44	12 32	1 09	10 19	19 43	23 28	20
21	19 55	10 34	N. 13	11 50	20 10	23 29	20 33	12 12	0 46	10 41	19 56	23 29	21
22	19 42	10 12	0 30	12 10	20 22	23 29	20 21	11 52	0 23	11 02	20 09	23 29	22
23	19 27	9 50	1 00	12 30	20 34	23 28	20 09	11 32	S. 01	11 23	20 22	23 28	23
24	19 13	9 28	1 24	12 50	20 45	23 27	19 57	11 11	0 24	11 44	20 34	23 27	24
25	18 59	9 06	1 47	13 10	20 56	23 26	19 44	10 51	0 48	12 05	20 46	23 26	25
26	18 43	8 44	2 11	13 29	21 07	23 24	19 31	10 30	1 11	12 26	20 58	23 24	26
27	18 28	8 21	2 34	13 48	21 17	23 22	19 18	10 09	1 35	12 46	21 09	23 21	27
28	18 12	7 59	2 58	14 07	21 27	23 19	19 04	9 48	1 58	13 07	21 19	23 19	28
29	17 56		3 21	14 26	21 37	23 17	18 50	9 26	2 21	13 27	21 30	23 15	29
30	17 40		3 44	14 44	21 46	23 13	18 36	9 0	2 45	13 47	21 40	23 12	30
31	17 23		4 08		21 55		18 21	8 43		14 06		23 08	31
Days	° /	° /	° /	° /	° /	° /	° /	° /	° /	° /	° /	° /	Days
	S. Decl.	S. Decl.	N. Decl.	N. Decl.	N. Decl.	N. Decl.	N. Decl.	N. Decl.	S. Decl.	S. Decl.	S. Decl.	S. Decl.	
	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	

1052. A TABLE of the Sun's Declination

For the years 1756, 1760, 1764; each being Leap-year.

Days	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Days
	Decl. S.	Decl. S.	Decl. S.	Decl. N.	Decl. N.	Decl. N.	Decl. N.	Decl. N.	Decl. N.	Decl. S.	Decl. S.	Decl. S.	
	° /	° /	° /	° /	° /	° /	° /	° /	° /	° /	° /	° /	
1	23 03	17 11	7 19	4 48	15 17	22 09	23 06	17 55	8 05	3 26	14 40	21 57	1
2	22 58	16 53	6 56	5 11	15 34	22 17	23 02	17 39	7 43	3 49	14 50	22 06	2
3	22 52	16 36	6 33	5 34	15 52	22 24	22 57	17 23	7 21	4 13	15 18	22 14	3
4	22 46	16 18	6 10	5 57	16 09	22 31	22 52	17 07	6 59	4 36	15 37	22 22	4
5	22 40	16 00	5 46	6 20	16 26	22 38	22 46	16 51	6 37	4 59	15 55	22 30	5
6	22 33	15 42	5 23	6 42	16 43	22 44	22 40	16 35	6 14	5 22	16 13	22 37	6
7	22 25	15 23	5 00	7 05	17 00	22 50	22 34	16 18	5 52	5 45	16 30	22 43	7
8	22 18	15 05	4 36	7 27	17 16	22 55	22 27	16 00	5 29	6 08	16 48	22 50	8
9	22 10	14 45	4 13	7 50	17 32	23 00	22 20	15 43	5 06	6 31	17 05	22 55	9
10	22 01	14 26	3 49	8 12	17 48	23 05	22 12	15 26	4 43	6 54	17 26	23 01	10
11	21 52	14 07	3 26	8 34	18 03	23 09	22 04	15 08	4 21	7 16	17 39	23 05	11
12	21 42	13 47	3 02	8 56	18 18	23 12	21 56	14 50	3 58	7 39	17 55	23 10	12
13	21 32	13 27	2 39	9 17	18 33	23 16	21 47	14 31	3 35	8 01	18 11	23 14	13
14	21 22	13 06	2 15	9 39	18 47	23 19	21 38	14 13	3 11	8 24	18 26	23 17	14
15	21 11	12 46	1 51	10 00	19 01	23 22	21 28	13 54	2 48	8 46	18 41	23 20	15
16	20 50	12 25	1 28	10 22	19 15	23 24	21 19	13 35	2 25	9 08	18 57	23 23	16
17	20 48	12 05	1 04	10 43	19 29	23 26	21 08	13 16	2 02	9 30	19 11	23 25	17
18	20 36	11 43	0 40	11 03	19 42	23 27	20 58	12 56	1 38	9 52	19 25	23 26	18
19	20 24	11 22	0 17	11 24	19 55	23 28	20 47	12 37	1 15	10 14	19 39	23 28	19
20	20 11	11 01	N. 07	11 45	20 07	23 29	20 36	12 17	0 52	10 36	19 53	23 29	20
21	19 58	10 39	0 31	12 05	20 19	23 29	20 24	11 57	0 28	10 57	20 06	23 29	21
22	19 45	10 18	0 54	12 25	20 31	23 28	20 12	11 37	0 05	11 18	20 19	23 28	22
23	19 31	9 56	1 18	12 45	20 43	23 27	20 00	11 16	S. 19	11 39	20 31	23 27	23
24	19 17	9 34	1 42	13 05	20 54	23 26	19 47	10 56	0 42	12 00	20 43	23 26	24
25	19 02	9 11	2 05	13 24	21 04	23 24	19 34	10 35	1 05	12 21	20 55	23 24	25
26	18 47	8 49	2 29	13 44	21 15	23 22	19 21	10 14	1 29	12 41	21 06	23 22	26
27	18 32	8 27	2 52	14 03	21 25	23 20	19 07	9 53	1 52	13 02	21 17	23 19	27
28	18 16	8 04	3 15	14 21	21 35	23 17	18 53	9 32	2 16	13 22	21 28	23 16	28
29	18 00	7 41	3 39	14 40	21 44	23 14	18 39	9 10	2 39	13 42	21 38	23 13	29
30	17 44		4 02	14 58	21 53	23 10	18 25	8 49	3 03	14 02	21 48	23 09	30
31	17 28		4 25		22 01		18 10	8 27		14 21		23 04	31
Days	Decl. S.	Decl. S.	Decl. N.	Decl. N.	Decl. N.	Decl. N.	Decl. N.	Decl. N.	Decl. S.	Decl. S.	Decl. S.	Decl. S.	Days
	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	

1053. A TABLE of the Sun's Right Ascension.

Days	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Days
	Right Asc. H.M.	Right Asc. H.M.	Right Asc. H.M.	Right Asc. H.M.	Right Asc. H.M.	Right Asc. H.M.	Right Asc. H.M.	Right Asc. H.M.	Right Asc. H.M.	Right Asc. H.M.	Right Asc. H.M.	Right Asc. H.M.	
1	18 48	21 01	22 49	0 43	2 34	4 37	6 41	8 46	10 42	12 30	14 26	16 30	1
2	18 53	21 05	22 53	0 47	2 38	4 41	6 45	8 50	10 45	12 34	14 30	16 35	2
3	18 57	21 09	22 57	0 50	2 42	4 45	6 49	8 53	10 49	12 37	14 34	16 39	3
4	19 02	21 13	23 01	0 54	2 46	4 49	6 53	8 57	10 53	12 41	14 38	16 43	4
5	19 06	21 17	23 04	0 57	2 49	4 53	6 57	9 01	10 56	12 45	14 42	16 48	5
6	19 10	21 21	23 08	1 01	2 53	4 57	7 02	9 05	11 00	12 48	14 46	16 52	6
7	19 15	21 25	23 12	1 05	2 57	5 01	7 06	9 09	11 04	12 52	15 50	16 57	7
8	19 19	21 29	23 15	1 08	3 01	5 05	7 10	9 13	11 07	12 56	14 54	17 01	8
9	19 23	21 33	23 19	1 12	3 05	5 10	7 14	9 17	11 11	12 59	14 58	17 05	9
10	19 28	21 37	23 23	1 16	3 09	5 14	7 18	9 20	11 14	13 03	15 02	17 10	10
11	19 32	21 41	23 26	1 19	3 13	5 18	7 22	9 24	11 18	13 07	15 06	17 14	11
12	19 37	21 44	23 30	1 23	3 17	5 22	7 26	9 28	11 22	13 10	15 10	17 19	12
13	19 41	21 48	23 34	1 27	3 20	5 26	7 30	9 32	11 25	13 14	15 15	17 23	13
14	19 45	21 52	23 38	1 30	3 24	5 30	7 34	9 35	11 29	13 18	15 19	17 27	14
15	19 49	21 56	23 41	1 34	3 28	5 34	7 38	9 39	11 32	13 21	15 23	17 32	15
16	19 54	22 00	23 45	1 38	3 32	5 39	7 42	9 43	11 35	13 25	15 27	17 36	16
17	19 58	22 04	23 48	1 41	3 36	5 43	7 46	9 47	11 40	13 29	15 31	17 41	17
18	20 00	22 08	23 52	1 45	3 40	5 47	7 50	9 50	11 43	13 33	15 35	17 45	18
19	20 07	22 12	23 56	1 49	3 44	5 51	7 54	9 54	11 47	13 36	15 39	17 50	19
20	20 11	22 15	23 59	1 53	3 48	5 55	7 58	9 58	11 50	13 40	15 44	17 54	20
21	20 15	22 19	0 03	1 56	3 52	5 59	8 02	10 02	11 54	13 44	15 48	17 58	21
22	20 19	22 23	0 07	2 00	3 56	6 04	8 06	10 05	11 58	13 48	15 52	18 03	22
23	20 23	22 27	0 10	2 04	4 00	6 08	8 10	10 09	12 01	13 52	15 56	18 07	23
24	20 28	22 31	0 14	2 08	4 04	6 12	8 14	10 13	12 05	13 55	16 00	18 12	24
25	20 32	22 34	0 18	2 11	4 08	6 16	8 18	10 16	12 08	13 59	16 05	18 16	25
26	20 36	22 38	0 21	2 15	4 12	6 20	8 22	10 20	12 12	14 03	16 09	18 21	26
27	20 40	22 42	0 25	2 19	4 16	6 24	8 26	10 24	12 16	14 07	16 13	18 25	27
28	20 44	22 46	0 28	2 23	4 20	6 29	8 30	10 27	12 19	14 11	16 17	18 30	28
29	20 48		0 32	2 26	4 24	6 33	8 34	10 31	12 23	14 15	16 22	18 34	29
30	20 52		0 36	2 30	4 29	6 37	8 38	10 35	12 26	14 19	16 26	18 38	30
31	20 56		0 39		4 33		8 42	10 38		14 22		18 43	31
Days	H.M. Right Asc.	H.M. Right Asc.	H.M. Right Asc.	H.M. Right Asc.	H.M. Right Asc.	H.M. Right Asc.	H.M. Right Asc.	H.M. Right Asc.	H.M. Right Asc.	H.M. Right Asc.	H.M. Right Asc.	H.M. Right Asc.	Days
	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	

1054. A TABLE of the Right Ascension and Declination of some of the principal Fixed Stars; as in the year 1752.

	Constellation	Place in the Constellat.	Stars names	Mark	Magnit.	Rt. Asc.		Declinat.	
						H.	M.	°	'
CONSTELLATIONS IN THE ZODIAC.	ARIES	Bright star of Aries		α	3	1	53	22	17N.
	TAURUS	The Bull's eye	ALDEBARAN	α	1	4	22	15	59N.
	GEMINI	{ Southern twin	POLLUX	β	2	7	28	28	36N.
		{ Northern twin	CASTOR	α	1	7	17	32	24N.
	CANCER	In Southern claw	ACUBENS	α	4	8	45	13	15N.
	LEO	{ Lion's heart	REGULUS	α	1	9	53	13	09N.
		{ In the Lion's tail	DENEK	β	2	11	35	15	55N.
	VIRGO	Virgin's spike		α	1	13	12	9	52 S.
	SCORPIO	Scorpion's heart	ANTARES	α	1	16	14	25	51 S.
	SAGITTARIUS	In the southern part of the bow		ϵ	2	18	06	34	28 S.
NORTHERN CONSTELLATIONS	ANDROMEDA	{ In the girdle	MIRACH	β	2	0	56	34	18N.
		{ In the foot	ALMAACH	γ	2	1	49	41	08N.
	PERSEUS	{ Medusa's head	ALGOL	β	2	2	52	39	59N.
		{ In Perseus's side	ALGENIB	α	2	3	07	48	58N.
	AURIGA	In the preceding should.	CAPELLA	α	1	4	59	45	43N.
	BOOTES	{ The brightest	ARCTURUS	α	1	14	03	20	30N.
		{ In the following thigh	MIRACH	ϵ	3	14	33	28	06N.
	NORTH CROWN	Brightest in the crown		α	2	15	22	27	33N.
	EAGLE	Eagle's heart	ALTAIR	α	2	19	39	8	14N.
	PEGASUS	{ In Pegasus's wing	MARKAB	α	2	22	53	13	53N.
		{ In Pegasus's leg	SCHBAT	β	2	22	52	26	43N.
	CASSIOPEIA	In the breast	SCHEDAR	α	3	0	27	55	11N.
	GREAT BEAR	{ Upper pointer	DUBHE	α	2	10	46	63	03N.
		{ Lower pointer		β	2	10	45	57	41N.
		{ First in the tail	ALIATH	ϵ	2	12	41	57	18N.
		{ Last in the tail	BENETNASCH	η	3	13	36	50	31N.
	HERCULES	In the head	RAS ALGETHI	α	3	17	03	14	40N.
	LYRA	In the harp	VEGA	α	1	18	29	38	34N.
	SWAN	In the tail	DENEK	α	2	20	33	42	25N.
	LITTLE BEAR	Pole star		α	2	0	42	88	00N.
SOUTHERN CONSTELLATIONS	ORION	{ Bright foot	REGEL	β	1	5	01	8	28 S.
		{ Right shoulder	BETELGEUSE	α	1	5	52	7	20N.
	GREAT DOG	In the mouth	SYRIUS	α	1	6	34	16	23 S.
	LITTLE DOG	In the flank	PROCYON	α	1	7	24	5	49N.
	HYDRA	In the heart		α	2	9	16	7	37 S.
	SOUTH FISH	The brightest	FOMALHAUT	α	1	22	42	30	55 S.
	ARGO	In the waves	CANOPUS		1	6	18	52	33 S.
	ERIDANUS	The last	ACHARNAR	α	1	1	26	58	29 S.
	CENTAURUS	Brightest		α	1	14	13	59	46 S.

1055. A TABLE for reducing the tables of the SUN'S DECLINATION from the meridian of London to any other meridian.

Daily diff. of Declination	Degrees of longitude from the meridian of London.											
	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
min.	min.	min.	min.	min.	min.	min.	min.	min.	min.	min.	min.	min.
2	00	00	00	00	00	00	01	01	01	01	01	01
3	00	00	00	00	01	01	01	01	01	01	01	01
4	00	00	00	01	01	01	01	01	02	02	02	02
5	00	00	01	01	01	01	01	02	02	02	02	02
6	00	00	01	01	01	01	02	02	02	02	03	03
7	00	01	01	01	01	02	02	02	03	03	03	03
8	00	01	01	01	02	02	02	03	03	03	04	04
9	00	01	01	01	02	02	03	03	03	04	04	04
10	00	01	01	02	02	02	03	03	04	04	05	05
11	00	01	01	02	02	03	03	04	04	05	05	05
12	00	01	01	02	02	03	03	04	04	05	05	06
13	01	01	02	02	03	03	04	04	05	05	06	06
14	01	01	02	02	03	03	04	05	05	06	06	07
15	01	01	02	02	03	04	04	05	06	06	07	07
16	01	01	02	03	03	04	05	05	06	07	07	08
17	01	01	02	03	04	04	05	06	06	07	08	08
18	01	01	02	03	04	04	05	06	07	07	08	09
19	01	02	02	03	04	05	06	06	07	08	09	09
20	01	02	02	03	04	05	06	07	07	08	09	10
21	01	02	03	03	04	05	06	07	08	09	10	10
22	01	02	03	04	05	05	06	07	08	09	10	11
23	01	02	03	04	05	06	07	08	09	10	11	11
24	01	02	03	04	05	06	07	08	09	10	11	12

1056. A TABLE of REFRACTIONS.

Alt. Deg.	Refr.		Alt. Deg.	Refr.		Alt. Deg.	Refr.		Alt. Deg.	Refr.		Alt. Deg.	Refr.		Alt. Deg.	Refr.	
	min.	sec.		min.	sec.		min.	sec.		min.	sec.		min.	sec.		min.	sec.
0	33	45	4	10	48	14	3	31	26	1	49	38	1	08	50	0	45
0 1/4	30	24	4 1/2	9	50	15	3	17	27	1	44	39	1	06	51	0	44
0 1/2	27	35	5	9	02	16	3	04	28	1	40	40	1	04	52	0	42
0 3/4	25	11	5 1/2	8	21	17	2	53	29	1	36	41	1	02	53	0	40
1	23	07	6	7	40	18	2	43	30	1	32	42	1	00	54	0	39
1 1/4	21	20	7	6	47	19	2	34	31	1	28	43	0	58	55	0	38
1 1/2	19	46	8	6	00	20	2	26	32	1	25	44	0	56	56	0	36
1 3/4	18	22	9	5	22	21	2	18	33	1	22	45	0	54	57	0	35
2	17	08	10	4	52	22	2	11	34	1	19	46	0	52	58	0	34
2 1/2	15	02	11	4	27	23	2	05	35	1	16	47	0	50	59	0	32
3	13	20	12	4	05	24	1	59	36	1	13	48	0	48	60	0	31
3 1/2	11	57	13	3	47	25	1	54	37	1	11	49	0	47	61	0	30

THE



THE
ELEMENTS
OF
NAVIGATION.

BOOK IX.
OF DAYS WORKS.

1057. **B**Y DAYS WORKS is here meant the practical methods of finding a ship's place every day at noon, and settling the course and distance she has then to sail.

The navigator requires for this purpose the following elements, or necessary things to be known.

- I. The means of measuring and correcting the run of the ship.
- II. The nature and use of the sea and azimuth compasses.
- III. How to find the sun's amplitude and azimuth.
- IV. How to find the variation of the compass, and thereby to correct the courses the ship has sailed.
- V. How to find the ship's lee-way, and thence correct the courses.
- VI. The nature and use of the instruments proper for taking the altitudes of celestial objects.
- VII. To find the latitude by observation.
- VIII. How to correct the current daily account of latitude and longitude.

SECTION

SECTION I.

1058.

Of the measuring a ship's run.

A great variety of methods have been proposed to measure the rate at which a ship sails, but that mostly in use at this time, especially among the English, is by the *log and half minute glass*.

The LOG is a piece of thin board of a quadrantal form, loaded in the circular side with lead sufficient to make it swim upright in the water: To this is fastened a line of about 150 fathoms long, called the LOG-LINE, which is divided into certain spaces called knots; and the log-line is wound on a reel which turns very easily.

The HALF MINUTE GLASS is of the make of an *hour glass*, and contains such a quantity of fine sand as will run thro' the hole in its neck in half a minute of time.

The making of the experiment to find the velocity of the ship, is called HEAVING THE LOG, which is thus performed.

Two men hold the reel between them, and another holds the half minute glass; an officer of the watch throws the log over the ship's stern, and when he observes the stray line is run off the reel (which is about 10 fathoms, this distance being usually allowed to carry the log out of the eddy of the ship's wake), and the first mark is going off, he cries *turn!* the glass holder answers *done!* who watching the glass, the moment it is run out says *stop!* the reel being immediately stopt, the last mark run off shews the number of knots, and the distance of that mark from the reel is estimated in fathoms: Then the knots and fathoms together shew the distance the ship has run the preceding hour, if the wind has been constant.

In the King's ships, India ships, and some others, it is usual to heave the log every hour: But coasters, and those using short voyages, heave the log once in two hours only.

1059. This practice of measuring a ship's rate of sailing, is founded upon the following principle.

That *the length of each knot, ought to be the same part of a sea mile; as half a minute is, of an hour.*

Therefore the length of each knot should be either

$$\left(\frac{1}{2} \times \frac{1}{60} =\right) \frac{1}{120} \text{ of a nautical mile.}$$

$$\text{Or } \left(\frac{1}{2} \times \frac{1}{60} \times \frac{1}{60} =\right) \frac{1}{7200} \text{ of a degree.}$$

$$\text{Or } \left(\frac{1}{2} \times \frac{1}{60} \times \frac{1}{60} \times \frac{1}{360} =\right) \frac{1}{259200} \text{ of a great circle of the earth.}$$

By Mr. Richard Norwood's experiment (see art. 900), it appears that a degree of a great circle on the earth contains 367200 English feet, and $\frac{1}{60}$ th thereof, or 6120 feet, are in one nautical mile.

Hence $\frac{1}{120}$ th of 6120, or 51 feet, should be the length of each knot.

But because it is safer to have the reckoning rather before the ship than after it, therefore 50 feet may be taken as the proper length of each knot.

And as most men can conveniently fathom with their arms about 5 feet; so a knot may be supposed to consist of 10 fathoms, each of 5 feet.

The

The length of the knots in the log-line used formerly were only of 42 feet; and it is much to be wished that not any line so divided was now in use, but custom in many things prevails over reason: This way of dividing the log-line was founded on the supposition that 60 miles, each of 5000 English feet, made a degree; for $\frac{1}{120}$ th of 5000 is $41\frac{2}{3}$, or in round numbers 42 feet. And altho' many mariners find by experience that this length of the knot is too short, yet rather than quit the *old way*, they use glasses for half minute ones that run but 24 or 25 seconds.

The method usually recommended to try if the glasses be accurate, is: On a round nail or peg, hang a string that has a musket ball fixed to one end, carefully measuring between the centre of the ball and the string's loop over the peg $39\frac{1}{8}$ inches: Then swing it, and count one for every time it passes under the peg, and the number of swings made during the time the glass is running out, shews the seconds it contains.

For experience shews, that the length of a second pendulum is about $39\frac{1}{8}$ inches.

The distance given by the log may be wrong on three accounts, namely, by there being an error either in the glass, or log, or in both.

1060. CASE I. *When the log-line is truly divided, and the glass is faulty.*

RULE. As the seconds run by the glass, is to 30 seconds:

So is the distance run by the log, to the true distance.

EXAM. Suppose a ship sails at the rate of $6\frac{1}{2}$ knots while a glass runs out, which is but 25 seconds: What is the true rate of sailing?

As 25 : 30 :: 6,5 : 7,8 miles, the true distance sailed in an hour.

1061. CASE II. *When the glass is true, and the log-line is faulty.*

RULE. As 50 feet, is to the distance measured between knot and knot:

So is the distance run by the log, to the true distance.

EXAM. Suppose a ship sails at the rate of $6\frac{1}{2}$ knots an hour, by a log-line which has only 44 feet to a knot: What is the true rate of sailing?

As 50 : 44 :: 6,5 : 5,72 miles, the true distance sailed in an hour.

1062. CASE III. *When both log-line and glass are faulty.*

RULE. Multiply thrice the measured length of a knot by the distance run per log; the product divided by five times the measured time of the glass, will give the true distance run.

EXAM. Suppose a ship runs 5 knots an hour, by a log-line of 45 feet to a knot, and a glass of 25 seconds: What is the true rate of sailing?

Then $3 \times 45 \times 5$, and divided by 5×25 , gives 5,4 miles = true dist.

For suppose K = meas. length of a knot; G = seconds per glass; D = run per log.

Then $G : 30 :: D : 30 \times D \div G$ = dist. run by a correct glass. (1060)

And $50 : K :: 30 \times D \div G : 30 \times D \times K \div G \times 50 = 3 \times D \times K \div 5 \times G$. (1061)

NOTE, The mark $\div$ signifies divided by.

S E C T I O N II.

Of the Mariner's compass.

1063. The MARINER'S COMPASS is an instrument used at sea to shew, or point out the way a ship is to go; and consists of three parts, namely, the box, the card or fly, and the needle.

The Box, which contains the card and needle, is a circular brass box hung within a square wooden one, by two concentric rings, so fixed by cross centres to the two boxes, that the inner one, or compass box, shall retain an horizontal position in all motions of the ship.

The CARD or FLY is a circle of stiff paper representing the horizon; and is divided into points and quarter points of the compass.

The NEEDLE is a piece of steel made magnetical; whereby it has the property of pointing one of its ends towards the north pole of the world.

The needle is fixed to the underside of the card, and in the centre is placed a conical socket, which is put on an upright pointed pin fixed in the bottom of the box; so that the card hanging on the pin, turns freely round its centre; one of the points being, by the property of the needle, always directed towards the north pole.

The top of the box is covered with a glass plane, whereby the card in its motion is not disturbed by the wind. And this apparatus together, is called the Compass.

1064. Before the invention of the compass, which is between 300 and 400 years ago, the navigating of ships was a very tedious and precarious operation: But the compass enables the mariner to hold his course over the seas in as direct and true a tract, as the land carrier directs his carriage in a well beaten road. Hence it might be reasonably imagined, that no necessary expence or care should ever be wanting in the construction of this most useful instrument: But it has so happened, that scarce one sea compass in ten is fit for the use for which it is made; and this has arose from their being fabricated by unskilful and ignorant workmen for the wholesale dealers in the shipping way; who generally pay no more regard to the construction of this instrument, whereon the success of the voyage and the lives of the men in a great measure depend, than they do to any indifferent thing of the same price.

There are however compasses used in the Royal Navy, and some few trading ships, which are constructed with more care, and on better principles; and are therefore vastly preferable to those commonly used in the merchant's service, which from their composition seem as tho' they had been contrived purposely to vary from what was to be expected of them: Such as joining the outside box together with iron nails; making the needle of soft wire, and disposing it in the form of a rhombus, in expectation that the magnetical forces of the sides would conspire to act in the diagonal: Making the pin and socket so badly, as to prevent the traversing of the card, &c. &c.

But it must be observed, that it is only within a very few years that sea compasses have been made free from the multitude of inconveniences to which the best constructed of them were liable before: Such as, first, needles

needles having several poles, occasioned by their irregular shape, the best kind being strait bars with flat ends. 2d. The needles being made of such a temper as was neither susceptible of receiving, or retaining half the virtue it was possible to give them; and consequently they were not with due strength and perseverance directed towards the poles of the world. 3d. The want of proper means to restore their loss of magnetism while on a voyage. 4th. The troublesome and inaccurate methods of rectifying a damaged pin on which the card turned, together with the expence attending the use of agate caps or sockets, than which none are so proper. 5th. The want of proper contrivances to hinder the card from being affected by the various motions of the ship. These, and several other imperfections, have been happily removed by the labours of the truly celebrated *Dr. Gower Knight*, F. R. S. whose admirable invention of giving magnetism to steel bars, greatly superior to any power they could derive from the natural loadstones, joined to a multitude of experiments which he has made for the marine service, have produced the means of constructing sea compasses so perfect, that there seems nothing farther to be wished for as necessary to this instrument. It may be reasonably expected that such correct compasses will readily come into general use, as there have been several trials made of them both in men of war and merchantmen: But, like several other useful improvements, it must wait till time has removed old prejudices.

1065. After the discovery of that most useful property of the magnet, namely, the giving hardened iron and steel a polarity, the compass was for many years used without knowing that its direction in any wise deviated from the poles of the world: And about 200 years ago, so certain were some of its inflexibly pointing to the north, that they treated with contempt the notion of the variation, which about that time began to be suspected. However, careful observations soon discovered that in England, and its neighbourhood, the needle pointed to the eastward of the true north: But the quantity of this deviation being known, mariners became as well satisfied as if the compass had none; because they imagined that the true course could be obtained by making allowance for the true variation.

From successive observations made afterwards, it was found that the deviation of the needle from the north was not a constant quantity; but that it gradually diminished, and at last, namely about the year 1660, it was found at London that the needle pointed due north, and has ever since been getting to the westward, and now the variation is more than 18 degrees to the westward of the north: So that it may be suspected the variation has a kind of libratory motion, traversing thro' the north to unknown limits eastward and westward. But the settling of this point must be left to time.

During the time of the said observations it was also discovered, that the variation of the needle was different in different parts of the world, it being west in some places when it was east in others; and in places where the variation was of the same name, yet the quantity of it greatly differed. It was therefore found necessary that mariners should every day, or as often as they had opportunity, make proper observations for an amplitude or azimuth, whereby they might be enabled to find the vari-

ation of the compass in their present place, and thence correct their courses.

1066. Beside the common sea compass, there is another called the AZIMUTH COMPASS, whose use is to take the bearing of any celestial object, either when it is at, or above the horizon. This compass in no respect differs from the other, only the circumference of the card or box is divided into degrees; and there is fitted to the box an index with two sights, which are upright pieces of brass placed diametrically opposite, having flits down their middles, thro' which the sun or star is to be viewed at the time of observation.

P R O B L E M.

To make an observation with the Azimuth compass.

1067.

I. FOR AN AMPLITUDE *.

Move the box about till the four cardinal points of the card coincide with the four lines in the sides of the compass box; keep the box steady, and turn the index till the sun at its rising or setting is seen thro' both the sights: Then the degrees intercepted between the north point of the compass and that end of the index pointing to the sun, will be the magnetic amplitude sought.

1068.

II. FOR AN AZIMUTH †.

Set the box so, that the four cardinal points of the card stand opposite to the four lines in the compass box; hold the box in this position, and move the index till (the sights being directed to the sun) the shadow of the string joining the sights falls directly along the middle line of the index: Then the degrees in the arc, intercepted between that end of the index next the sun and the north point of the card, will give the magnetic azimuth required.

If the sun does not shine strong enough to give a distinct shadow, look thro' one of the sights, and move the index till the string cuts the sun's centre; and then the intercepted arc, as before, shews the sun's azimuth.

What is here said of the sun, is in like manner to be applied to the stars.

1069. When there is a rough sea, and the ship yaws much, the observation is best performed by two persons, thus:

Let one look thro' the sights, and turn them till the thread cuts the centre of the sun, and be sure to keep a constant sight of it notwithstanding the motion of the ship, which being signified to the assistant by any word, let him that instant observe what degree of the card is under the middle line of the sights, and the arc intercepted between that point and the meridian will shew the quantity of the magnetic amplitude or azimuth. If the card should vibrate very much, let him take the middle degree between the limits to which the vibration reaches.

When the magnetic azimuth of any object is observed, its altitude must also be taken at the same time.

This instrument is also useful in setting the ship's wake, in order to find the leeway; and also to find the bearings of headlands and other objects.

* Amplitudes, are from observations made on objects when in the horizon. See articles 939, 986 to 992.

† Azimuths, are from observations made on objects when above the horizon. See articles 938, 993.

SECTION

SECTION III.

1070.

To work an Amplitude.

The true amplitude of a celestial object, is its distance from one of the cardinal points of the horizon, at the rising or setting of that object.

The amplitude is reckoned eastward in the morning; but it is named westward in the evening.

The customary way of accounting the amplitude is, in the morning to reckon it so much to the southward or northward of the east; and in the evening so much to the southward or northward of the west: But as bearings are generally taken from the meridian, therefore in the following pages the amplitudes will be chiefly estimated from the north point of the horizon.

The finding of the true amplitude at sea is of considerable importance in navigation; for the true courses the ship has steered is partly ascertained thereby.

1071.

PROBLEM.

Given the latitude of the place, and the declination of a celestial object: Required the true amplitude of that object.

SOLUTION.

Say: As the co-sine of the latitude,
To Radius;
So sine of the given declination,
To the sine of an horizontal arc.

Which arc taken from 90 degrees, if the declination is north, will give the sun's true amplitude from the north point of the horizon: But if the declination is south, that arc added to 90 degrees gives the amplitude from the north.

When the amplitude is reckoned from the east or west points of the horizon; then the said horizontal arc is the true amplitude, and is to be taken to the northward of those points, when the declination is north: But to the southward, when the declination is south.

1072.

SOLUTION BY THE TRAVERSE TABLE.

With the declination among the courses, find the departure to the distance 100.

With the given latitude among the courses, and said departure in column of latitude, find the distance.

In a column of departure, find the said distance against 100 in the column of distance, and the degrees in the corresponding course will give the horizontal arc sought.

This rule will always give the degrees as truly as by logarithms: And as the magnetic amplitude can be observed to degrees only by the compass, the true amplitude will be had sufficiently correct by this rule.

EXAM.

EXAM. I. *What is the sun's true amplitude on the 14th day of May, 1753, in the latitude of $38^{\circ} 25' N.$?*

By Logarithms.

As cos. lat. $= 38^{\circ} 25'$ 0,10595
 To Radius $= R$ 10,00000
 So sin. decl. $= 18\ 44$ 9,50673
 To sin. hor. arc $= 24\ 12$ 9,61268
 Subtract from $90\ 00$ for N. dec.
 Leaves true amp. $= 65\ 48$ from N.

By the Traverse table.

To decl. $18^{\circ} 45'$, and dist. 100, the dep. is 32.
 To lat. $38^{\circ} 25'$, and diff. lat. 32, the dist. is 41.
 To dist. 100, and dep. 41, the course is $24^{\circ} 15'$. Therefore the horizontal arc is 24° .

In this and the following examples, the nearest deg. and min. to the declin. that can be found in the Traverse table, is taken for the declin.

EXAM. II. *Required the sun's true amplitude in the lat. of $51^{\circ} 32' N.$, on the 25th day of November, 1753?*

By Logarithms.

As cos. lat. $= 51^{\circ} 32'$ 0,20617
 To Radius $= R$ 10,00000
 So sin. decl. $= 20\ 52$ 9,55169
 To sin. hor. arc $= 34\ 56$ 9,75786
 Add to $90\ 00$ for S. decl.
 Gives true amp. $= 124\ 56$ fr. the N.

By Traverse table.

To decl. $20^{\circ} 45'$, and dist. 100, the dep. is 35,4.
 To lat. $51^{\circ} 30'$, and diff. lat. 35,4, the dist. is 57.
 To dist. 100, and dep. 57, the course is $34^{\circ} 48'$. Therefore 35° may be taken for the horizontal arc sought.

EXAM. III. *On the 20th of January, 1753, what was the sun's amplitude at the Cape of Good Hope, in lat. $34^{\circ} 15' S.$*

By Logarithms.

As cos. lat. $= 34^{\circ} 15'$ 0,08271
 To Radius $= R$ 10,00000
 So sin. decl. $= 20\ 01$ 9,53440
 To sin. hor. arc $= 24\ 28$ 9,61711
 Which add to $90\ 00$ for S. decl.
 Gives true amp. $= 114\ 28$ fr. the N.

By Traverse table.

To decl. $20^{\circ} 00'$, and dist. 100, the dep. is 34,3.
 To lat. $34^{\circ} 15'$, and diff. lat. 34,3, the dist. is 41,5.
 To dist. 100, and dep. 41,5, the course is $24^{\circ} 37'$. Therefore 24° may be taken for the horizontal arc sought.

EXAM. IV. *On what point of the horizon does the star Arcturus rise and set at Buenos Aires in lat. $34^{\circ} 35' S.$?*

By Logarithms.

As cos. lat. $= 34^{\circ} 35'$ 0,08444
 To Radius $= R$ 10,00000
 So sin. decl. $= 20\ 30$ 9,54432
 To sin. hor. arc $= 25\ 10$ 9,62876
 Which add to $90\ 00$ for S. dec.
 Gives true amp. $= 115\ 10$ fr. the N.

By Traverse table.

To decl. $20^{\circ} 30'$, and dist. 100, the dep. is 34,8.
 To lat. $34^{\circ} 30'$, and diff. lat. 34,8, the dist. is 42.
 To dist. 100, and dep. 42, the course is $25^{\circ} 00'$. Therefore the horizontal arc may be taken as 25 degrees.

SECTION

SECTION IV.

1073.

To work an Azimuth.

The true azimuth of a celestial object, is its bearing, when above the horizon, from the north or south points thereof.

In the forenoon the azimuth is reckoned eastward, and in the afternoon westward.

The azimuth is usually estimated from the south, when to the southward of the east or west points of the horizon; and when to the northward of those points, it is reckoned from the north: But it may be more convenient to account it always from the north.

The use of the azimuth is to find the variation of the compass, in order to correct the courses the ship has steered.

1074.

P R O B L E M.

The latitude of the place, the declination of the object, and its altitude being known: Required the true azimuth of that object.

SOLUTION.

When the latitude and declination are of the same name, let the co-declination be called A.

But when of contrary names, add the declination to 90 deg., and call the sum A.

Let the difference between the co-lat. and co-alt. be called D.

Also, find the half sum and half difference of A and D.

Then write down these four logarithms under one another.

Namely: The arith. comp. of the log. sine of the co-latitude.

The arith. comp. of the log. sine of the co-altitude.

The logarithm sine of the foresaid half sum of A and D.

The logarithm sine of the foresaid half diff. of A and D.

Add the four logarithms together, and take half their sum, which seek among the sines for the corresponding degrees and minutes, and these being doubled will give the true azimuth required.

And is to be reckoned from the north, in north latitude.

But in south latitude, take those degrees from 180°, and the remainder will give the true azimuth from the north.

The working of an azimuth by logarithms, altho' as short as any numeral solution can be expected, is one of the most tedious elements that enter into a day's work: But by Gunter's scale it may be readily done, sufficiently correct.

1075.

To work an azimuth by the Gunter's scale.

Find the co-lat., co-alt., the half sum, and half difference as above.

Then, On the scale of log. sines, set one point of the compasses on the half sum, extend the other point to the co-lat., observing whether the second point falls to the right or left of the first, and take the compasses off, without altering their extent.

Set one point on the co-alt., and let the second point fall where it will (the same way it did before), there fix it, and open the compasses till the other point falls on the half difference, then take them off without altering their extent.

Now

Now set one point of the compasses on the brass pin at the beginning, or right hand end of the scale of versed fines (marked V. S.), turn the other point to the left, and observe what degrees and minutes in the versed fines it falls on; those degrees and minutes will be the azimuth, if the latitude is south: But if north, take them from 180° , and the remainder is the azimuth sought.

If the first extent, when applied from the co-alt. to a fourth, should reach beyond the right hand end of the scale, as it will sometimes happen, then hold the Gunter flat on a board, and thereon let the over point rest, and thence take the extent to the half difference, and work as before.

These directions, with those at articles 379 and 380, will remove all the seeming difficulties that beginners may meet with in working an azimuth by the Gunter's scale.

EXAM. I. *Being at sea, in latitude $40^\circ 28' N.$, in the afternoon the sun's altitude was observed to be $20^\circ 46'$, when his declination was $17^\circ 10' S.$: Required the sun's true azimuth at that time?*

By Logarithms.

The co-alt. is $69^\circ 14'$

The co-lat. is $49^\circ 22'$

The diff. D = $19^\circ 52'$

Now $107^\circ 10' = A = 90^\circ + \text{decl.}$

And $19^\circ 52' = D.$

$\begin{array}{r} 127 \ 02 \\ 2) \end{array} \left| \begin{array}{l} 63^\circ \ 31' \\ \hline \end{array} \right. = \frac{1}{2} \text{ sum.}$

$\begin{array}{r} 87 \ 18 \\ 2) \end{array} \left| \begin{array}{l} 43^\circ \ 39' \\ \hline \end{array} \right. = \frac{1}{2} \text{ diff.}$

Ar. co-log. fin. co-lat. = $49^\circ \ 22' \ 0,11982$

Ar. co-log. fin. co-alt. = $69^\circ \ 14' \ 0,02917$

Log. fin. $\frac{1}{2}$ sum = $63^\circ \ 31' \ 9,95179$

Log. fin. $\frac{1}{2}$ diff. = $43^\circ \ 39' \ 9,83901$

The sum of the four logs. $19,93979$

Their $\frac{1}{2}$ sum, gives $68^\circ \ 55'$ $9,96989$

The double, is $137^\circ \ 50'$, which is the sun's true azim. from the N. point of the horizon.

By Gunter's scale.

On the fines, the extent from $63^\circ 31'$ to $49^\circ 22'$, will reach from $69^\circ 14'$ to a little more than 52° : Let the point rest there, and extend the other to $43^\circ 39'$; this extent will reach from the beginning of the versed fines to about 42° , which taken from 180° leaves 138° for the azimuth, and is very near what was found by the logarithms.

EXAM. II. *What is the sun's true azimuth in the latitude of $26^\circ 30' N.$, when his altitude in the forenoon is $24^\circ 28'$, and his decl. is $22^\circ 40' N.$?*

By Logarithms.

The co-alt. is $65^\circ 32'$

The co-lat. is $63^\circ 30'$

The diff. D = $2^\circ 02'$

Now $67^\circ 20' = A$, is 90° less

And $2^\circ 02' = D.$ [decl.]

$\begin{array}{r} 69 \ 22 \\ 2) \end{array} \left| \begin{array}{l} 34^\circ \ 41' \\ \hline \end{array} \right. = \frac{1}{2} \text{ sum.}$

$\begin{array}{r} 65 \ 18 \\ 2) \end{array} \left| \begin{array}{l} 32^\circ \ 39' \\ \hline \end{array} \right. = \frac{1}{2} \text{ diff.}$

Ar. co-log. fine co-lat. = $63^\circ \ 30' \ 0,04821$

Ar. co-log. fine co-alt. = $65^\circ \ 32' \ 0,04086$

Log. fine $\frac{1}{2}$ sum = $34^\circ \ 41' \ 9,75514$

Log. fine $\frac{1}{2}$ diff. = $32^\circ \ 39' \ 9,73200$

The sum of the four logs. $19,57621$

Their $\frac{1}{2}$ sum, gives $37^\circ \ 52'$ $9,78810$

The double, is $75^\circ \ 44'$, which is the sun's true azim. from the north point of the compass.

B.

By Gunter's scale.

On the log. lines, the extent from $34^{\circ} 41'$ to $63^{\circ} 30'$, will reach from $65^{\circ} 32'$ to some point beyond the right hand end of the scale; let the point rest there, and extend the other point to the line of $32^{\circ} 39'$, and this extent will, on the versed lines, reach from the beginning, or 0 deg., to $104^{\circ} 20'$, whose supplement $75^{\circ} 40'$ is the azimuth, nearly, as found by logarithms.

EXAM. III. *At the Island of St. Helena, in lat. $16^{\circ} 00'$ S., the sun's altitude was observed in the forenoon to be $30^{\circ} 22'$, when his declination was $22^{\circ} 58'$ S.: What was the sun's azimuth at that time?*

The co-lat. is $74^{\circ} 00'$	Ar. co-log. sine co-lat. = $74^{\circ} 00'$ 0,01716
The co-alt. is $59^{\circ} 38'$	Ar. co-log. sine co-alt. = $59^{\circ} 38'$ 0,06409
	Log. sine $\frac{1}{2}$ sum = $40^{\circ} 42'$ 9,81431
The diff. D = $14^{\circ} 22'$	Log. sine $\frac{1}{2}$ diff. = $26^{\circ} 20'$ 9,64698
Now $67^{\circ} 02' = A$, is 90° less	The sum of the four logs. 19,54254
And $14^{\circ} 22' = D$. [decl.	
$81 \quad 24 \quad 40^{\circ} 42' = \frac{1}{2}$ sum.	Their $\frac{1}{2}$ sum, gives $36^{\circ} 12'$ 9,77127
2) $52 \quad 40 \quad 26^{\circ} 20' = \frac{1}{2}$ diff.	Which doubled, is $72^{\circ} 24'$, for the sun's true
	azim. from the south, or $107^{\circ} 36'$ from north.

Or thus: The extent from $40^{\circ} 42'$ to $74^{\circ} 00'$ on the lines, will reach from the line of $59^{\circ} 38'$ to a point beyond the limits of the scale; then the extent from that point to the line of $26^{\circ} 20'$ will, on the versed lines, reach from 0 deg. to about $107^{\circ} 40'$, the azimuth from the north.

EXAM. IV. *On what point of the horizon, at the Cape of Good Hope, does the star Aldebaran bear, when its altitude is $22^{\circ} 25'$?*

The co-alt. is $67^{\circ} 35'$	Ar. co-log. sin. co-lat. = $55^{\circ} 45'$ 0,08271
The co-lat. is $55^{\circ} 45'$	Ar. co-log. sin. co-alt. = $67^{\circ} 35'$ 0,03412
	Log. sin. $\frac{1}{2}$ sum = $58^{\circ} 54'$ 9,93261
The diff. D = $11^{\circ} 50'$	Log. sin. $\frac{1}{2}$ diff. = $47^{\circ} 05'$ 9,86472
Now $105^{\circ} 59' = A$, is 90° & decl.	The sum of the four logs. 19,91416
And $11^{\circ} 50' = D$.	
$117 \quad 49 \quad 58^{\circ} 54' = \frac{1}{2}$ sum.	Their $\frac{1}{2}$ sum, gives $64^{\circ} 57'$ 9,95708
2) $94 \quad 09 \quad 47^{\circ} 05' = \frac{1}{2}$ diff.	Which doubled, is $129^{\circ} 54'$, for the sun's true
	azim. from the south, or $50^{\circ} 06'$ from the N.

Or thus: The extent from about 59° to about $67^{\circ} \frac{1}{2}$ on the lines reaches from about $55^{\circ} \frac{3}{4}$ to about 63° : Then the extent from about 63° to about 47° , reaches from the beginning of the versed lines to about 50° for the azimuth sought.

These examples sufficiently shew, that an azimuth may be worked by the Gunter's scale, not only with great readiness, but also as accurately as the magnetic azimuth can be taken by the compass.

The manner of working amplitudes and azimuths have been pretty largely insisted on, in order to render them familiar to mariners; who, for the generality, esteem these operations as very troublesome; and it is on this account that many writers have contrived various tables to remove the complained of trouble: But from a due consideration of what is here done, it is apprehended such tables will not hereafter be so much wanted.

SECTION V.

1076.

Of the Variation of the Compass.

The *magnetic poles*, are those points of the world, towards which the north and south ends of the needle in the sea compass are directed.

Magnetical meridians, are imaginary lines passing thro' the magnetic poles.

The *variation of the compass*, is the difference between the bearings of the *true* and *magnetic* north poles; and is measured by an arc of the horizon contained between the true and magnetic points thereof.

The variation is named *east*, when the magnetic north point is to the eastward of the true north: But when to the westward, it is named *west* variation.

The magnetic amplitude of a celestial object, is its bearing by the compass when in the horizon.

The magnetic azimuth of a celestial object, is its bearing by the compass when above the horizon.

The variation is found by comparing together the true and magnetic amplitudes or azimuths of celestial objects.

The use of the variation is to correct the courses a ship has steered by the compass.

1077.

PROBLEM.

Given the true and magnetic amplitudes, or azimuths, of a celestial object: Required the variation of the compass.

SOLUTION.

Let the true and magnetic amplitudes, or azimuths, be reckoned from the north; and named east when the observation is made in the forenoon; but when made in the afternoon, let it be named west.

Then the difference between the true and magnetic amplitudes, or azimuths, is the variation of the compass.

And is of the same name as the true amplitude, or azimuth, if the true is greater than the magnetic: But when the true is less than the magnetic, the variation is of a contrary name to the true amplitude, or azimuth.

For, let N represent the true north point of the horizon, and the arc NW the true amplitude or azimuth: Now it is evident, that if the arc OW , representing the magnetic amplitude or azimuth, be less than OWN ; then the magnetic north point of the compass will fall on the same side the true north, that the amplitude or azimuth is on: But when greater, n falls on the contrary side. Plate XI. Fig. 8.

EXAM.

EXAM. I. The sun was observed to rise 70° from the north by the sea compass, when his true amplitude was $N. 81^{\circ} 45' E.$: What was the variation, and which way?

True amplitude = $81^{\circ} 45' E.$

Magnetic amplitude = $70^{\circ} 00'$

The variation = $11^{\circ} 45' E.$

EXAM. III. Suppose the sun's true azimuth to be $N. 84^{\circ} 40' W.$, when by the compass his bearing was $W. b. S.$: Required the quantity and quality of the variation?

True azimuth = $84^{\circ} 40' W.$

Mag. azimuth = $101^{\circ} 15' = 9 \text{ pts.}$

The variation = $16^{\circ} 35' E.$

EXAM. II. The sun was observed by the compass to set $WSW.$ when his true amplitude was $N. 126^{\circ} 17' W.$: Required the variation of the compass, and which way?

True amplitude = $126^{\circ} 17' W.$

Mag. amplitude = $112^{\circ} 30' = 10 \text{ pts.}$

The variation = $13^{\circ} 47' W.$

EXAM. IV. When the sun was observed to bear $ESE.$, his true azimuth was found to be $N. 88^{\circ} 46' E.$: What was the quantity and name of the variation?

True azimuth = $88^{\circ} 46' E.$

Magnetic azim. = $112^{\circ} 30'$

The variation = $23^{\circ} 44' W.$

The manner of finding the variation being known, the courses may be corrected by the following rules.

1078.

PROBLEM.

Given the course set, and the variation of the compass:

Required the true course the ship steers.

SOLUTION.

If the variation is west, call the NW. quarter the 1st, the SW. the 2d, the SE. the 3d, and the NE. the 4th: But if the variation be east, call the NE. quarter the 1st, the SE. the 2d, the SW. the 3d, and the NW. the 4th.

Then, if the course set be in the first or third quarters, add the variation to the points or degrees in the given course: But if in the second or fourth quarters, subtract; and the sum in the former, or difference in the latter case, will be the course corrected by the variation *.

EXAM. I. Suppose the variation is $1\frac{1}{2}$ point $W.$; to correct these courses, $NNW.$; $SW. \frac{1}{2} W.$; $SE. b. S. \frac{3}{4} E.$; $ENE. \frac{1}{4} E.$?

Courses set $NNW. 2 \text{ pts.}$ | $SW. \frac{1}{2} W. 4\frac{1}{2} \text{ pts.}$ | $SE. b. S. \frac{3}{4} E. 3\frac{3}{4} \text{ pts.}$ | $ENE. \frac{1}{4} E. 6\frac{1}{4}$

Variation Add $1\frac{1}{2} \text{ pts.}$ | Subtr. $1\frac{1}{2}$ | Add $1\frac{1}{2}$ | Subtract $1\frac{1}{2}$

Cor. cou. $NW. b. N. \frac{1}{2} W. 3\frac{1}{2} \text{ pts.}$ | $SW. b. S. 3$ | $SE. b. E. \frac{1}{4} E. 5\frac{1}{4}$ | $NE. \frac{3}{4} E. 4\frac{3}{4}$

EXAM. II. The variation is $18^{\circ} E.$; to correct the following courses?

Courses set $NE. b. N. = 33^{\circ} 45'$ | $S. b. E. = 11^{\circ} 15'$ | $SW. b. W. = 56^{\circ} 15'$ | $NW. = 45^{\circ} 00'$

Variation Add $18^{\circ} 00'$ | Subtr. $18^{\circ} 00'$ | Add $18^{\circ} 00'$ | Subtr. $18^{\circ} 00'$

Cor. cour. $N. 51^{\circ} 45' E.$ | $S. 6^{\circ} 45' W.$ | $S. 74^{\circ} 15' W.$ | $N. 27^{\circ} 00' W.$

* For where the compass has a variation, the course set is not so far removed from the true north as it should be, by the quantity of the variation: But the quantity of the variation being added in the 1st and 3d quadrants, and subtracted in the 2d and 4th, puts the course so much farther from the true north, counting from thence, the same way with the variation, round the compass.

SECTION VI.

1079. *Of finding the Ship's drift, or leeway.*

When a ship turns to windward, or is close hauled on a wind, she generally runs to the leeward of her intended course.

LEEWAY is the angle which the ship's real course makes with her intended course, occasioned by contrary winds and a rough sea.

The leeway made by different ships, under the same circumstances of wind and sails, will be different; and even the same ship, with different lading, and having more or less sail abroad, will make more or less leeway. However, the following precepts for allowing for the leeway have been generally given by most writers on this subject within the last 50 years; they were given by Mr. John Buckler to Mr. William Jones, who first published them about the year 1702.

1080. *Allowances for Leeway.*

I. When a ship is close hauled, has all her sails set, the water smooth, and a moderate gale of wind, she is then supposed to make little or no leeway.

II. Allow one point, when it blows so fresh that the small sails are taken in.

III. Allow two points, when the top-sails must be close reefed.

IV. Allow two and a half points, when one top-sail must be handed.

V. Allow three and a half points, when both top-sails are to be taken in.

VI. Allow four points, when the fore course is handed.

VII. Allow five points, when trying under the main-sail only.

VIII. Allow six points, when both main and fore courses are taken in.

IX. Allow seven points, when the ship tries a hull, or all sails handed.

When the wind has blown hard in either quarter, and shifts across the meridian into the next quarter, the leeway will be lessened.

But in all these cases, respect must be had to the roughness of the sea, with the trim of the ship; and hence the mariner will be able to correct his course.

1081. *P R O B L E M.*

Given the place of the wind, the course set, and the leeway:

Required the true course the ship makes.

RULE. Count the nearest way of the compass from the wind to the course set, and as much beyond as is the leeway, and it gives the point for the true course.

EXAM. I. *The wind NW. b. N., the course set NE. b. N., leeway 1 point: What is the course made good?*

The wind NW. b. N.

Course set NE. b. N. = 3 points.

Add the leeway = 1 point.

True course NE. = 4 points.

Here, counting the nearest way in the compass from NW. b. N. to NE. b. N., the leeway going beyond it towards the east must be added to drive it so much farther from the wind.

EXAM. II. *The wind SE. b. S., a ship is close hauled with her starboard tacks aboard, and both top-sails handed: What is her true course?*

The wind SE. b. S.

Course set E. b. N. = 7 points.

Subtract leeway = $3\frac{1}{2}$ points.

True cou. NE. b. N. $\frac{1}{2}$ E. = $3\frac{1}{2}$ points.

The ship being close hauled, is supposed 6 pts. from the wind, and must be in the NE. quarter, being on the starboard tacks: Then counting 6 pts. from SE. b. S., gives E. b. N. 7 points; from which the leeway is to be taken.

SECTION

SECTION VII.

Of Instruments for taking of Altitudes.

The altitude of the sun, or any other celestial object, may be taken by instruments contrived after various ways; those only, best adapted for use at sea, will be here considered.

1082. *The description and use of Davis's quadrant.* Plate XI. Fig. 9.

The instrument most in use at sea, by us called DAVIS's * QUADRANT, and by foreigners the ENGLISH QUADRANT, consists of two arches (making together ninety degrees, and therefore called a quadrant), fixed and braced, the one above, the other below a strait bar, whose length is equal to the radius of the lower arch, and is about thrice the length of the radius of the upper arch.

The upper, or, as it is commonly called, the GREATER ARCH, usually contains 65 degrees, not subdivided into smaller parts; and the LESSER ARCH contains 25 degrees, which on account of their larger size are subdivided into smaller parts, such as $\frac{1}{3}$ or $\frac{1}{6}$ parts of degrees; and these by the help of diagonal divisions, and 10 concentric circles, are usually subdivided so, as to shew every one or two minutes.

There are three pieces called vanes, so fitted to the instrument as to stand at right angles to its plane.

First, The HORIZON VANE is fitted to slip on the end of the bar or radius, close to the centre of the two arches: In this vane there is a long slit, thro' which is to be seen the horizon, that is the line where the sky and water apparently meet.

Secondly, The SIGHT VANE, which is so fitted as to slide along the lesser, or 25 arch, having a sharp edge to cut the division it stands at; and in a line with this edge is a small hole, thro' which the horizon is to be seen thro' the slit in the horizon vane.

Thirdly, The SHADE VANE, which is fitted to slide on the greater, or 65 arch: The upper line of the shade cast by this vane is to fall on the edge of the slit in the horizon vane during the time of observation.

Instead of the shade vane there is generally used one called the GLASS VANE, it having a convex lens (or as usually called a burning glass) fitted to it, which collects the sun's rays into a bright spot on the horizon vane.

AB, is the bar or radius. BC, the lesser or 25 arch. DE, the greater or 65 arch. *ab*, *cg*, *hi*, are braces which fix the arches to the radius. G the horizon vane. H the sight vane. F the shade or glass vane. $\odot$ FA a ray of the sun. $\angle DAC$ a right angle, or 90° .

* The instrument now called *Davis's quadrant*, is somewhat different from that mentioned by Capt. John Davis in his treatise called the Seaman's secrets, published in the year 1594.

1083. To observe the sun's altitude or zenith distance by Davis's quadrant.

First. Put on the horizon vane close to the shoulder or notch at A, in the bar AB: Set the shade or glass vane on the arch DE to an even degree, about 10 or 15 degrees less than what the zenith distance may be then guessed at; and put the sight vane H about the middle of the arch BC; observing that the shade and sight vanes be thrust close to the backs of the arches.

Secondly. Turn your back to the sun, hold the plane of the instrument in an upright position, look thro' the hole in the sight vane, and raise or lower the quadrant until the bright spot, cast by the sun thro' the glass vane, falls on the $\ominus$ in the line drawn thro' the slit in the horizon vane; then if the horizon is seen thro' the slit in the horizon vane, even with the $\ominus$, the quadrant will give the sun's altitude or zenith distance at that time.

Thirdly. But if the sky appears thro' the slit instead of the horizon, slide the sight vane H higher, till the horizon coincides with $\ominus$; and if the sea appears instead of the horizon, slide the sight vane H lower till the spot and horizon come together, and the observation at that time is made.

Fourthly. To the degrees between D and F add those between C and H, their sum gives the zenith distance: But the degrees between B and H added to those between F and E, shews the altitude. Plate XI. Fig. 9.

For the line HA is horizontal; therefore the arc fBH gives the height of the sun above the horizon: But $fBH = FE + BH$.

Also, The arc dfc is 90° , and zen. dist. = comp. of altitude.

Therefore the arc $df + \text{arc } HC$, are together the comp. of arc fH .

But $df = DF$; therefore $DF + HC = \text{zenith distance}$.

1084. Now to find the latitude, the *meridian* altitude, or zenith distance of the sun is wanted: Therefore begin some time before noon, and having taken an observation as before directed, wait a minute or two and observe again; and if the sea appears thro' the horizon vane, the sun has got higher, therefore slide the sight vane lower till the solar spot and horizon coincide; and thus continue, until the sky appearing instead of the sea, shews the sun has past the meridian; therefore let the vane stand as at the last alteration, and add the degrees in the greater arc to the degrees and minutes in the lesser, their sum gives the meridional zenith distance.

If instead of the bright solar spot, the upper border of the shade is made to cut the horizon line, at the time the horizon is abreast that line; then the sum of the arcs gives the zenith distance of the sun's upper edge or limb, to which add 16 minutes, and the sum is the distance of the sun's centre from the zenith when he is upon the meridian. * But in most quadrants there are on the back of the greater arch a scale of degrees, where the sun's semidiameter is allowed for.

* For in Pl. XI. Fig. 10. if AB represents the sun whose centre is c, and its rays pass over the top of an upright wall DE: Then it is evident the ray cc only, gives the true alt. HC, or zenith distance zc: Also, that the sun must descend for the ray Aa to come to cc, and ascend for the ray Bb to come to cc: Therefore the sun's semidiameter must be added in zen. distances, and subtracted in altitudes.

1085. One great objection against this instrument, is the trouble and time lost in sliding the sight vane up or down, which cannot be conveniently done without taking the quadrant from the eye, whereby an opportunity is sometimes lost of making the observation: But this defect is easily removed by having a long index or ruler fitted to the quadrant; one end moving round the centre to which the horizon vane is fixed, and having the sight vane fixed to the other end. By this contrivance the sight vane may be readily raised higher, or lowered, by the motion of the index about its centre, which may be done without taking the instrument from the eye.

1086. There is another instrument called a FORE STAFF, or CROSS STAFF, that has been a long time in use for taking the altitudes of celestial objects, particularly the stars. With this instrument, the observer holds one end of the staff, which is a three feet rod, close to one side of his eye, and slides a transum or cross piece, which is in an upright position, along the staff till one end of the cross cuts the star, and the other end at the same time cuts the horizon; then the division of the staff, which the cross stands against, gives the altitude.

But the great trouble in the use of this instrument, joined to other inconveniencies attending it, make it now of little note, especially since the use of a lately invented instrument is become more general; which far surpasses all others for the taking of altitudes or zenith distances of celestial objects at sea, not only for the accuracy with which it shews the altitude, but also for the readiness and ease with which they are taken; whose description and use here follows.

1087. *The description and use of HADLEY'S QUADRANT.* Pl. XI. Fig. II.

This instrument consists of the following particulars.

I. An octant, or $\frac{1}{8}$ part of a circle, ABC.

II. The index D.

III. The speculum E.

IV. Two horizon glasses, F and G.

V. Two screens, K, K.

VI. Two sight vanes, H, I.

I. *Of the Octant.*

The octant consists of the two radius's or limbs AB, AC; the arch BC; also of the two braces L, M, which are to strengthen and prevent it from warping. The arch contains only the eighth part of the circumference of a circle, or 45 degrees, but is divided into 90 primary divisions, each of which stands for degrees, and are numbered 0, 10, 20, 30, &c. to 90, beginning at each end of the arch for the convenience of numbering both ways, either for altitudes or zenith distances. Every degree is subdivided into one or two minutes, either by the method of diagonals, or by (another which is much better, and called the) *Vernier's* division, or scale.

II. *Of the Index.*

The index D is a flat rod or bar, moveable round the centre of the instrument; that part of the index which slides over the graduated arch

EC,

BC, has either a sharp edge to cut the diagonal divisions, or has Vernier's scale on it.

From the bottom of the index turns up a piece of brass with a screw in it, serving to fasten the index against any division.

III. *Of the Speculum.*

This is a piece of flat glass, quicksilvered on one side (like common looking glass), and set in a brass box perpendicular to the plane of the instrument, against whose centre stands the middle of the glass, its surface coinciding with the middle line of the index. Now as the index slides along the arch, the position of the speculum is altered, it being fixed to the index.

The use of the speculum, is to receive the rays from the object observed, and reflect them on the horizon glasses.

IV. *Of the Horizon glasses.*

These are small pieces of looking glass placed on one of the limbs with their faces turned obliquely to the speculum, from whence they receive the reflexed rays of observed objects.

The glass F has only its lower part quicksilvered, and set in brass work; the upper part being left transparent to see the horizon thro' it.

In the middle of the glass G is a transparent slit, thro' which the horizon is to be seen.

Both of these glasses are so mounted as to have their positions varied, should the wood work of the instrument be warped.

V. *Of the Screens.*

These are two pieces of coloured glass set in frames, and are to prevent the strength of the sun's rays from hurting the eye during the time of observation: When they are used with the glass F, they are to be where the figure shews them; but when used with the glass G, they are to be set at the hole *r* above G.

VI. *Of the Sight vanes.*

That at H is used with the glass G in back observations: And the sight vane I, which has two holes, is used with the glass F in fore observations.

For with this instrument an observation may be made with the face either turned from the object, and called a back observation; or with the face turned towards the object, and called a fore observation.

1088. *To take the sun's altitude with Hadley's quadrant.*

Put the stem of the screens K, K, into the hole *r*, and according to the faintness or strength of the sun's rays, let neither, or either, or both, of the frames of these glasses be turned close to the face of the limb; turn your back to the sun; with your left hand hold the instrument by the braces L, M, so that it be in a vertical position, and the arch downward; put the eye close to the hole in the vane H; look for the horizon thro'

thro' the transparent slit in the horizon glass G; with the right hand move the index D until the reflected image of the sun is seen in the quicksilvered part of the glass G, and in a line with the horizon; swing your body to and fro, and if the sun's image be observed to brush the horizon, then is the observation well made, and the degrees, reckoned from that end of the arch farthest from your body, will give the sun's altitude at that time; subtracting 16 minutes if the sun's lower edge be used; and adding 16 minutes to the altitude when it is taken by the upper edge: But if taken by the centre, there is no need of this correction.

1089. *To take the altitude of a star by Hadley's quadrant.*

Thro' the vane H, and the transparent slit in the glass G, look directly at the star; at the same time move the index till the image of the horizon behind you, being reflected by the great speculum, is seen in the quicksilvered part of G, and meets the star; and the index will then shew the degrees of altitude.

These methods are both by the back observation; but the altitude of any celestial object may be taken by a fore observation (which is mostly practised if the horizon towards the object be conspicuous): For which, and the methods of rectifying the instrument, the reader is referred to the little book generally given with these quadrants at the shops where they are sold.

The directions already given for the taking of altitudes at sea would be sufficient, were it not for two corrections necessary to be made, before the altitude can be accurately assigned: The first of these arises on account of the observer's eye being raised above the plane of the horizon; and the second on account of the refraction occasioned in small altitudes by the haziness of the atmosphere.

1090. *Of the correction for the horizontal dips.* Pl. XI. Fig. 12.

The eye being raised above the level of the sea, observes an horizon below the level of the true one.

Thus let asf be the surface of the sea, Es the height of the eye above that surface, and AF the observer's horizon: Then the eye looking along Ea for the horizon, it dipping below the true one, by the arc Aa , the apparent altitude aB of an object B must evidently be lessened by the arc Aa : But if the observer has his back towards the object B ; then looking along the line Ef , the apparent horizon now becoming the line fEc , and dipping below the true one by the arc Ff , or Ac ; it is plain the apparent altitude cB is too little, and must be augmented by the arc $Ac = Ff$.

Consequently in the fore observations, the altitude must be lessened by the horizontal dip; and in the back observations the altitude must be increased by the dip; but in zenith distances, add the dips forwards, and subtract them backwards.

1091.

To compute the horizontal dips.

Let c be the earth's centre, E the eye, and the tangent Ef meeting the surface in f , where the sky and water seem to meet; then cfE is a triangle right angled at f , wherein are known $cf = cs = 20899059$ feet, the earth's semidiameter; and $CE = (cs + se)$ the height of the eye added to the earth's radius: Then as $CE : \text{Rad} :: cf : \sin. \angle CEf$, whose complement is the $\angle FEf$, the dip sought.

And in this manner was the table here annexed calculated to the different elevations of the eye therein specified.

1092.

Of the correction caused by refraction.

The haziness, or vapours, in the atmosphere causes the sun and stars to appear higher, or with greater altitudes than what they really have: These refractions have been collected by eminent astronomers; and as far as observations at sea may be affected by the atmosphere, the refractions are inserted in the annexed table. Hence it is evident the apparent altitude is to be lessened by the refraction; but it must be added to the zenith distance.

And because the corrections of altitude may appear at sight, therefore the horizontal dips, and the refractions, are both exhibited in the following table.

1093. *A TABLE, shewing the corrections to be made to altitudes taken at sea, both on account of the height of the eye above the horizon, and of the refraction of the atmosphere.*

Heights of the eye	Correc- tions	Altitud. observ'd	Correc- tions	Altitud. observ'd	Correc- tions
Feet	Minutes	Degrees	Minutes	Degrees	Minutes
5	2	1	23	12	4
10	3	2	$17\frac{1}{2}$	15	$3\frac{1}{2}$
15	4	3	14	20	$2\frac{1}{2}$
20	5	4	11	25	2
25	$5\frac{1}{2}$	5	9	30	$1\frac{1}{2}$
30	6	6	8	35	$1\frac{1}{3}$
35	$6\frac{1}{2}$	7	7	40	1
40	7	8	6	50	$0\frac{3}{4}$
45	$7\frac{1}{2}$	9	$5\frac{1}{2}$	60	$0\frac{1}{2}$
50	8	10	5	70	$0\frac{1}{3}$

To have the sun's or star's altitude, or zenith distance, as truly as the instruments will take them, the observation must always be corrected by these numbers, whether it is made by Davis's or Hadley's quadrants, or by any other instrument.

General

General rules for using the Table of corrections.

1094. *In the fore observations.*

Add the sum of the corrections to the observed zenith distance, for the true zenith distance.

Or, Take the sum of the corrections from the observed altitude, for the true altitude.

1095. *In the back observations.*

For altitudes, add the dips and subtract the refractions.

And for zenith distances, subtract the dips and add the refractions.

Also, when the edge of the sun is observed by Hadley's quadrant.

1096. *In the fore observation.*

To the observed altitude add 16 minutes, if the sun's lower edge is brought to the horizon: Or, subtract 16 minutes, when the horizon and upper edge coincide.

1097. *In the back observation.*

To the observed altitude add 16 minutes, if the sun's apparent upper edge comes to the horizon: Or, subtract 16 minutes, if its lower edge is brought in a line with the horizon.

EXAM. I. *By a back observation, the altitude of the sun's lower edge was found by Hadley's quadrant to be $25^{\circ} 12'$; the eye being 30 feet above the horizon: Required the correct altitude?*

The dip on 30 feet high is 6 min.; and the refrac. on 25° is 2 min.

Alt. $\odot$ lower edge	$25^{\circ} 12'$	Ap. alt. $\odot$	$24^{\circ} 56'$	From	$25^{\circ} 02'$
Subtract semidiam.	0 16	Add dip	0 06	Take refrac.	0 02
App. alt. $\odot$ centre	$24 56$	Sum	$25 02$	Cor. alt.	$25 00$

And in like manner must all the observations of the sun's altitude be corrected, before the latitude is found.

EXAM. II. *By a fore observation, the altitude of the sun's lower edge was found by Hadley's quadrant to be $30^{\circ} 20'$; the eye being 20 feet above the horizon: Required the correct altitude?*

Alt. $\odot$ lower edge	$30^{\circ} 20'$	Ap. alt. $\odot$	$30^{\circ} 36'$	From	$30^{\circ} 31'$
Add semidiam.	0 16	Take dip	0 05	Take refrac.	0 01
App. alt. $\odot$ centre	$30 36$	Remains	$30 31$	Cor. alt.	$30 30$

1098.

To make an artificial Horizon.

One great inconvenience that mariners have to struggle with at sea, is the frequent want of an horizon: For tho' the atmosphere may, at the height of 10 or 12 degrees and upwards, be clear enough to give a view of the sun or other objects, yet all below that height is oft so hazy as to hinder a distinct sight of the horizon; and consequently an observation made at such a time cannot have the correctness wished for: But this defect is quite removed by a kind of horizontal speculum invented by Mr. *Serfon*, who was lost with the *Victory* man of war, wherein he was sent to make trial of his machine.

The principle whereon the ingenious author founded this instrument, was derived from the consideration of a top while spinning; for he observed, that the top had a very considerable degree of steadiness in, and force to acquire an upright motion; whether the body which supported it was in motion or at rest, in an horizontal or an inclined situation: He therefore judged, that if a circular machine, whose upper surface was a flat polished speculum, was to have a swift circular motion communicated to it, that speculum, by acquiring an horizontal situation, would shew all objects which it reflected, as much below the horizon, as they really are above it, in the same manner as in a common looking glass; consequently if the image of the sun, as seen reflected from the speculum, was made to coincide with the sun's image, seen in a Hadley's quadrant, the angle given by the quadrant would in all cases be the double of the real altitude. Mr. *Serfon* also found, that to confine the speculum to one place while it was spinning, it was necessary to let the point run in a hollow cup; for the horizontality of the speculum would not be altered, whatever position might be given to the cup, provided it touched the *Top* only at the point it spun on.

1099. This curious and useful instrument, as it is now improved by Mr. *John Smeaton*, F. R. S. consists in a well-polished metal speculum of about $3\frac{1}{2}$ inches in diameter, inclosed within a circular rim of brass; so fitted, that the centre of gravity of the whole shall fall near the point whereon it spins; that is, the end of a steel axis running thro' the centre of the speculum, above which it finishes in a square, for the convenience of fitting a roller to it, when it is to be set a going by the means of a piece of tape wound round the roller. The cup in which it spins is made of polished agate, flint, or other hard substance, and is placed on a small pillar, which is fixed to the bottom of a box, whose pyramidal cover is composed of glass planes; whereby an observation can be made on the speculum as well covered as uncovered, and thereby it will be prevented from tarnishing by the moist air or spray of the sea.

1100. When the *Top* is to be used in a meridian observation, it is convenient to know about what time it is proper to set it up; this may be had near enough by taking the sun's bearing from the meridian with an azimuth compass, allowing for the variation; and if it has about 5 degrees to

to run before it culminates, it is time to spin the *Top*, which will generally run 12 or 15 minutes fit for observation: But if it is likely to go down, or spin too weak, before the observation can be finished, the tape should in the mean time be got ready wound about the roller, that the *top* may be set up again without loss of time; and if the box be held steady upon something nearly level when the tape is drawn away, the speculum will be fit for observation commonly in less than two minutes.

As soon as the *Top* is set up, put on the glass roof if necessary; let an assistant hold the box as steady and level as he can, and in such a position as the observer may look directly thro' the glass roof: Then let the observer place himself as near the box as he can conveniently, and look down on the sun's image in the horizontal speculum, and bring the sun's image seen in the quadrant to agree with it, so that their centres coincide, and the quadrant will give the double altitude, without any allowance for the height of the ship, or the sun's semidiameter. And altho' there may appear in the horizontal image a kind of trembling, yet when the centres of the solar images are brought to coincide, the observation may be taken as truly as if there was no such trembling.

1101. When the sun is above 45 degrees high, the observer must look thro' that sight of the quadrant which is used for a back observation: But he must look down on the horizontal image, or that in the top, as if it was the back horizon; and then making the solar images to agree, the quadrant, according as it is numbered, will give the double altitude, or double zenith distance.

As this latter observation is sometimes found at first more difficult to make than the former, the observer would do well to exercise himself therein in good weather, when he has an opportunity of comparing his observations with those taken from the horizon properly corrected.

These *speculum tops* are as useful by night as by day; for as the images of the smallest stars may be seen in the speculum, consequently any object that can be seen reflected from the glasses of the quadrant, may be observed by the *top*; and these are all the stars of the first magnitude, the planets Venus, Mars, Jupiter, Saturn, and the moon: So that by having the declinations of these bodies in an Ephemeris, they may be used in observations as well as the sun.

As the great distresses to which ships are sometimes drove in several parts of the world for want of an horizon to observe by, are by this most ingenious contrivance quite removed. It is to be hoped, that when the use of this instrument is more generally known, few ships will be without one, altho' the expence should amount to five guineas.

SECTION VIII.

1102.

To find the Latitude by observation.

Find the meridian altitude or zenith distance of the sun or star by article 1084 or 1088.

Correct the altitude or zen. dist. as shewn in arts. 1094, 95, 96, 97.

Seek the sun's or star's declination in the Tables at art. 1052, 1054; observing to correct the sun's decl. for places more than 12° to the E. or W. of London, as at art. 963.

In latitudes near the poles, when the sun's declination exceeds the co-latitude, the sun then does not set; and therefore two altitudes may be taken on the meridian; the least of which is called the altitude below the pole: And the same may be understood of any other celestial object, which does not set.

Let the declination and the altitude, or zenith distance, be always named either north or south, according to the season of the year, and the situation of the object; and when it so happens that they are of no magnitude, they may then be named either north or south at pleasure.

These remarks being premised, all the varieties of working an observation for the latitude are contained in the solution to the following problem,

P R O B L E M.

The declination of the sun or a star, and also its meridian altitude, or zenith distance being known; to find the latitude of the place of observation.

WHEN THE OBJECT RISES AND SETS.

1103. CASE I. *If the zen. dist. and decl. have the same name.*

RULE. Their difference shews the latitude required.

Like the declination, when it is greater than the zenith distance: But unlike, when it is less than the zenith distance.

1104. CASE II. *If the zen. dist. and decl. have contrary names.*

RULE. Their sum shews the latitude required.

And is always like the declination.

When altitudes are given, take them from 90° for zenith distances.

1105.

WHEN THE OBJECT DOES NOT SET.

If the greatest meridian altitude or zenith distance can be taken, the preceding rules will serve: But when the least, or that below the pole, can only be taken. Then

RULE. To the meridian altitude below the pole, add the co-declination, the sum shews the latitude, of the same name with the declination.

The demonstrations of these rules will be shewn after the examples.

EXAMPLES

EXAM.

EXAM. I. *The sun's meridional zenith distance was observed to be $48^{\circ} 20'$ southerly, when he entered the sign Aries: Required the lat. of the place?*

The zenith distance $48^{\circ} 20' S.$
Declination $00^{\circ} 00' S.$

Latitude sought $48^{\circ} 20' N. (1103)$

EXAM. III. *Being at sea, the sun was observed in the zenith, when his declination was $22^{\circ} 10' N.$: What is the latitude of that place?*

The zenith distance $00^{\circ} 00' N.$
Declination $22^{\circ} 10' N.$

Lat. sought $22^{\circ} 10' N. (1103)$

EXAM. V. *The sun was observed to be $47^{\circ} 25'$ distant from the zenith in the south, when his declination was $14^{\circ} 18' S.$: What is the lat. of the place of observation?*

The zenith distance $47^{\circ} 25' S.$
Declination $14^{\circ} 18' S.$

Lat. sought $33^{\circ} 07' N. (1103)$

EXAM. VII. *December 12th, 1753, the sun's meridional zenith distance was observed to be $8^{\circ} 25' S.$: Required the latitude of the place?*

The zenith distance $8^{\circ} 25' S.$
Declination $23^{\circ} 07' S.$

Lat. sought $14^{\circ} 42' S. (1103)$

EXAM. IX. *August 25th, 1753, the sun's meridional zenith distance was observed to be $32^{\circ} 57' S.$: What is the latitude of the place?*

The zenith distance $32^{\circ} 57' S.$
Declination $10^{\circ} 51' N.$

Lat. sought $43^{\circ} 48' N. (1104)$

EXAM. II. *When the sun entered the sign Libra, his meridional altitude was observed to be $53^{\circ} 32'$ in the north: What is the latitude of that place?*

The co-alt. or zen. dif. $36^{\circ} 28' N.$
Declination $00^{\circ} 00' S.$

Lat. sought $36^{\circ} 28' S. (1104)$

EXAM. IV. *The sun's meridional zenith distance was observed $21^{\circ} 08' S.$, when his decl. was the same: Required the latitude of the place of observation?*

The zenith distance $21^{\circ} 08' S.$
Declination $21^{\circ} 08' S.$

Lat. sought $00^{\circ} 00' S. (1103)$

EXAM. VI. *The sun's meridional altitude was observed to be $54^{\circ} 27'$ north, when his declination was $19^{\circ} 57'$ north: Required the latitude of the place?*

The co-alt. or zen. dif. $35^{\circ} 33' N.$
Declination $19^{\circ} 57' N.$

Lat. sought $15^{\circ} 36' S. (1103)$

EXAM. VIII. *June 12th, 1754, observed the sun's meridional altitude to be $79^{\circ} 23' N.$: What is the latitude of the place of observation?*

The co-alt. or zen. dif. $10^{\circ} 37' N.$
Declination $23^{\circ} 13' N.$

Lat. sought $12^{\circ} 36' N. (1103)$

EXAM. X. *Suppose the meridional altitude of the sun was observed to be $48^{\circ} 14' N.$, when his declination was $17^{\circ} 38' S.$: Req. the lat. of that place?*

The co-alt. or zen. dif. $41^{\circ} 46' N.$
Declination $17^{\circ} 38' S.$

Lat. sought $59^{\circ} 24' S. (1104)$

In the same manner, the latitude may be found by observations of the altitude or zenith distance of any known star, or such whose declination is known; for which see the table at art. 1054.

EXAM.

EXAM. XI. Observed the meridian zenith distance of Sirius to be $36^{\circ} 25' N.$: Required the latitude of the place of observation?

The zenith distance	$36^{\circ} 25' N.$
Declination	$16 \ 21 \ S.$
Lat. sought	$52 \ 46 \ S.$

EXAM. XIII. June 12th, 1753, the sun was observed on the meridian, when his altitude below the pole was $8^{\circ} 45' N.$: What is the latitude of the place of observation?

The altitude	$8^{\circ} 45' N.$
Co-declin.	$66 \ 50 \ N.$
Lat. sought	$75 \ 35 \ N.$

EXAM. XII. The meridian altitude of the star Arcturus was observed to be $68^{\circ} 39' S.$: What is the latitude of the place of observation?

The co-alt. or zen. dist.	$21^{\circ} 21' S.$
Declination	$20 \ 38 \ N.$
Lat. sought	$41 \ 59 \ N.$

EXAM. XIV. December 18th, 1754, the sun's meridian altitude below the pole was found to be $10^{\circ} 25' south$: Required the latitude of the place of observation?

The altitude	$10^{\circ} 25' S.$
Co-declination	$66 \ 34 \ S.$
Latitude sought	$76 \ 59 \ S.$

1106.

Demonstration of the preceding rules.

See Plate XI. Figures 13, 14, 15.

Let the circle HZRN represent the meridian of the place, HR the horizon, and z its zenith or pole: Also let PS represent the axis, P being the north, and s the south pole; the line EQ the equator, O the place of the sun, the arc ZO the zenith distance, and EO the declination.

Now the latitude is expressed either by the height of the pole above the horizon, or by the distance of the equator from the zenith; consequently the arc $RP = HS$, or the arc ZE, being known, will give the latitude.

But the arc ZE is ever equal to the sum of the arcs ZO and EO, when of contrary names; or to their difference when of the same name.

When the object does not set, then its least altitude RO added to the arc PO, or co-declination, gives RP, the height of the pole above the horizon, or the latitude sought: And the same may be understood, was the least altitude above the south point of the horizon.

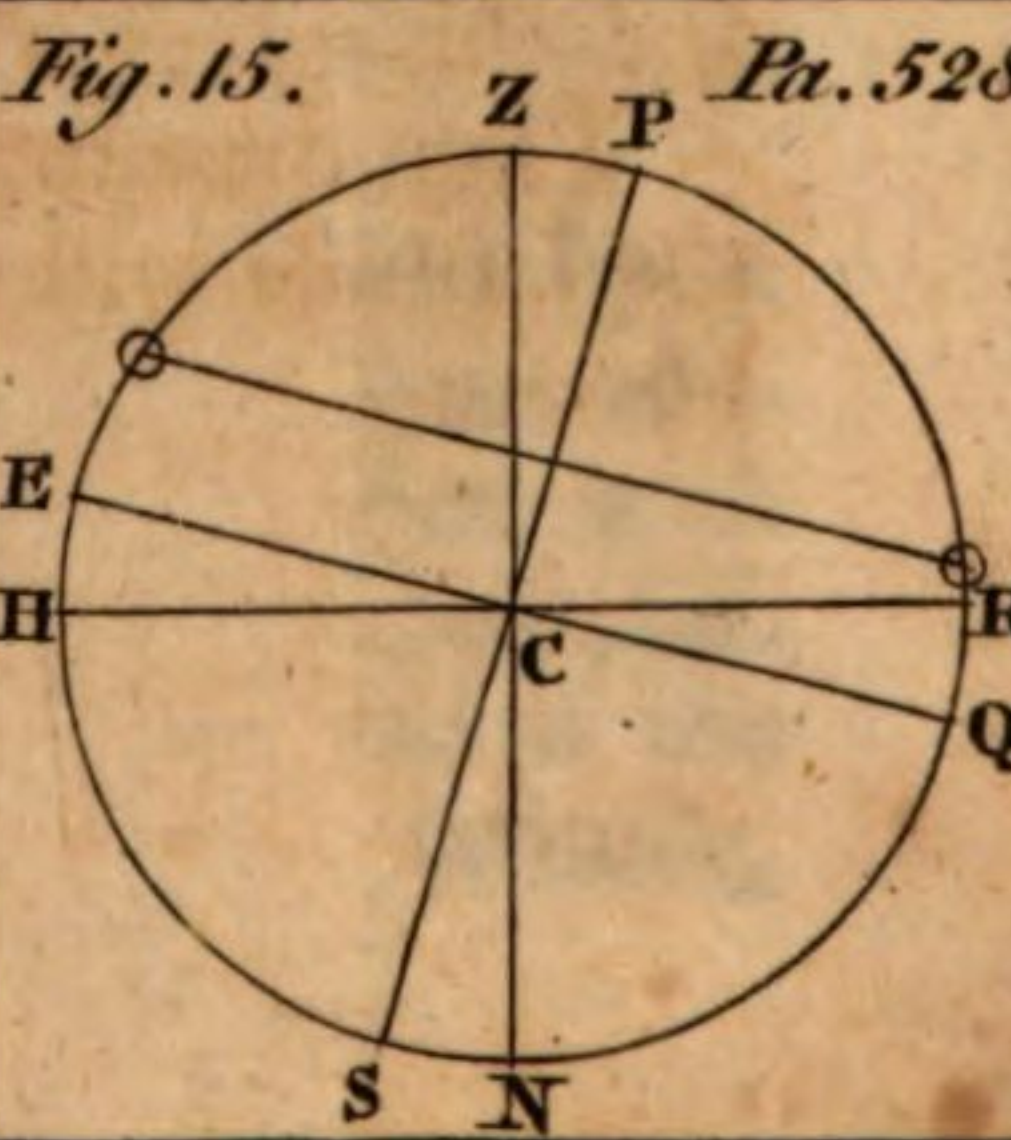
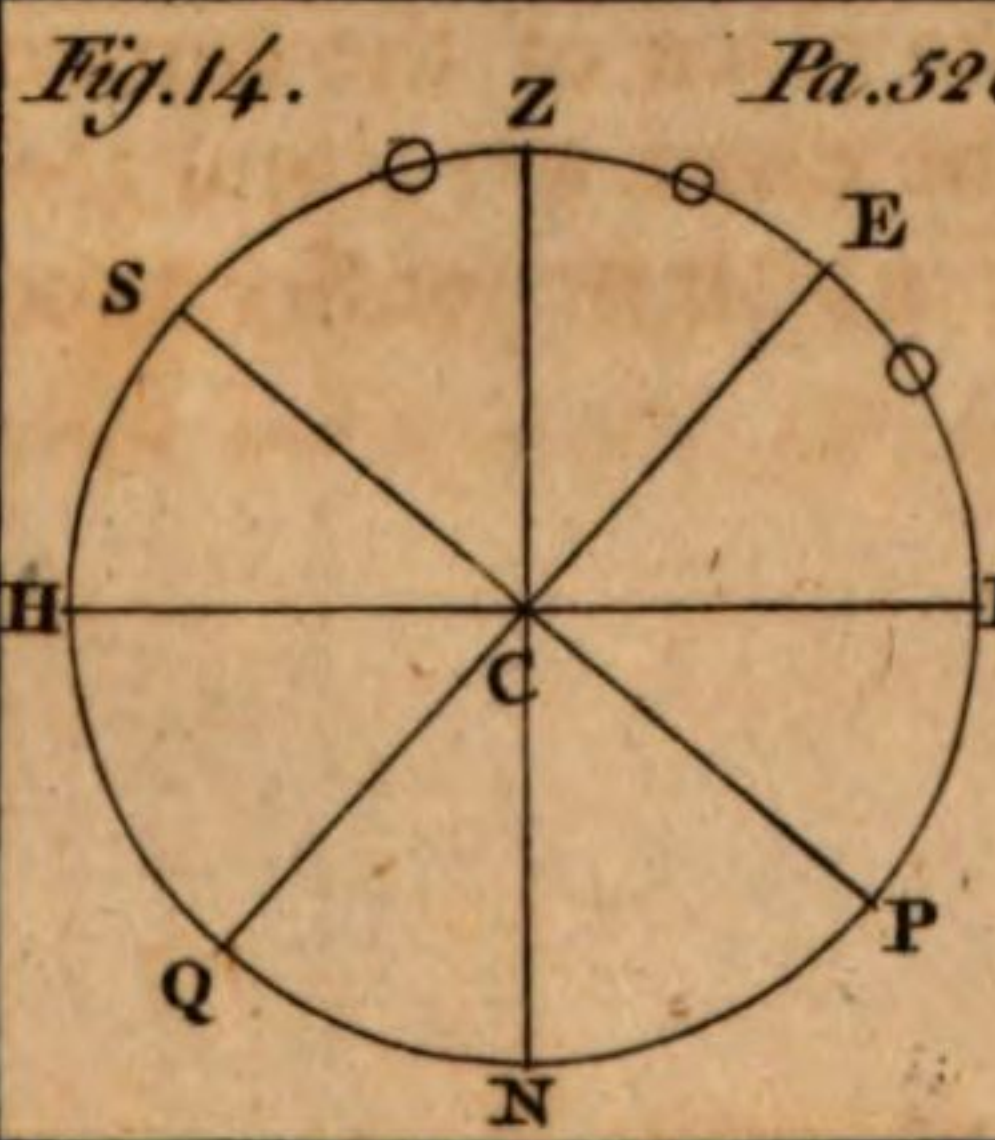
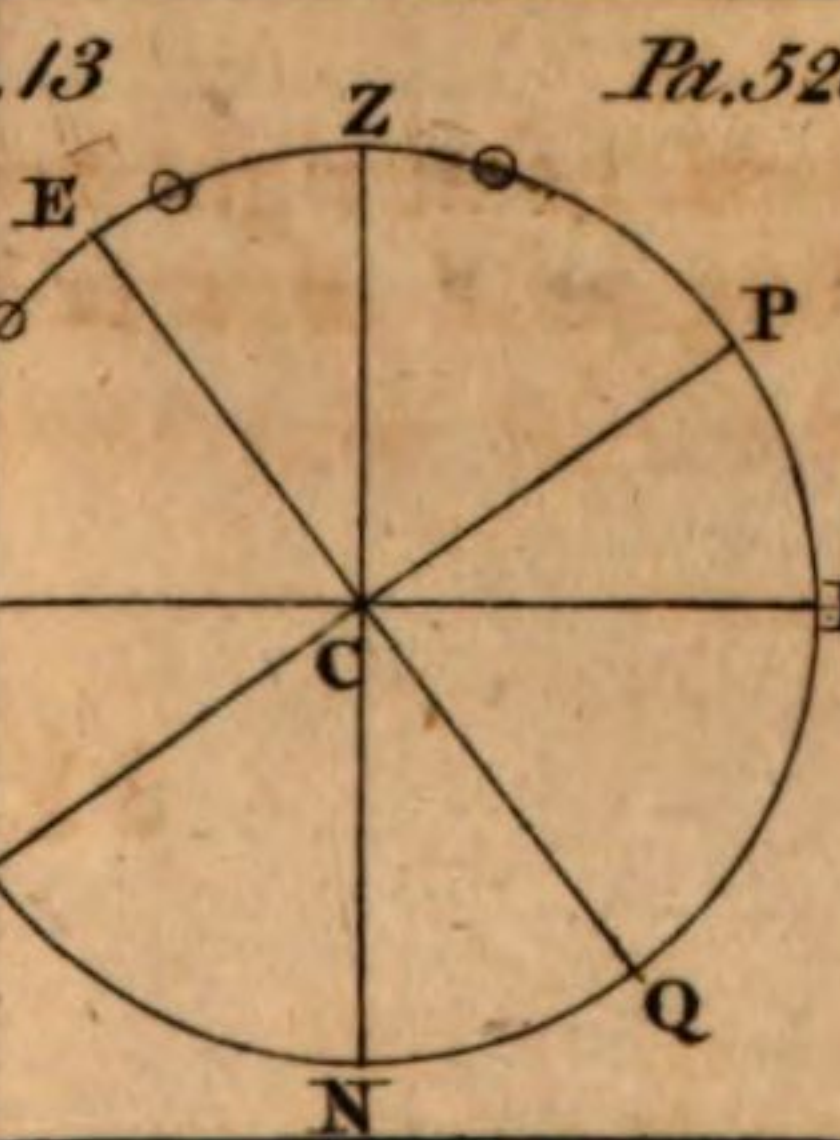
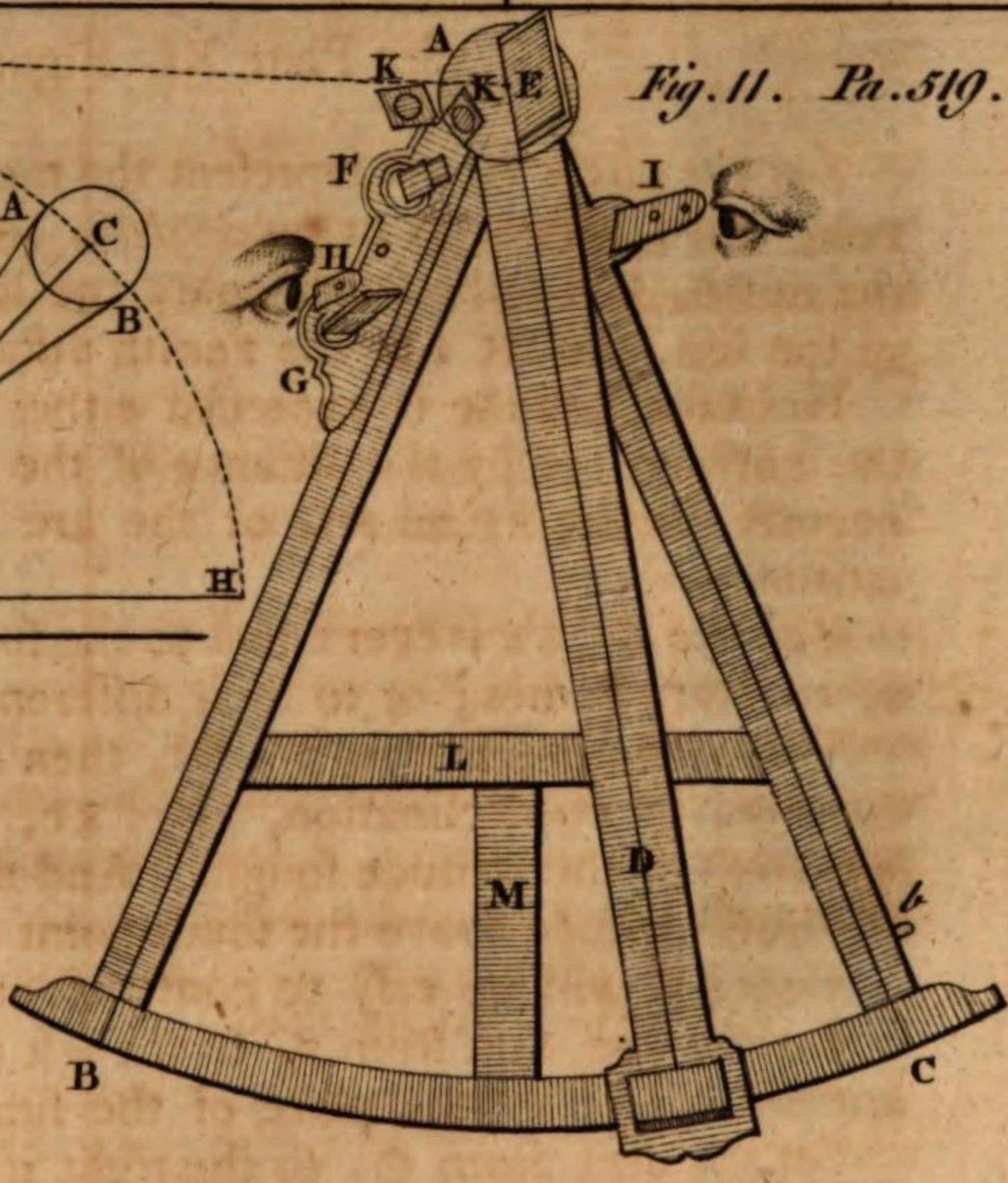
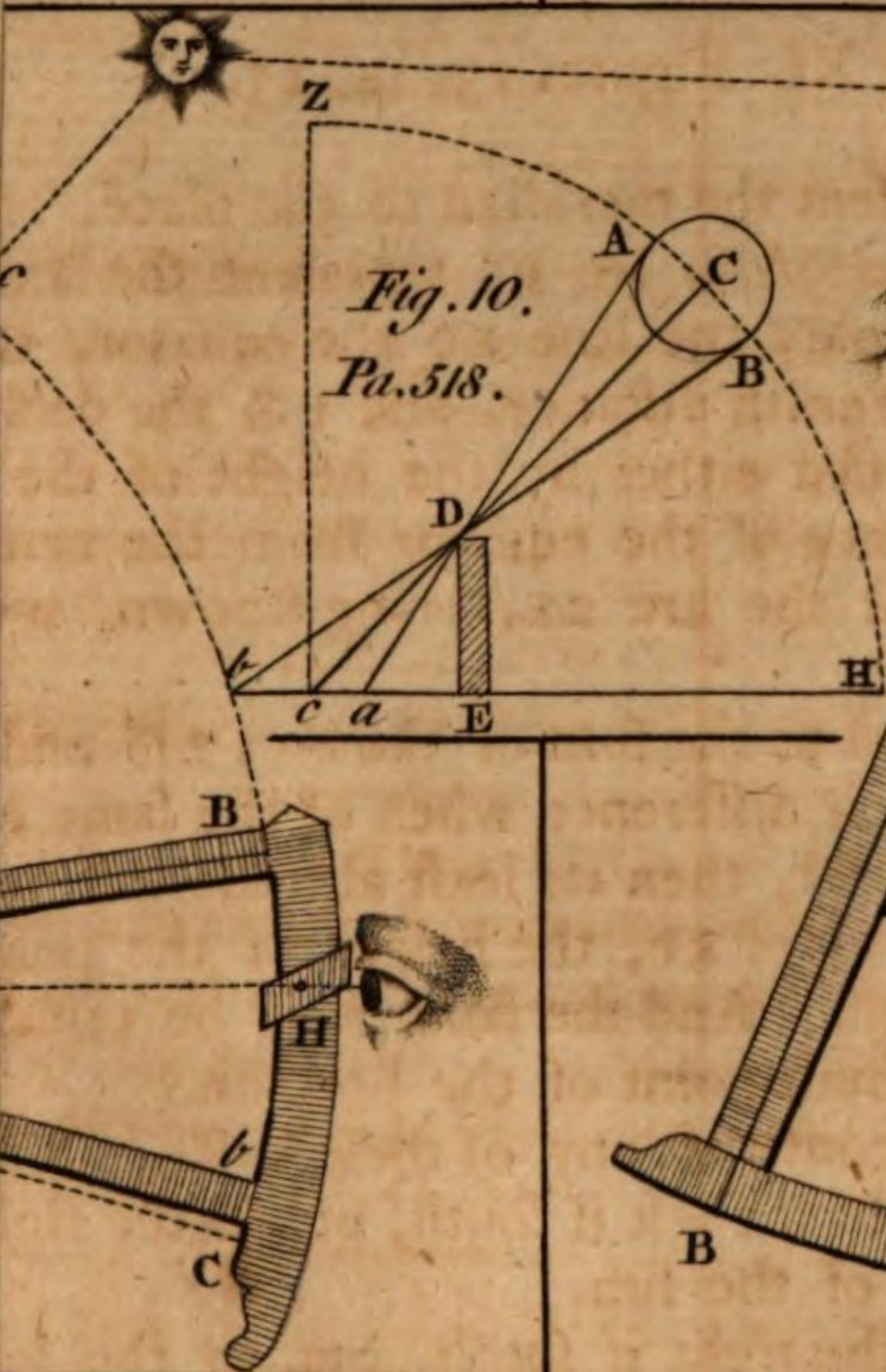
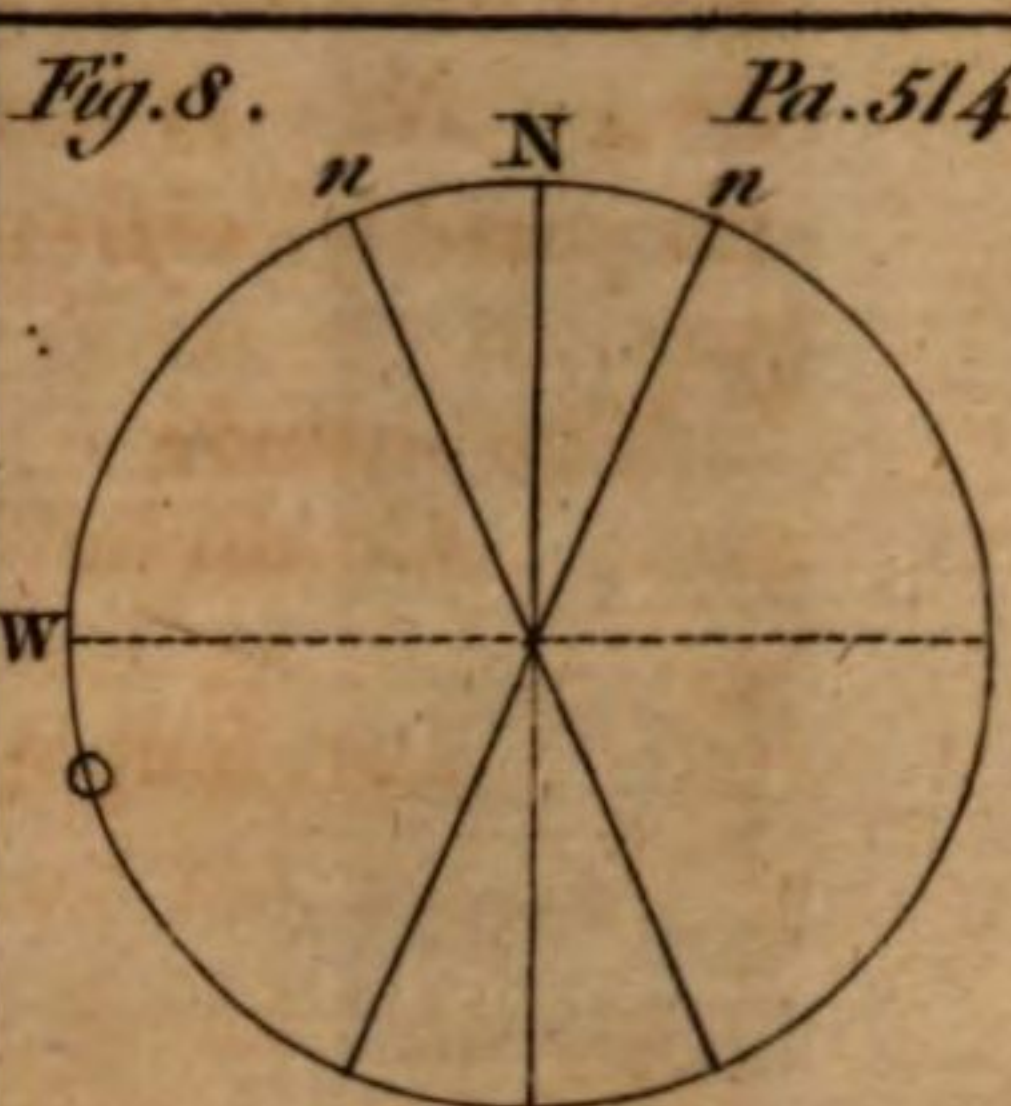
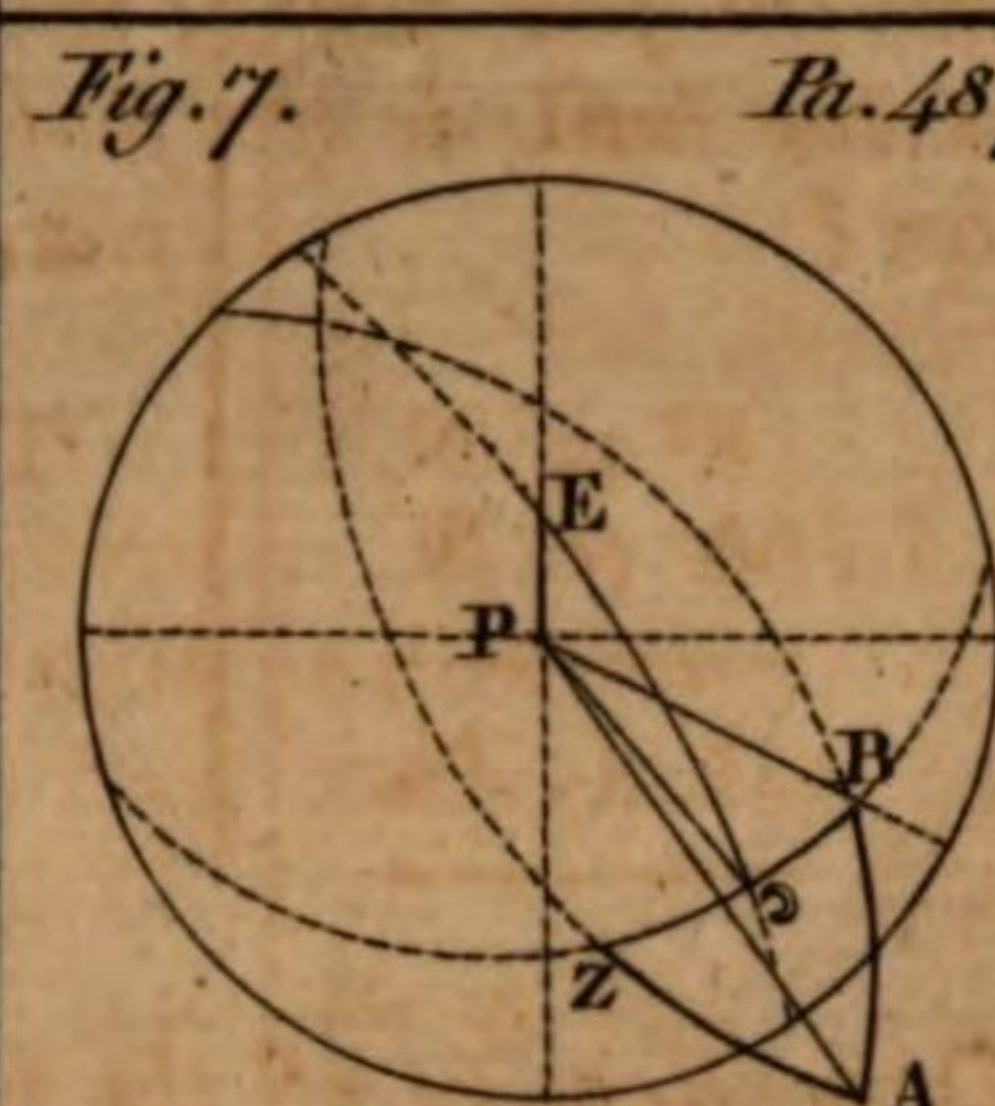
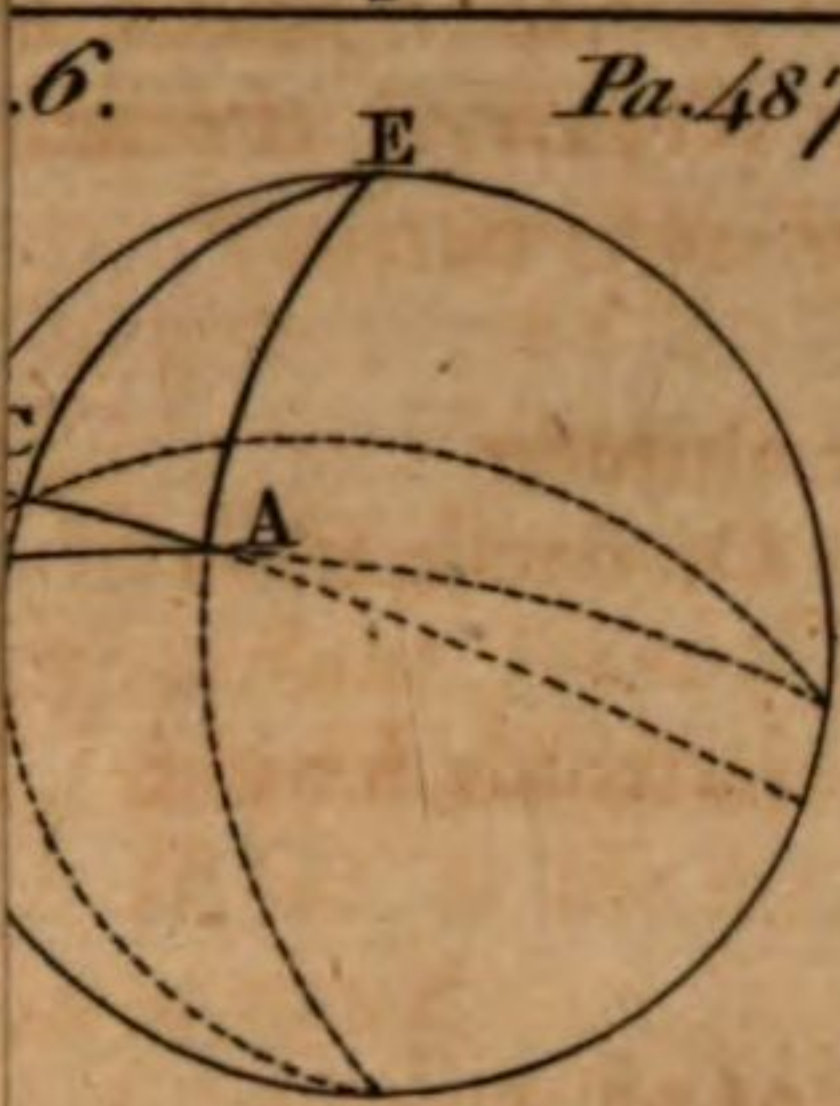
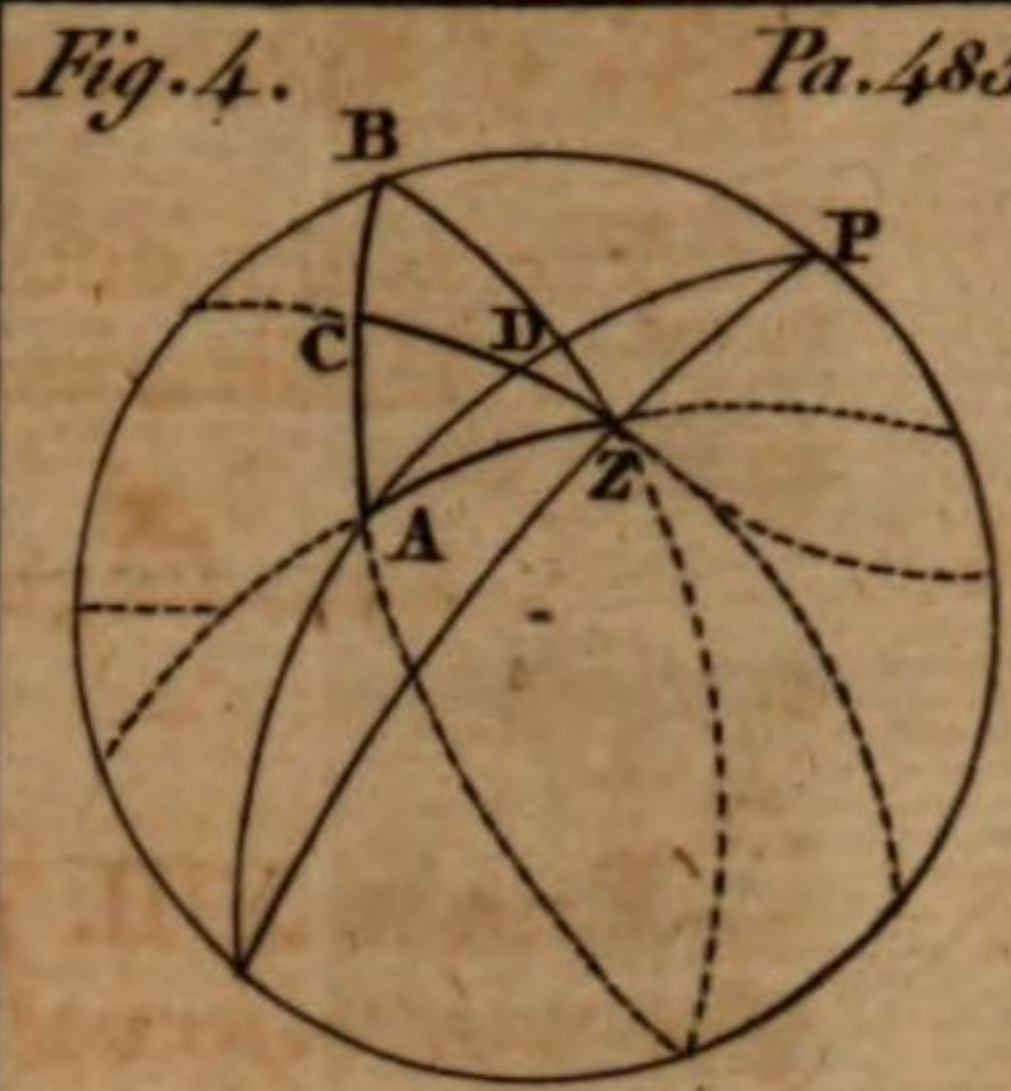
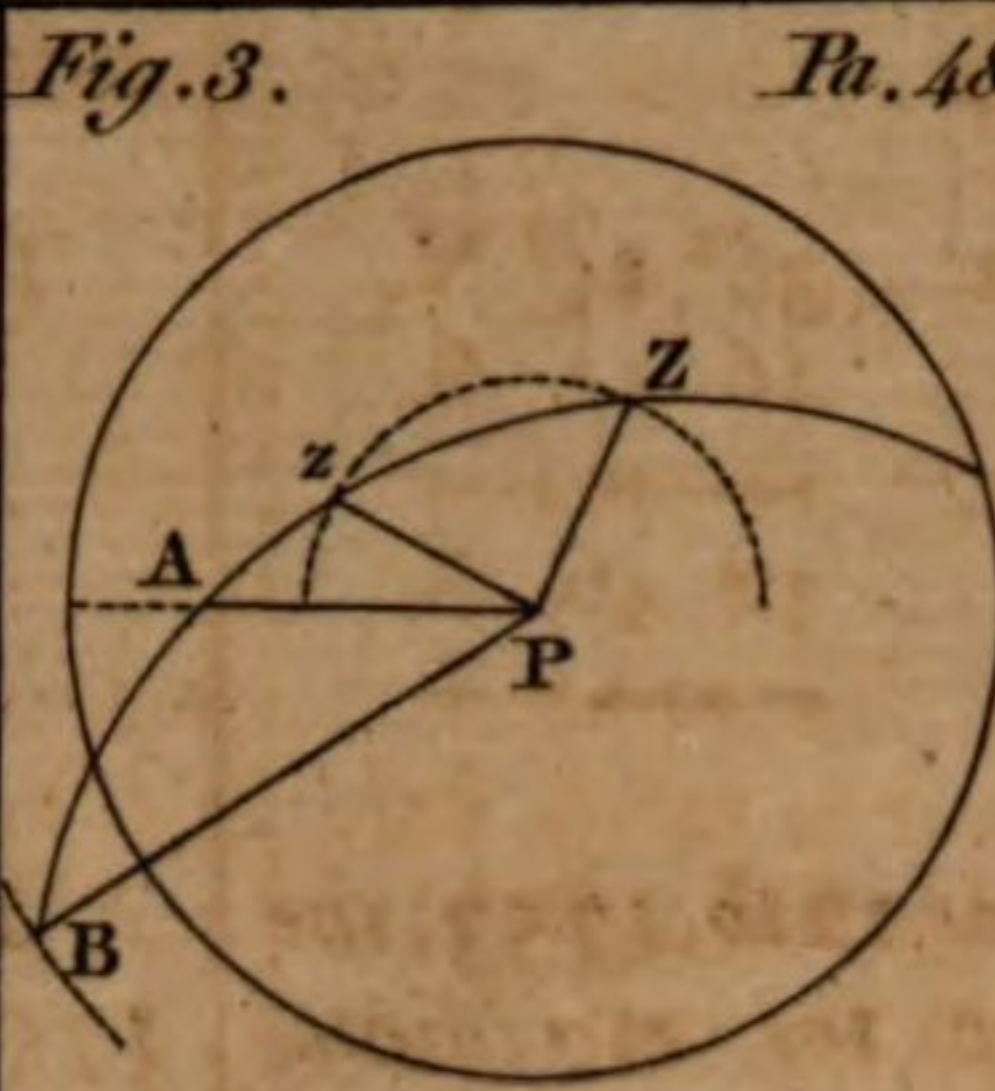
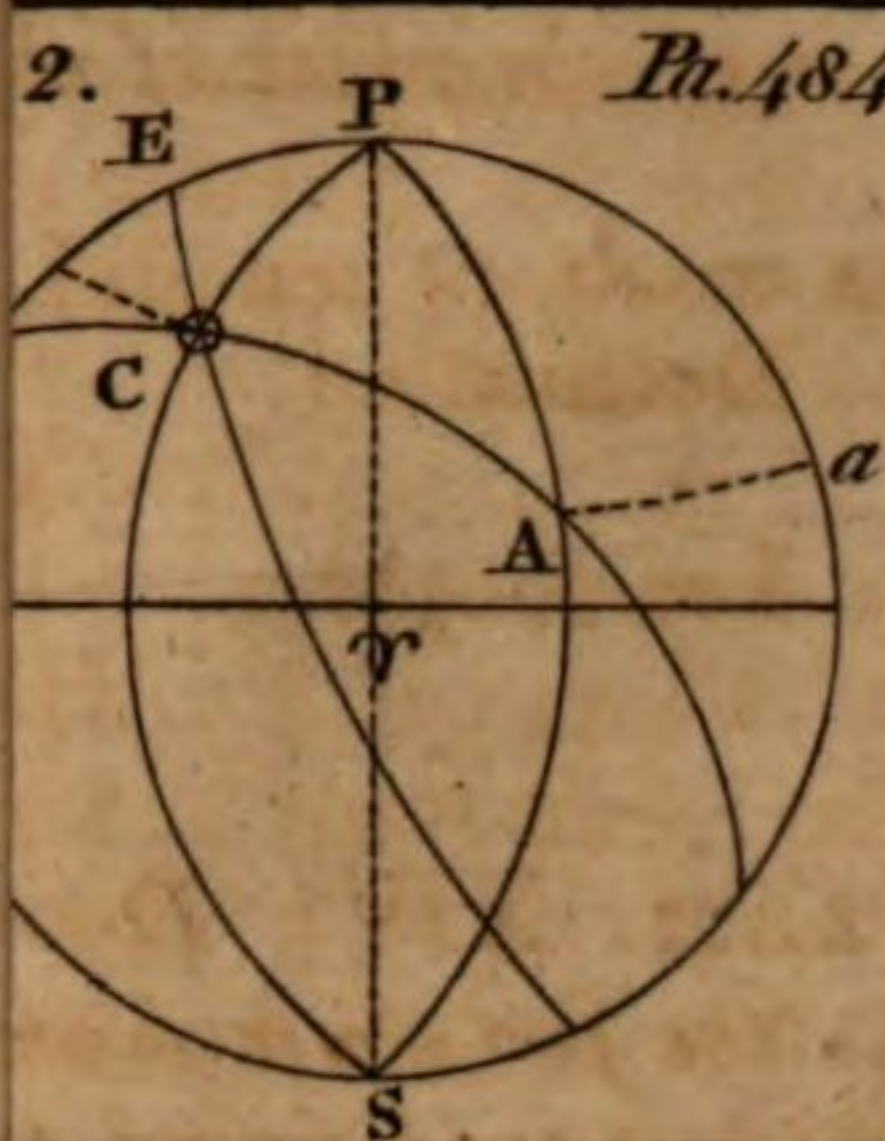
Hence it will be easy to construct any of these cases. Thus:

Set the zen. dist. from z, to the left if south, but to the right if north; and this gives O the place of the sun.

Set the decl. from O, to the right if south, but to the left if north; and this gives E the place of the equator.

Then drawing the diam. EQ, and the diam. PS at right angles to EQ, the latitude will be north or south, according as P or s falls above the horizon.

SECTION



SECTION IX.

Of a Ship's Reckoning.

1106. A SHIP'S RECKONING is that account, whereby at any time it may be known where the ship is, and on what course or courses she is to steer to gain her port.

DEAD RECKONING, is that account deduced from the occurrences wrote on the log-board.

The LOG-BOARD is a painted black board, whereon is wrote in chalk whatever is thought worthy of note from day to day; and for a proper disposition of such notes, the board is divided into five columns: The left hand one contains the 24 hours from the noon of one day to the noon of the next; in the 2d and 3d columns are the knots and fathoms the ship is found to run per hour, set against the hours when the log was hove; the 4th column contains the courses the ship steers; and in the 5th, or right hand column, is wrote the winds, the alteration of the sails, the business doing aboard, and what other remarks the officer of the watch thinks proper to insert: For it should be observed, that it is usual to divide a ship's company into two parts, called the starboard and larboard watches, who alternately do the duty of the ship for four hours and four hours.

1107. The LOG-BOOK is a book ruled like the log-board, in order to contain the daily copies of the remarks wrote on the log-board, which is the only authentic record of the ship's transactions; and these are by the persons who keep *Journals* transcribed every day at noon into their log-books, from whence they make the necessary deductions relative to the ship's place.

1108. But as the ship's motion is liable to be disturbed from a variety of causes, such as a continual deflection from the course set, by the ship's playing to the right and left round her centre of gravity, and the unequal care of the *helmsmen*; the distance being faulty, on account of tumbling seas rolling with or against the ship; the unsteadiness of the wind; unknown currents; sudden storms, and many other impediments which seem impossible to be surmounted; consequently her place, according to the dead reckoning, may be justly doubted, and therefore mariners try every day to find the latitude their ship is in by observations on the sun or stars; and if the latitude obtained by observation, and that found by dead reckoning, agree, it is presumed the ship's place is well determined: But if they disagree, the account of longitude must be corrected; and for the latitude, that found by observation is always to be depended on.

1109. *Various methods of correcting the Reckoning of longitude.*

Consider whether the difference may not have been occasioned by a current, and if it is possible, try it (by art. 706); or make such an estimate of its setting and drift as may be judged reasonable: Then with the setting and drift, as a course and distance, find the diff. lat. and diff. long., with which increase or diminish the dead reckoning; and if the

Y y y

latitude,

latitude, thus corrected, agrees with the latitude by observation, then the longitude thus corrected may be safely taken as true, and so the ship's place determined.

EXAM. Suppose a ship in 24 hours finds by her dead reckoning she has made 96 miles of diff. lat. N., and 38 miles of departure west; but by observation, finds her difference of latitude is 112, and on trial finds a current, which in 24 hours make 16 miles of diff. lat. N., and 10 miles of dep. E.: Required the ship's departure?

Diff. lat. by account	96 m. N.	Departure by account	38 m. W.
Diff. lat. by current	16 m. N.	Departure by current	10 m. E.
True diff. lat.	112 m. N.	True departure	28 m. W.

Here the dead reckoning corrected by the current gives the diff. lat. 112 miles, which is the same as found by observation; therefore the departure 28 is taken as the true one.

1110. The error in the dead reckoning is usually attributed to unknown currents: For by various causes, yet indetermined, there are many counter-motions of the water in the open seas, as well as those observed near the shores where the motions may be tolerably well accounted for. Some of the observed currents in the great seas may perhaps be owing to the tide's following the moon, and to the libratory motion the waters may have thereby; and the unsettled setting and drift of these currents may possibly depend on the change in the moon's declination: However, it is well known from observations that the trade winds occasion a considerable current within their limits, particularly within the Torrid zone, where the motion is perpetually towards the west, at the rate of about 8 or 10 miles a day: But at the extremities of the trade winds, or near the latitudes of 30 degrees north and south, it is likely that the currents are compounded of the said western motion, and of one towards the equator (by obs. 2d. art. 607): Therefore all ships sailing within these limits should allow a course each day for this current.

1111. If when all currents have been allowed for, there should still be a difference between the observed latitude and that given by account; then a farther correction must be made.

The business of correcting a sea reckoning is a very precarious operation, and at best is little more than guessing; for by ought that appears, the ship may be either to the eastward or westward of the point wherein the reckoning places her, and the mariner not be able to pronounce with certainty whether: However, the following methods are usually taken.

If the difference of latitude is much more than the departure, or the direct course has been within three points of the meridian; then it is most likely the error is in the distance.

And if the departure is much greater than the diff. lat., or the direct course is within three points of the parallel, or more than five points from the meridian, the error may be ascribed to the course.

But if the courses are in general near the middle of the quadrant, the error may be either in the course, or in the distance, or in both.

For,

For, to cause an alteration in the diff. lat., the first of these cases requires a greater error in the course than can well be supposed to have been committed; in the second case, the distances must be so faulty as would scarce escape observation; and in the last, it is often doubtful whether to attribute the error to the course or distance; but it is usually corrected in both.

1112.

FIRST METHOD.

When, by the dead reckoning, the diff. lat. is more than once and a half the departure: Or, which is the same, when the course is less than three points.

To the diff. lat. and the departure by account, find the course.

With this course, and the diff. lat. by observation, find a new depart.

With the new depart., and the co-mid. lat., find the diff. long.

EXAM. Yesterday noon we were in lat. $39^{\circ} 18' N.$; and by an observation, are this day noon in lat. $37^{\circ} 48' N.$; our dead reckoning gives 106 miles of southing, and 64 miles of easting: Required the ship's true difference of lon.?

BY THE TRAVERSE TABLE.

The diff. lat. 106, and the dep. 64, gives the course $2\frac{3}{4}$ points.

The course $2\frac{3}{4}$ points, and the diff. lat. by observ. 90, gives a dep. 54.

The co-mid. lat. $51^{\circ} 27'$, and true dep. 54, gives the diff. long. 69.

1113.

SECOND METHOD.

When, by the dead reckoning, the departure is more than once and a half the diff. lat.: Or, the course is more than five points.

With the diff. lat. and dep. by account, find the distance.

With this distance, and the diff. lat. by observ. find the true depart.

With the co-mid. lat. and true dep. find the diff. longitude.

EXAM. Yesterday noon we were in lat. $48^{\circ} 52' N.$, and were to day at noon in lat. $50^{\circ} 18' N.$; our dead reckoning shews we have made 68 miles of northing and 112 miles of westing: Required the true difference of longitude the ship has made?

BY THE TRAVERSE TABLE.

The diff. lat. 68 and dep. 112, gives 132 for the distance.

The dist. 132, and diff. lat. by observ. 86, gives the true depart. 100.

The co-mid. lat. $40^{\circ} 25'$, and true dep. 100, gives 155 miles for diff. longitude.

1114.

THIRD METHOD.

When the diff. lat. and departure by account are nearly equal, or the direct course has been between three and five points of the meridian.

To the diff. lat. and depart. by account, find the distance.

With this dist. and diff. lat. by observation, find another departure.

Take the half sum of the two departures for the true one.

With the co-mid. lat. and true dep. find the diff. longitude.

EXAM. These last 24 hours we have made 84 miles of northing, and 76 of easting; we were yesterday noon in lat. $52^{\circ} 40' N.$, and are this noon in lat. $54^{\circ} 22' N.$: What diff. longitude has the ship made?

BY THE TRAVERSE TABLE.

To the diff. lat. 84 and dep. 76, the distance is 113 miles.

With the dist. 113 and diff. lat. by observ. 102, find a dep. 47,7.

The sum of the two departures 76 and 47,7 is 123,7, its half is 61,8.

To the co-mid. lat. $36^{\circ} 29'$ and true dep. 61,8, the diff. long. is 103.

1115. The difficulty of finding the longitude at sea has inclined many persons to attempt the solution of this most useful problem: Some, with no other view than the great advantage which the art of navigation would derive from their labours; others, and by much the greatest number, who have exhibited their thoughts on this subject, were visionary whimsical men, only lured by the hopes of the appointed reward: But Dr. Halley having collected a great multitude of observations made on the variation of the needle in many parts of the world, he by the help of the latitude and longitude of the places of those observations, was enabled to draw on a Mercator chart certain lines, shewing the variation of the compass in all those places over which they passed at the time he published, in the year 1700; and consequently the longitude of any of those places could be found by the chart, having its latitude and the variation of the needle in that place given.

1116.

FOURTH METHOD.

To find the longitude at sea by the variation chart.

Find the variation of the compass. For which see art. 1076.

Draw a parallel of latitude on the chart thro' the lat. found by observation.

And the point where it cuts the curved line, whose variation is the same with that observed, will be the ship's place.

EXAM. *A ship finds by a good observation that she is in the lat. of $18^{\circ} 20'$ N.; and that the variation of the compass is 4° west: Required the ship's place?*

Take from the graduated meridian the distance between the equator and the lat. of $18^{\circ} 20'$ N.: Lay a ruler along the equator, and slide one point of the compasses along its edge till the other point cuts the curve of 4° west variation, and the intersection gives the ship's place, whose longitude will be found to be about $27^{\circ} 10'$ west from London.

1117. There are two considerable inconveniences which attend this method.

First. That wherever the variation lines run east or west, or nearly so, this way of finding the longitude becomes imperfect: But among all the trading parts of the world, this imperfection is at present found chiefly on the west coasts of Europe between the latitudes of 45° and 53° ; on the eastern shores of North America, and in some parts of the western ocean and Hudson's bay, lying between the said shores; therefore, for the other parts of the world, a variation chart may be esteemed as of the greatest use.

1118. But the variation curves, even where they run east and west, may be sometimes applied to good use in correcting the latitude, when
meridian

meridian observations cannot be had, as it frequently happens on the northern coasts of America, in the western ocean, and about Newfoundland: For if the variation can be obtained correctly, then the east and west curve answering to that variation on the chart will shew the latitude.

Secondly. As the deflection of the magnetical meridian from the true one is subject to continual alteration; therefore a chart, to which the variation lines are fitted for any year, must in time become useless, unless new lines, shewing the state of the variation at that time, be drawn on that chart: But as the change in the variation is very slow, therefore new variation charts being published every 10 or 12 years, will answer the purpose wanted.

In the year 1746 Mr. William Mountaine, F. R. S. and Mr. James Dodson fitted the variation lines to Dr. Halley's chart, for the year 1744. As these gentlemen had collected several thousands of observations to work on, they were enabled to perform this business with great correctness; insomuch, that wherever their chart has been used it was found of very great importance, even to the saving a ship in the Indian ocean; and it is much to be wished they were in more general use.

1119.

FIFTH METHOD.

To find the longitude from celestial observations.

It was shewn at article 657, that the difference of longitude between two places could be determined, by knowing the difference between the times that any remarkable appearance in the heavens was seen in those places.

For as these appearances consist in the *appulses*, that is, the approaches of the heavenly bodies to one another, or their passage by one another; and these appulses when they happen, are seen at the same instant of absolute time to all parts of the earth where they are visible: Therefore by knowing the relative times of the day when such appearances are seen in two distant places, the difference between those times is known, and consequently the difference of longitude between those places.

There are every year published several Almanacs, or Ephemerides, wherein are noted the eclipses of the sun and moon, the times of the planets rising, setting, and southing; the eclipses of Jupiter's satellites, and the times when other celestial appearances may be seen at the places for which those Ephemerides are calculated: Now a mariner having one of these Almanacs, if he observes at what time any of these appearances happen in any part of the world he may be then in, will readily know his longitude.

1120. Among the satellites of Jupiter, one or more is eclipsed almost every night; for they disappear either in going behind Jupiter, or in passing before him; and the instants of such immersions and emersions may be seen by a refracting telescope of about 8 or 9 feet long, or by a reflecting one of about 9 inches focal length. The reflecting telescope is much more commodious at sea, on account of its shortness; but the metals are very subject to be tarnished by the moist air.

1121. To determine the longitude of any place from celestial observations, it is necessary that the time of the day be well known in that place

place where the observation is made; and among the various methods proposed for this purpose, none seems more fit to be practised at sea than the method by *equal altitudes*; for no regular deductions can be made without knowing the precise time of noon. Now as clocks and watches undergo considerable alterations from the motion of a ship and the change of the climate, they ought to be carefully examined, and their gain or loss per day ascertained whenever they are to be used in the correcting of the longitude; and this need not be done very often, only at such times when the ship may be supposed thro' storms or bad weather to be considerably out in her dead reckoning.

At such times, the ship lying by, let the sun's altitude be taken at any time in the forenoon, set down that time and altitude: In the afternoon, watch for the sun's having the same altitude exactly, and note down that time; then the half sum of these two times, is the time shewn by the clock or watch when it was noon in that place.

As it is possible that an altitude, equal to one taken on the eastern side of the meridian, cannot be observed on the western side, by the interposition of clouds; it is therefore best to take several in the forenoon, or on the east side, in order to have the greater probability of securing a corresponding altitude on the western side or afternoon; and if several observations of equal altitudes can be made on both sides of the meridian, it will be best to take the mean time corresponding to each pair of equal altitudes; and if the means all agree, it is an evident sign that the observations were well made: But should they disagree half a minute or so; then it is best to add together all the times of noon, and divide the sum by the number of times, and the quotient may be taken for the true time of noon in that place, as shewn by the clock or watch.

In the taking of these equal altitudes of the sun, it will be most convenient to have them at a distance from one another not exceeding 4 or 5 hours; because then there will be no occasion to correct the latter altitude by the alteration in declination during that time.

1122. The true time of noon being obtained, then the time that any planet, or other celestial body passes the meridian, may be thus known: On the evening following, let one or more altitudes of the planet be taken, which set down with their respective times; and on the next morning note down the times when that planet has the same altitudes: The half sum of these two times will be the time when the observed planet passed the meridian of that place; and if several observations have been taken, find the mean of them, as before for the sun: But if the planet observed be the moon, whose declination may be considerably altered between the times of the observations; then let a proportional part of her daily variation in declination be added to, or subtracted from, the time found for her passing the meridian, according as the moon is receding from, or approaching to, the elevated pole.

And if there is reason to suspect that the clock or watch gains or loses every day; let the times of two equal altitudes of the sun be taken next day, and the noon thereby determined: Then the difference between the times of noon both days, as shewn by the clock or watch, will be its daily variation; and the time of the planet's passing the meridian must be increased or diminished by a proportional part of that variation.

1123. Now by having a table or an Ephemeris, containing the times when the moon passes the meridian of a certain place; and by having from observation the time when she passes the meridian of any other place, the difference between the longitudes of those two places may be thus found.

Seek in the Ephemeris for the time of her southing on the given day, and that of the following day, and take their difference; also take the difference between the times of the tabular southing and the observed one, on the same day; then say:

As the daily difference, is to the observed difference;

So is 360 degrees, to the difference of longitude sought.

FOR, As the whole difference arises in a day, or by the running thro' 360°; consequently any part of that difference will require a proportional part of 360 degrees.

EXAM. Suppose at sea, the moon was observed to pass the meridian on the 18th day of June, 1753, at 57 min. after one in the morning: Required the longitude of that place?

At London the moon passes the meridian on the 18th of June, at 1 h. 47 m., and on the 19th day, at 2 h. 31 m.; the difference is 44 min.; and the observed time is 10 min. later than at London.

Then as 44 min. : 10 min. :: 360° : ($\frac{360 \times 10}{44} =$) 81° 49'. Which shews that place to be 81° 49' to the west of the meridian of London.

1124. As the Tables of the moon's motions are not yet arrived at the wished for accuracy, consequently the times of her passing the meridian of any place, as shewn by an Ephemeris, are not so true as they might be, and therefore the diff. long. found by the time of the moon's southing may err about 2 degrees; and should the time of her passing the meridian be ill observed, the error may be much greater: It has therefore been proposed to keep an account of longitude by a time-keeper, or kind of clock, that should always shew the true time under the meridian of some one place; consequently was the time of the day, found in any other place, compared with the time then shewn by such a clock, the difference of longitude between those places would be determined: But the difficulties attending the construction of such an instrument have been hitherto found too great to be overcome. Indeed the ingenious Mr. Harrison has removed some of, if not all, the capital obstructions; and his judicious friends suspect he will intirely perfect his most elaborate machine: He has received some gratuities from the public for the advances he has already made towards the solution of this intricate problem of the longitude; and it is wished he may merit the whole reward allotted by the Government for the discoverer of this so much desired acquisition in navigation.

1125. Upon the whole, it appears that there are many means by which a ship's place may be found at sea, nearly exact enough for navigating her to places whose longitudes are almost as incorrectly known as the ship's place; and was the method of finding the longitude at sea correctly known, the ships must then keep as good a look out for land as they do now, and also keep their account of dead reckoning: Therefore would seamen put in practice all the precepts that are given them for this purpose, they might proceed on their respective voyages, and (which is suspected now to be the case) have very little concern about the perfecting the discovery of the longitude.

SECTION

SECTION X.

1126. *Of a Sea Journal.*

A SEA JOURNAL is a book wherein is truly entered the most remarkable daily occurrences relating to the ship during her voyage outward and homeward.

There are various ways of keeping sea journals, according to the different notions of mariners concerning what articles are to be entered.

Some writers direct the keeping such a kind of journal, which is only an abstract of each day's transactions, specifying the weather, what ships or lands were seen, accidents on board, the latitude, longitude, meridional distance, course and run: These particulars are to be drawn from the ship's log-book, or that kept by the Person himself, for most good mariners keep private log-books.

Other authors recommend the keeping but of one account, including the log-book and all the work of each day, with the deductions drawn therefrom: This method is used in this treatise, on account of representing to the beginner the whole of each day's work; but when he is well versed in this method, he may abridge what part of it he pleases: Altho' it is conceived, that a journal neatly kept, with all the particulars of the work commodiously ranged, would give more satisfaction to those who might have occasion afterwards to inspect it, than a journal containing the heads only, and all the intermediate parts suppressed.

The method here chose to introduce the young mariner into this most capital part of navigation, is, by shewing him first how to work a few separate days works independent one of the other, and then to proceed to a continued journal of several successive days works; for the doing of which it was judged necessary to premise the following observations.

1127. I. That the day is supposed to begin at noon, and the day's work relates to the transactions done in the time between the noon of one day and the noon of the following day.

II. If there is an observation for an amplitude or an azimuth; let that amplitude or azimuth be worked as shewn between art. 1070, and 1076; and then find the variation as shewn at art. 1077; taking care in these operations that the declination of the day be fitted to the proper time, and longitude, as shewn arts. 964, 965.

III. Correct each course by the variation thus found, as at art. 1078; also correct these corrected courses by the proper allowance for leeway, as shewn at arts. 1080, 1081.

IV. Write these corrected courses in a Traverse table, sum up the knots and fathoms belonging to those hours the ship kept on the same course, and write the several sums as distances in the Traverse table against their respective courses.

V. Complete the Traverse table as by art. 695, find the present latitude and longitude as shewn in art. 786, 787, and the following examples, and then will the ship's place be obtained by dead reckoning.

1128. EXAMPLE I. August 25th, 1753. Yesterday noon we were in the latitude of $46^{\circ} 28' N.$, longitude $22^{\circ} 18' W.$, and have sailed till this day noon as by the log-board, the tide having all the time set S. b. E. $2\frac{1}{2}$ miles an hour: Required the ship's place, and the direct course and distance made good?

LOG-BOARD.					TRAVERSE TABLE.					
H.	K.	F.	Courses	Winds	Courses	Dif.	N.	S.	E.	W.
1	6	3	NNE	West	NNE.	31	28,6		11,9	
2	6	2			ENE.	35	13,4		32,3	
3	6	5			E. b. S.	30		7,0	35,3	
4	6	4			SSE.	51		47,1	19,5	
5	6	0			S. b. E.	60		58,8	11,7	
6	6	1	ENE.	NW.			42,0	112,9	110,7	Dep.
7	6	6						42,0		
8	5	8					D. lat.	70,9		
9	5	6								
10	5	4								
11	5	5								
12	5	3	E. b. S.	North						
1	5	9								
2	6	2								
3	6	0								
4	6	3								
5	6	4								
6	7	0	SSE.	NW.						
7	6	8								
8	7	3								
9	7	5								
10	7	1								
11	7	9								
12	7	3								

Depart. lat.	=	$46^{\circ} 28' N.$	M. P.	=	3156
Diff. lat.	=	1 11 S.			
Pref. lat.	=	45 17 N.	M. P.	=	3054
Sum lats.	=	91 45	M. D. lat.	=	102
Middle lat.	=	45 52			
Co-middle lat.	=	44 08			
Dep. longitude	=	22 18 W.			
Difference long.	=	2 39 E.	Or,	$2^{\circ} 40'$	
Present long.	=	19 39 W.			
Direct course	=	S. $57^{\circ} 15'$ E.			
Distance	=	131 miles.			

The courses and winds in the log-board being examined, it appears that the ship goes large, and has no leeway; therefore the several courses from the log-board are entered in the Traverse table without alteration.

Next, the fathoms and knots belonging to each course are summed up, and the results are put in the column of distances in the Traverse table; and to these courses and distances the whole difference of latitude and departure are found. (695)

Then the present lat. is found by art. 655; the co-mid. lat. by 758; the present long. by 777; and the course and distance by 692.

Or by Mercator's. Having found the present lat., find the meridional diff. lat. = 102 by art. 765; and the course and dist. by 692.

Then the diff. long. being found by art. 778, will be $2^{\circ} 40'$, differing by one mile from that found by middle latitude.

Z z z

1129.

1129. EXAMPLE II. May 24, 1753. Being yesterday noon in the latitude of $49^{\circ} 27' N.$, and longitude $16^{\circ} 02' W.$, we have sailed till this day noon, as by the log-board, in a current setting NW. $2\frac{1}{4}$ miles an hour: Required the ship's place, together with her direct course and distance sailed?

LOG-BOARD.					TRAVERSE TABLE.					
H	K	F	Courses	Winds	Courses	Dif	N.	S	E.	W.
1	5	2	SWb.W	NWbW	SW.	29		20.5		20.5
2	5	3			SE.	43		30.4	30.4	
3	5	2			NE. b. N	42	34.9		23.3	
4	4	4			N. b. E.	20	25.5		5.1	
5	4	3			NW.	54	38.2			38.2
6	4	2					98.6	50.9	58.8	58.7
7	5	5	SE. b. S	SW. b. S.			50.0		58.7	
8	5	1					47.7	D. lat.	0.1	Dep.
9	6	4								
10	6	2								
11	7	2								
12	6	3								
1	6	2								
2	7	4	NE.	ESE.						
3	7	4								
4	7	6								
5	6	4								
6	6	3								
7	6	5								
8	5	2	NNE	East						
9	5	3								
10	5	4								
11	5	5								
12	4	2								

Dep. lat.	=	$49^{\circ} 27' N.$	Mer. pts.	=	3423
Diff. lat.	=	$0 \ 48 \ N.$			
Pref. lat.	=	$50 \ 15 \ N.$	Mer. pts.	=	3498
Sum lats.	=	$99 \ 42$	M. D. lat.	=	75
Middle lat.	=	$49 \ 51$			
Co-mid. lat.	=	$40 \ 09$			
Dep. long.	=	$16 \ 02 \ W.$			
Diff. long.	=	$00 \ 00 \ W.$			
Pref. long.	=	$16 \ 02 \ W.$			
Course is due North.					
Distance 48 miles.					

As each course on the log-board is 6 points distant from the wind, therefore one point leeway is allowed to each, and the courses thus corrected (art. 1081) are put in the column of courses in the Traverse table.

Then the Traverse table is completed to the distances belonging to the respective courses, and the whole diff. lat. is found to be 47.7 miles of northing, and but $\frac{1}{10}$ th part of a mile of departure; therefore it may be concluded that the ship would have got to her present place by a course due north, and consequently she has not at all altered her longitude.

In these two examples, and the following ones, it was not thought necessary to take any more than one fractional place in the several differences of latitudes and departures, altho' the Traverse table gives two such places.

1130. EXAMPLE III. *June 27, 1753. Being yesterday noon in latitude $25^{\circ} 30' S.$, and longitude $10^{\circ} 15' E.$, we have sailed till this day noon, as per log-board, in a current setting south $2\frac{1}{2}$ miles an hour, the variation $1\frac{1}{2}$ point west: Required the ship's place?*

LOG-BOARD.					TRAVERSE TABLE.					
H.	K.	F.	Courses	Winds	Courses	Dif	N.	S.	E.	W.
1	6	0	SW.	WNW.	S. b. W $\frac{1}{2}$ W	30		28,7		8,7
2	6	2			S. b. E. $\frac{1}{2}$ E.	32		30,6	9,3	
3	6	4			S. $\frac{1}{2}$ E.	30		29,9	2,9	
4	6	0			SE. b. E $\frac{1}{2}$ E.	39		18,4	34,4	
5	5	3			S b E $\frac{1}{2}$ E.	60		57,4	17,4	
6	6	0	S. b. W.	W. b. S.			D. lat	105,0	64,0	8,7
7	5	1							8,7	
8	5	4							55,3	Dep.
9	5	2								
10	5	3								
11	5	5								
12	5	2	SSW.	West						
1	5	2								
2	5	0								
3	4	6								
4	5	0								
5	5	1								
6	5	2	SE. b. S	SW. b. S.						
7	5	4								
8	5	4								
9	6	0								
10	6	0								
11	5	4								
12	5	5								

Diff. lat.	=	$2^{\circ} 45' S.$		
Dep. lat.	=	$25^{\circ} 30' S.$	Mer. pts.	1583
Pref. lat.	=	$28^{\circ} 15' S.$	Mer. pts.	1768
Sum lats.	=	$53^{\circ} 45'$	M. diff. lat.	185
Mid. lat.	=	$26^{\circ} 52'$		
Co-mid. lat.	=	$63^{\circ} 08'$		
Dep. long.	=	$10^{\circ} 15' E.$		
Diff. long.	=	$1^{\circ} 02' E.$	Or, $1^{\circ} 01\frac{1}{2}' E.$	
Pref. long.	=	$11^{\circ} 17' E.$		
Course is		$S. 18^{\circ} 30' E.$		
Distance is		174 miles.		

The courses and winds in the log-board being examined, it appears that the ship is close hauled on each tack, and one point leeway being allowed (1081) reduces the courses to SW. b. S., S. b. W., SE., and take a course for the current S.; these several courses being corrected by art. 1078 for the variation $1\frac{1}{2}$ point west, give those in the Traverse table; to which the whole difference of latitude and departure is to be found as before.

And hence the present latitude and longitude may be found either by middle latitude, or Mercator's sailing: For as the ship is so near the equator, the difference will be almost insensible.

If a greater accuracy is required, which is very rarely wanted, especially in such small latitudes, the diff. of long. may be found to each course, as shewn in the arts. 789, 790, and thence the whole diff. long. may be collected; but there will be no material difference gained by such operation.

1131. EXAMPLE IV. Yesterday noon we were in lat. $14^{\circ} 40' N.$, long. $57^{\circ} 25' W.$; we have sailed till this day noon, as per log-board, in a current setting WSW. $\frac{1}{2} W.$ $\frac{1}{2}$ mile an hour; the variation was N. $\frac{1}{2}$ point west: Required the ship's place, and her course and distance to the Island of Barbadoes?

LOG-BOARD.					TRAVERSE TABLE.					
H.	K.	F.	Courses	Winds	Courses	Dif.	N.	S.	E.	W.
1	5	0	SWbS $\frac{1}{2} W.$	WNW $\frac{1}{2} W.$	SSW.	21		19.4		8.0
2	5	2			SW.	52		30.8		36.8
3	5	2			S b. W $\frac{1}{2} W.$	41		39.2		11.9
4	6	0			SE. b. S.	44		36.6	24.5	
5	6	3	SWb. W $\frac{1}{2} W.$	NW. $\frac{1}{2} W.$	WSW.	12		4.6		11.1
6	6	4					D. lat.	136.6	24.5	67.8
7	6	3								24.5
8	7	0							Dep.	43.3
9	7	3								
10	6	0								
11	6	2								
12	6	4								
1	7	0	SW.	WNW.						
2	7	0								
3	7	1								
4	7	3								
5	6	0								
6	6	2								
7	6	4	S.b.E. $\frac{1}{2} E.$	SW. $\frac{1}{2} W.$						
8	7	0								
9	7	4								
10	8	0								
11	8	2								
12	7	4								

Diff. lat.	=	$2^{\circ} 17' S.$
Dep. lat.	=	$14^{\circ} 40' N.$
Pref. lat.	=	$12^{\circ} 23' N.$
Sum lats.	=	$27^{\circ} 03'$
Mid. lat.	=	$13^{\circ} 31'$
Co-mid. lat.	=	$76^{\circ} 29'$
Dep. long.	=	$57^{\circ} 25' W.$
Diff. long.	=	$0^{\circ} 45' W.$
Pref. long.	=	$58^{\circ} 10' W.$

Here the ship's place is found; namely, in lat. $12^{\circ} 23' N.$, and longit. $58^{\circ} 10' W.$
Now to find her course and distance to Barbadoes.

Barbadoes's lat.	=	$13^{\circ} 00' N.$	Barbadoes's long.	=	$59^{\circ} 50' W.$
Present lat.	=	$12^{\circ} 23' N.$	Present long.	=	$58^{\circ} 10' W.$
Diff. lat.	=	$0^{\circ} 37' N.$	Diff. longit.	=	$1^{\circ} 40' = 100 m.$
Sum lats.	=	$25^{\circ} 23'$	The departure	=	97.5
Mid. lat.	=	$12^{\circ} 41'$	The course is	=	$N 69^{\circ} 15' W.$
Co-mid. lat.	=	$77^{\circ} 19'$	The distance	=	104 miles.

In this example, the manner of correcting the courses and finding the ship's place is the same as in the preceding ones.

But the course and distance she has still to go to arrive at her port, is found by the Traverse table according to the directions in art. 769. Or, when the latitudes concerned are above 40° , the operation may be best performed by the directions for the use of the Traverse table in art. 770.

1132. EXAMPLE V. Yesterday noon we were in lat. $33^{\circ} 40' N.$, long. $16^{\circ} 20' W.$, the sun was observed to set $50^{\circ} 18'$ from the north point of the compass; we have sailed till this day noon, as per log-board, in a current setting *WSW*. $1\frac{1}{2}$ miles an hour: Required the ship's place, and her course and distance to the west end of the Island of Madeira, June 4th, 1753?

LOG-BOARD.					TRAVERSE TABLE.					
H	K	F	Courses	Winds	Courses	Dis.	N.	S.	E.	W.
1	6	2	S. b. W	West	S. $01^{\circ} E.$	40		40,0	0,7	
2	6	0			S. $10 W.$	70		68,9		12,2
3	6	3			S. $44 W.$	58		41,7		40,3
4	7	0			S. $55 W.$	36		20,6		29,5
5	7	2					D. lat.	171,2	0,7	82,0
6	7	3								0,7
7	7	2	SW.b.S.	W.b.N.					Dep.	81,3
8	7	2								
9	7	4								
10	7	6								
11	7	4								
12	8	1								
1	8	0								
2	8	5								
3	8	2								
4	7	5	SWb.W	NW.						
5	7	3								
6	6	6								
7	6	4								
8	6	0								
9	6	2								
10	6	1								
11	6	3								
12	6	1								

Before the courses can be corrected to put in the Traverse table, the variation of the compass must be found from the sun's true amplitude; which may be found by art. 1070, or by working the proportion in art. 1071, either by logarithms or the Gunter.

June 4th, 1753, the declination is $22^{\circ} 30' N.$
 As $\cos. 33^{\circ} 40' : \text{Rad} :: \sin. 22^{\circ} 30' : \sin. 27^{\circ} 22'.$
 So that the true amplitude = $62^{\circ} 38' W.$
 Mag. amplitude = $50 18$
 Variation = $12 20 W.$

The courses in the log-board being corrected by this variation and the leeway, will give the courses fitted for the Traverse table.

Dep. lat. = $33^{\circ} 40' N.$
 Diff. lat. = $2 51 S.$
 Pref. lat. = $30 49 N.$
 Sum lats. = $64 29$
 Mid. lat. = $32 14$
 Co-mid. lat. = $57 46$
 Dep. long. = $16 20 W.$
 Diff. long. = $1 36 W.$
 Pref. long. = $17 56 W.$

M. P.
 Madeira's lat. = $32^{\circ} 25' N.$ 2058
 Pref. lat. = $30 49 N.$ 1945
 Diff. lat. = $1 36$ 113
 Sum lats. = $63 14$
 Mid. lat. = $31 37$
 Co-mid. lat. = $58 23$
 Madeira's long. = $17 21 W.$
 Pref. long. = $17 56 W.$
 Diff. long. = $0 35 W.$
 The course (770) = $N. 17^{\circ} 15' E.$
 The distance = 101 miles.

In the work for the amplitude, the lat. at sun-set was taken the same as at noon; for altho' there were about 46 m. of southing in that time, and so the lat. at sun-set was about $32^{\circ} 54'$, yet the amplitude being only about $15'$ less, the alteration in variation would scarcely affect the diff. lat. and dep. found from the courses so corrected

1133. EXAMPLE VI. August 3d, 1753. Being yesterday noon in lat. $19^{\circ} 30'$ S., long. $0^{\circ} 10'$ E.; this forenoon we observed the sun's altitude to be $10^{\circ} 40'$, when he was $80^{\circ} 30'$ from the north point of the compass; we have sailed till this day noon, as per log-board, in a current setting by the compass WNW. $\frac{1}{2}$ mile an hour: Required the ship's place, with her direct course and distance to the Island of St. Helena?

LOG-BOARD.					TRAVERSE TABLE.					
H.	K.	F.	Courses	Winds	Courses	Dif.	N.	S.	E.	W.
1	0	7	N. b. E.	E. b. N.	N. $13^{\circ} 30'$ W.	38	36.9			8.9
2	6	2			N. $24^{\circ} 45'$ W.	38	34.5			15.0
3	6	4			N. $47^{\circ} 15'$ W.	76	51.7			55.6
4	6	3			N. $81^{\circ} 00'$ W.	12	1.0			11.8
5	6	1					125.0			92.2
6	6	0					D. lat.			Dep.
7	5	8	North	ENE.						
8	5	4								
9	5	0								
10	5	3								
11	5	6								
12	5	9								
1	5	7								
2	6	4	NNW.	NE.						
3	6	8								
4	7	0								
5	7	3								
6	7	6								
7	7	5								
8	7	0								
9	7	2								
10	7	4								
11	6	3								
12	6	0								

Before the courses can be corrected, an azimuth must be worked as in arts. 1074, 1075.

	$90^{\circ} 00'$	$90^{\circ} 00'$	$90^{\circ} 00'$
Decl.	$17^{\circ} 27'$	Lat. $19^{\circ} 30'$	Alt. $10^{\circ} 40'$
A =	$107^{\circ} 27'$	Co-lat. $70^{\circ} 30'$	Co-alt. $79^{\circ} 20'$
D =	$8^{\circ} 50'$		Co-lat. $70^{\circ} 30'$
	$116^{\circ} 17'$	$58^{\circ} 08' = \frac{1}{2} \text{sum}$	D = $8^{\circ} 50'$
	$98^{\circ} 37'$	$49^{\circ} 19' = \frac{1}{2} \text{diff.}$	

Then by the Gunter, the extent from $58^{\circ} 08'$ to $79^{\circ} 20'$ on the fines, will reach from $70^{\circ} 30'$ to some point beyond 90° ; rest one foot there, and extend the other to $49^{\circ} 19'$; then this extent will reach from the beginning of the versed fines to the versed fine of 67° , the azimuth sought, as the latitude is south. Hence the variation will be $13^{\circ} 30'$ W.; with which the courses in the log-board are to be corrected.

For the ship's place.

Dep. lat.	=	$19^{\circ} 30'$ S.
Diff. lat.	=	$2^{\circ} 05'$ N.
Pref. lat.	=	$17^{\circ} 25'$ S.
Sum lats.	=	$36^{\circ} 55'$
Mid. lat.	=	$18^{\circ} 27'$
Co-mid. lat.	=	$71^{\circ} 33'$
Dep. long.	=	$0^{\circ} 10'$ E.
Diff. long.	=	$1^{\circ} 36'$ W.
Pref. long.	=	$1^{\circ} 26'$ W.

For the bearing and dist. of Helena.

St. Helena's lat.	=	$16^{\circ} 00'$ S.	M.P. 973
Pref. lat.	=	$17^{\circ} 25'$ S.	1061
Diff. lat.	=	$1^{\circ} 25'$	88
Sum lats.	=	$33^{\circ} 25'$	
Mid. lat.	=	$16^{\circ} 42'$	
Co-mid. lat.	=	$73^{\circ} 18'$	
St. Helena's lon.	=	$5^{\circ} 53'$ W.	
Pref. long.	=	$1^{\circ} 26'$ W.	
Diff. long.	=	$4^{\circ} 27'$	= 267 m.
Course N.	=	$71^{\circ} 30'$ W.	(770)
Distance	=	270 miles.	

Here

Here the azimuth is worked with yesterday's lat.; but had it been work'd with this day's, the variation would have been $12^{\circ} 30'$, the diff. lat. 126 m., and the dep. 90 m.

1134. In the following Journal are introduced almost all the circumstances that can attend a day's work in any voyage: The ship is taken from the river *Thames*, conducted thro' the *Downs*, down the *English channel*, and from thence over the *Ocean* till she is brought within sight of the port she is bound to, which is *Fonchal* in the island of *Madeira*. In this navigation, care has been taken to point out the usual business done in like circumstances of wind and weather; the courses and distances run down the Channel are such as produce the bearings and distances of the places mentioned, and 'tis not unusual for many vessels to keep a track not very different from what is here assigned. Indeed the runs inserted in the log-boards, after the ship has taken her departure from the Lizard, are rather somewhat greater than can be expected from any vessel in like weather: But such long days runs were used to save 4 or 5 days works, which must have been added to bring the ship to her port, had the usual runs been introduced. All the astronomical observations are fitted to the times and places; and 'tis apprehended that the whole is conducted in such a manner as 'tis possible to have happened.

To the observations in article 1127 it may be proper to annex the following ones.

1135. I. That as each day's account is settled at the noon ending that day, therefore the sun's declination of the day following is used: Thus, in settling Sunday's account, the declination of Monday is taken; because the latitude by observation at the end of Sunday is the same as at the beginning of Monday.

II. The present longitude each day is found by middle latitude; but the bearings and distances to the Lizard and Madeira are found by Mercator's sailing; the one being rather preferable, for its ease of working, in the former case, the other in the latter.

III. As the greatest trouble in using the Traverse table is the finding therein a corresponding difference of latitude and departure, or meridional difference of latitude and difference of longitude, nearly equal to two given numbers, it may perhaps be found most convenient, as the Traverse table to the points of the compass at art. 711 lies in two openings or folio's, first to find the nearest corresponding numbers in one of those folio's, marking what course they stand with, and then to turn to the degrees nearest that course in the large Traverse table, art. 712, and near that place the given numbers will be found more accurate, with but little trouble. Thus to find the course to 672 meridional diff. lat. and 353 diff. long. In page 307, the nearest numbers 67,03 and 35,82 stand with $2\frac{1}{2}$ points, or about 28 deg. Then turning to 28 deg., and not finding them there, turn to 27° , and the numbers 67,26 and 35,39 answer to $27^{\circ} 45'$ course.

In most Journals, the week days are marked by the planets characters: Thus Sunday is $\odot$, Monday D , Tuesday δ , Wednesday g , Thursday u , Friday f , Saturday h .

1136. *A JOURNAL of a voyage in the year 1752. In the Hope-
well of London, from England towards Madeira. LUCAS
TRIMMER, Commander. Kept by FRANCIS DRAKE, Mate.*

Winds.	Remarks on board.
SSW.	♂. 26th September. At 5 A. M. the pilot came on board ; then weigh'd and sail'd from Cherry-Garden. At 11 came to with the best bower * at Blackwall.
From SSW. to NNW.	♀. 27th September. Fresh gales and cloudy weather with rain. At 5 A. M. weigh'd and sail'd ; and at 9 came to an anchor at Gravesend, and clear'd ship.
W. b. S.	♂. 28th. September. At 4 P. M. weigh'd and sail'd, moderate weather : At 9 came to with the best bower at the Nore in $9\frac{1}{2}$ fathoms ; fresh gales : At 4 A. M. weigh'd and sail'd. At 11 came to anchor in the Downs in 7 fath. Deal-castle bearing W. $\frac{1}{2}$ S. distant 3 miles.
W. b. S.	♀. 29th September. At 1 P. M. sent the pilot ashore. These 24 hours, the first and middle parts moderate and fair, the latter part strong gales and cloudy.
Variable from S. b. W. to West.	♂. 30th September. Strong gales and cloudy. At 2 P. M. veer'd out the long service † of the best bower ; got top-gallant yards down. At 4 P. M. struck yards and top-masts. These 24 hours, had very hard gales of wind.
SW.	♂. 1st October. These 24 hours, for the most part fresh gales. At 4 A. M. hove up best bower, and let go small bower *. At 9, hove up small bower, and let go best bower again. People employed in making points and gaskets.
W. b. N.	♂. 2d October. At 6 P. M. strong gales with heavy rain. At 8, veer'd out long service, and let go the sheet anchor * under At 9 A. M. hove up sheet anchor. [foot.
NW.	♂. 3d October. These 24 hours, for the first and middle parts moderate and fair ; the latter part strong gales.
NW.	♀. 4th October. These 24 hours, fresh gales and fair. At 3 P. M. got top-gallant masts down. At 10 A. M. got yards and top-masts up.

* NOTE, It is customary in most ships to have three kinds of anchors, namely :
The small bower, used to fix the ship in still water and calm weather.
The best bower, a larger anchor, for riding in rough weather.
The sheet anchor, or largest, for fixing the ship in stormy weather.
† The first 30 fathoms of cable next the anchor is called *short service* ; and
when the remaining part is veered out, it is called the *long service* Four-

Journal from England towards Madeira.

LOG-BOARD.					Remarks on board.
H.	K.	F.	Courses	Winds	
2			S.b.W $\frac{1}{4}$ W	N. $\frac{1}{2}$ W.	24. 5th October. At 2 P.M. hove short and hoisted in the long-boat.
4					At 4, weigh'd and sail'd in company with a 20 gun man of war and 40 sail of merchantmen.
6					At 6, South-Foreland bore NNW. dist. 4 m.
8					At 2 A. M. Fairlee bore N. dist. 6 miles.
10					At 6, Beachy bore N. b. W. 6 miles.
12			W.	N. b. W.	At 8, Beachy bore NE. b. E. 9 miles.
2			SW b W $\frac{1}{4}$ W		Fresh gales and clear. Several ships standing up Channel. Close reefed both top-fails.
4					At 12, Bembridge point bore WNW. 27 m.
6			WNW $\frac{1}{2}$ W.	N. $\frac{1}{2}$ W.	Still in company with the fleet.
8			WSW.	N. b. E.	
10					
12					

LOG-BOARD.					Remarks on board.
H.	K.	F.	Courses	Winds	
2			WSW.	N. b. E.	2. 6th October. Fresh gales and clear.
4	4	6	W. $\frac{1}{4}$ S.		At 4 P. M. parted with the fleet, they being bound to Spithead. Dunnose bearing WNW. distance 21 miles.
6	5	5			At 5, let out one reef of each top-fail.
8	5	0			At 7 A. M. Portland lights bore NNW. 9 m.
10	5	1			At 10 ditto bore NE. 12 miles.
12	4	6			Fourteen sail in sight.
2	5	0			
4	4	5			
6	4	4			
8	4	0			
10	4	2	SbW. $\frac{1}{2}$ W.	NNE.	
12	5	0			

LOG-BOARD.					Remarks on board.
H.	K.	F.	Courses	Winds	
2	5	6	Sb, W $\frac{1}{2}$ W.	NNE.	5. 7th October. Cloudy weather, fresh gales.
4	5	5			At 4 P. M. Start bore W. b. N. dist. 12 m.
6	4	0			At 8 ditto bore E. b. N. 9 m.
8	3	5	Wb. N $\frac{1}{2}$ W	North	At 4 A. M. Edystone bore NE. b. E. $\frac{1}{2}$ E. 12 m.
10	4	1			At 6, got up both top-gallant masts and yards.
12	4	3			At 7 $\frac{1}{2}$, Lizard bore W. b. N.
2	3	0			Several ships standing up Channel.
4	3	6	SW. $\frac{1}{4}$ W.	NW. b. W.	At noon Lizard bore N. b. W. dist. 21 miles.
6	4	5			For, From 7 $\frac{1}{2}$ h. to 12 h. is 4 $\frac{1}{2}$ hours.
8	5	2			4 $\frac{1}{2}$ hours, at 5, 4 miles an hour, is 24 miles.
10	5	4			Between SW. $\frac{1}{4}$ W. and W. b. N. is 4 $\frac{3}{4}$ points.
12	5	6			Between SW. $\frac{1}{4}$ W. and N. b. W. is 5 $\frac{1}{4}$ points.

The other angle is 6 points.
 Then 6 pts. : 4 $\frac{3}{4}$ pts. :: 24 m. : 21 m. (340)
 A a a a *Journal*

LOG-BOARD.				
H.	K.	F.	Courses	Winds
2			Foggy & lay by WSW. $\frac{1}{4}$ W.	WSW.
4				NW.b.N. [$\frac{1}{4}$ W.]
6	6	c		
8	6	4		
10	6	2		
12	7	5		
2	7	c		
4	7	3		
6	7	1		
8	7	c		
10	7	c		
12	7	c		

Remarks on board.

☉. 8th October. These 24 hours, first part foggy, the latter part moderate gales and clear weather.

At 4 P. M. fog clear'd up. Shifted the Channel compass, where the variation was allow'd for, and put another into the bit-tacle.

Lizard bore N. $\frac{1}{2}$ E. distance 21 miles.

Variation $1\frac{1}{2}$ point to the west.

Leeway $\frac{1}{2}$ point.

Unbent our cables and stowed anchors.

At 2 A. M. set top-gallant sails.

Observed sun's merid. zen. dist. = $54^{\circ} 33' S.$

Declination = $6^{\circ} 31' S.$

TRAVERSE TABLE.					
Courses	Dif.	N.	S.	E.	W.
S. b. E.	21		20,6	4,1	
SW. $\frac{1}{4}$ W.	140		94,0		103,7
		D. 1	114,6	4,1	103,7
					4,1
				Dep	00,6

Lat. per observation = $48^{\circ} 02' N.$

Dep. lat. = $49^{\circ} 57' N.$

Diff. lat. = $1^{\circ} 55' S.$

Pref. lat. = $48^{\circ} 02' N.$

Sum lats. = $97^{\circ} 59'$

Middle lat. = $48^{\circ} 59'$

Dep. long. = $5^{\circ} 14' W.$

Diff. long. = $2^{\circ} 12' W.$

Pref. long. = $7^{\circ} 26' W.$

Lizard bears N. $41^{\circ} E.$

Distance 152 miles.

Here the lat. by account agreeing with the lat. by observation, it is judged that the ship's place is truly ascertained.

Ship's lat. = $48^{\circ} 02' N.$

Madeira's lat. = $32^{\circ} 38' N.$

Diff. lat. = $15^{\circ} 24'$

Or 924 miles.

Ship's longit. = $7^{\circ} 26' W.$

Madeira's long. = $16^{\circ} 51' W.$

Diff. long. = $9^{\circ} 25' = 565 \text{ miles.}$

Island of Madeira bears S. $24^{\circ} 45' W.$ Distance 1015 miles.

Here the ship is supposed to sail from the Lizard; and as the Lizard bore N. $\frac{1}{2}$ E., the course must be on the opposite point, which is S. $\frac{1}{2}$ W.; and this course corrected by the variation, as at art. 1078, gives S. b. E.; also the other course WSW. $\frac{1}{4}$ W. being corrected both for the variation and leeway (arts. 1078, 1081), the Traverse table, and other work for the present latitude and longitude, is performed by the observations in art. 1127.

Then the bearing and distance to Funchal in the Island of Madeira is found, as in art. 770, working by the Traverse table.

Journal

Journal from England towards Madeira.

LOG-BOARD.					Remarks on board.
H	K.	P.	Courses	Winds	
2	7	6	West	S. b. W.	D. 9th October. These 24 hours stiff gales with drizzling rain.
4	8	4			Variation $1\frac{1}{2}$ point west.
6	8	6			At 4 P. M. spoke with the Charming Molly bound from Carolina to London.
8	8	9			At 8, took in top-gallant fails.
10	9	0			At 4 A. M. took single reef in each T. S. *
12	8	6			People employ'd in making points.
2	8	3			Observ'd sun's merid. zen. dist. = $54^{\circ} 09' S.$
4	8	4			Declination = $6 \quad 54 S.$
6	8	1			
8	8	6			
10	8	3			Latit. by observation = $47 \quad 15 N.$
12	8	0			

TRAVERSE TABLE.					
Courses	Diff.	N.	S.	E.	W.
WSW $\frac{1}{2}$ W	202		58,6		193,3

Dep. lat. = $48^{\circ} 02' N.$ or last day's lat.
 Diff. lat. = $0 \quad 59 S.$ as by Trav. tab.
 Pref. lat. = $47 \quad 03 N.$ as by account.

Here the lat. by observation differs 12 minutes from that by account.

Therefore, before the longitude can be found, the dead reckoning must be corrected by art. 1113, as the course is nearer the parallel than the meridian.

Then to the diff. lat. 58,6 and depart. 193,3, the dist. is 205.

And to the dist. 205, and diff. lat. by observ. 47, the dep. is 199,5.

Now Dep. lat.	= $48^{\circ} 02' N.$	Dep. long.	= $7^{\circ} 26' W.$
Pref. lat. by obs.	= $47 \quad 15 N.$	Diff. long.	= $4 \quad 50 W.$
Sum lats.	= $95 \quad 17$	Pref. long.	= $12 \quad 16 W.$
Middle lat.	= $47 \quad 38$		

		M. P.			M. P.
Lizard's lat.	= $49^{\circ} 57' N.$	3470	Madeira's lat.	= $32^{\circ} 38' N.$	2073
Ship's lat.	= $47 \quad 15 N.$	3225	Ship's lat.	= $47 \quad 15 N.$	3225
Diff. lat.	= $2 \quad 42$	245	Diff. lat.	= $14 \quad 37$	1152

Or 162 miles.

Lizard's long. = $5 \quad 14 W.$
 Ship's long. = $12 \quad 16 W.$

Diff. long. = $7 \quad 02 = 422 m.$
 Lizard bears N. $59^{\circ} 45' E.$
 Distance 324 miles.

Or 877 miles.

Madeira's long. = $16 \quad 51 W.$
 Ship's long. = $12 \quad 16 W.$

Diff. long. = $4 \quad 35 = 275 m.$
 Madeira bears S $13^{\circ} 30' W.$
 Distance 903 miles.

* Mariners in their Journals use abbreviations of words; such as T. S. for top fails, B. T. S. both top fails, and many others.

Journal from England towards Madeira.

LOG-BOARD.				
H.	K.	F.	Courses	Winds
2	9	2	S. $\frac{1}{2}$ E.	NNW.
4	10	0		
6	9	5		
8	10	0		
10	10	6		
12	10	2		
2	10	4		
4	9	6		
6	10	0		
8	10	4		
10	9	8		
12	10	0		

Remarks on board.

♂. 10th October.
 These 24 hours hard gales and clear weather.
 At 6 P. M. close reef'd the top sails.
 At 6 A. M. a sail standing to northward.
 At noon Cape Finisterre bore N. 85° E.,
 distant 25 miles.
 Variation $1\frac{1}{2}$ W.
 Observed sun's mer. zen. dist. $50^{\circ} 33'$ S.
 Declination $7^{\circ} 16'$ S.
 Latitude by observation $43^{\circ} 17'$ N.
 Dep. lat. $47^{\circ} 15'$

TRAVERSE TABLE.					
Course.	Diff.	N.	S.	E.	W.
SSE.	240		221,8	91,8	

Diff. lat. $3^{\circ} 58' = 238$ m.
 Departed lat. $= 47^{\circ} 15'$ N.
 Diff. lat. $= 3^{\circ} 42'$ S.
 Pref. lat. $= 43^{\circ} 33'$ N. by acct.
 Pref. lat. $= 43^{\circ} 17'$ N. by obser.

Here the latitude by account falls short of the latitude by observation. Therefore before the longitude can be found, the account of departure must be corrected by art. 1112, as the course is only two points from the meridian.

Now to diff. lat. 221,8 and dep. 91,8, the course is SSE.

To course SSE, and diff. lat. 238, the correct departure is 99,5.

Then, Sum lats.	$= 90^{\circ} 32'$	Dep. long.	$= 12^{\circ} 16'$ W.
Middle lat.	$= 45^{\circ} 16'$	Diff. long.	$= 2^{\circ} 23'$ E.
Co-middle lat.	$= 44^{\circ} 44'$	Pref. long.	$= 9^{\circ} 53'$ W.

Lizard's lat.	$= 49^{\circ} 57'$ N.	M. P.	3470	Madeira's lat.	$= 32^{\circ} 38'$ N.	M. P.	2073
Ship's lat.	$= 43^{\circ} 17'$		2886	Ship's lat.	$= 43^{\circ} 17'$		2886
Diff. lat.	$= 6^{\circ} 40'$		584	Diff. lat.	$= 10^{\circ} 39'$		813
Or	$= 400$ miles:			Or	$= 639$ miles.		
Lizard's long.	$= 5^{\circ} 14'$ W.			Madeira's lon.	$= 16^{\circ} 51'$ W.		
Ship's long.	$= 9^{\circ} 53'$ W.			Ship's longit.	$= 9^{\circ} 53'$		
Diff. long.	$= 4^{\circ} 39'$	$= 279$ m.		Diff. longit.	$= 6^{\circ} 58'$	$= 418$ m.	
Lizard bears N. $25^{\circ} 30'$ E.				Madeira bears S. $26^{\circ} 45'$ W.			
Distance 443 miles.				Distance 715 miles.			

Journal from England towards Madeira.

LOG-BOARD.					Remarks on board.
H.	K.	F.	Courses	Winds	
2	10	6	W. b. S $\frac{1}{4}$ W	Sb. W $\frac{1}{4}$ W.	8. 11th October. These 24 hours very hard gales and squally with rain. Top fails close reefed. At 10 P. M. spoke with the <i>Princessa</i> now bound for London. Observ. the sun to set 66° from N. westward. Hence find the true amplitude. (art. 1072.) The lat. $43^{\circ} 17' N.$ Declin. $7^{\circ} 16' S.$ To course $7^{\circ} 15'$ and dist. 100, the dep. is 12,62. To course $43^{\circ} 15'$ and d. lat. 12,62, the dist. is 17. To dist. 100 and dep. 17, the course is 10°. Therefore, the true ampl. is N. $80^{\circ} 00' W.$ Mag. ampl. $66^{\circ} 00'$ Variation $14^{\circ} 00' W.$ Or, allow $1\frac{1}{4}$ point variation. Obs. sun's mer. zen. dist. = $48^{\circ} 59' S.$ Declination = $7^{\circ} 39' S.$ By observ. Latitude = $41^{\circ} 20' N.$ Dep. lat. = $43^{\circ} 17'$ Diff. lat. = $1^{\circ} 57'$ Or = 117 m. Here the lat. by acct. having out-run the lat. by obser. the dep. must be corrected by art. 1114, as diff. lat. and dep. are nearly equal. Then to d. lat. 131,2 and dep. 138,1, the distance will be 190. To dist. 190 and true diff. lat. 117, find the dep. 149,7. The sum of both dep. 138,1 and 149,7 is 287,8. The half sum 143,9 = corrected departure. M. P. Madeira's lat. = $32^{\circ} 38' N.$ 2073 Ship's lat. = $41^{\circ} 20' N.$ 2728 Diff. lat. = $8^{\circ} 42'$ 625 Or = 522 miles. Madeira's lon. = $16^{\circ} 51' W.$ Ship's long. = $13^{\circ} 07' W.$ Diff. long. = $3^{\circ} 44' W.$ = 224. Madeira bears S. $19^{\circ} W.$ Distance 552 miles.
4	10	2	Leeway	1 point	
6	11	0	SSW.	W.	
8	10	3	Leeway	1 point	
10	9	7			
12	9	6	W. b. N $\frac{1}{4}$ W	SSW $\frac{3}{4}$ W	
2	10	2	Leeway	1 point	
4	10	5			
6	10	8	SW. $\frac{3}{4}$ W.	NWbW $\frac{1}{4}$ W	
8	10	0	Leeway	1 point	
10	10	1			
12	9	6			

TRAVERSE TABLE.					
Courses	D.	N.	S.	E.	W.
W. b. S.	42		8,2		41,2
S. $\frac{1}{4}$ E.	52		61,9	3,0	
W. b. N.	51	12,1			60,8
SSW $\frac{1}{2}$ W.	83		73,2		39,1
		12,1	143,3	3,0	141,1
			12,1		3,0
			131,2		138,1

Dep. lat.	=	$43^{\circ} 17' N.$
Diff. lat.	=	$2^{\circ} 11' S.$
Lat. by account	=	$41^{\circ} 06' N.$
Lat. by observation	=	$41^{\circ} 20' N.$
Sum lats.	=	$84^{\circ} 37'$
Mid. lat.	=	$42^{\circ} 18'$
Dep. long.	=	$9^{\circ} 53' W.$
Diff. long.	=	$3^{\circ} 14' W.$
Pres. long.	=	$13^{\circ} 07' W.$

Lizard's lat.	=	$49^{\circ} 57' N.$	3470
Ship's lat.	=	$41^{\circ} 20' N.$	2728
Diff. lat.	=	$8^{\circ} 37'$	742
Or	=	517 miles.	
Lizard's long.	=	$5^{\circ} 14' W.$	
Ship's long.	=	$13^{\circ} 07' W.$	
Diff. long.	=	$7^{\circ} 53' W.$	= 473 m.
Lizard bears N. $32^{\circ} 30' E.$			
Distance 614 miles.			

Here the lat. by acct. having out-run the lat. by obser. the dep. must be corrected by art. 1114, as diff. lat. and dep. are nearly equal.			
Then to d. lat. 131,2 and dep. 138,1, the distance will be 190.			
To dist. 190 and true diff. lat. 117, find the dep. 149,7.			
The sum of both dep. 138,1 and 149,7 is 287,8.			
The half sum 143,9 = corrected departure.			
M. P.			
Madeira's lat.	=	$32^{\circ} 38' N.$	2073
Ship's lat.	=	$41^{\circ} 20' N.$	2728
Diff. lat.	=	$8^{\circ} 42'$	625
Or	=	522 miles.	
Madeira's lon.	=	$16^{\circ} 51' W.$	
Ship's long.	=	$13^{\circ} 07' W.$	
Diff. long.	=	$3^{\circ} 44' W.$	= 224.
Madeira bears S. $19^{\circ} W.$			
Distance 552 miles.			

Journal from England towards Madeira.

LOG-BOARD.				
H.	K.	F.	Courses	Winds
2	9	6	NWbW $\frac{1}{2}$ W.	SW. $\frac{1}{2}$ W.
4	9	2	Leeway	2 points
6	9	5	S. $\frac{3}{4}$ W.	NE. $\frac{1}{2}$ E.
8	8	5	Leeway	0 point
10	8	7	WSW $\frac{1}{4}$ W.	S. $\frac{1}{4}$ W.
12	7	2	Leeway	2 points
2	8	3	NNE $\frac{1}{4}$ E.	S. $\frac{1}{2}$ W.
4	8	7	Leeway	0 point
6	7	6	SW.	SSE.
8	7	2	Leeway	1 $\frac{1}{2}$ point
10	7	0		
12	6	4		

Remarks on board.

24. 12th October.

These 24 hours hard gales and cloudy weather, accompanied with hard rains.

Handed fore-top sails.

At 10 P. M. heard a gun at a dist.

At 2 A. M. saw to the NNE. $\frac{1}{2}$ E. the flash of a gun, and after 56 beats of the pulse at the wrist we heard the sound, by which we judged the dist. to be about 11 miles (1140), and a ship in distress. Bore away to speak with her.

At 6, having greatly overrun the dist. and finding nothing, concluded she was gone down, so hauled our wind, and stood to the SW.

No observation this day.

Variation 1 $\frac{1}{4}$ point west.

TRAVERSE TABLE.					
Courses	D	N.	S.	E.	W.
NW. $\frac{3}{4}$ W.	37	22,0			29,7
S. $\frac{1}{2}$ E.	36		35,8	3,5	
W. b. S.	30		6,6		33,6
N. b. E.	34	33,6		6,6	
SW. $\frac{1}{4}$ W.	56		37,6		41,5
		55,6	80,0	10,1	104,8
			55,6		10,1
		D. 1.	24,4	Dep.	94,7

Depart. lat. = 41° 20' N.

Diff. lat. = 0 24 S.

Pref. lat. = 40 56 N.

Sum lats. = 82 16

Mid. lat. = 41 08

Depart. long. = 13 07 W.

Diff. long. = 2 06 W.

Pref. long. = 15 13 W.

		P. M.
Lizard's lat.	= 49° 57' N.	3470
Ship's lat.	= 40 56 N.	2696
Diff. lat.	= 9 01	774
Or	= 54 $\frac{1}{2}$ miles.	
Lizard's long.	= 5° 14' W.	
Ship's long.	= 15 13 W.	
Diff. long.	= 9 59	= 599 m.
Lizard bears N. 37° 45' E.		
Distance 685 miles.		

		M. P.
Madeira's lat.	= 32° 38' N.	2073
Ship's lat.	= 40 56 N.	2696
Diff. lat.	= 8 18	623
Or	= 49 $\frac{1}{2}$ miles.	
Madeira's lon.	= 16° 51' W.	
Ship's long.	= 15 13 W.	
Diff. long.	= 1 38	= 98 m.
Madeira bears S. 9° 00' W.		
Distance 505 miles.		

Journal from England towards Madeira.

LOG-BOARD.					Remarks on board.
H.	K.	F.	Courses	Winds	
2	6	7	S. $\frac{1}{4}$ W.	E.	♀. 13th October.
4	8	0	Leeway	0 point	These 24 hours variable winds and hard gales, with cloudy and foggy weather.
6	8	6	E. $\frac{1}{4}$ S.	SSE.	Set fore-top fails.
8	9	2	Leeway	1 point	A great sea setting to the northward, for which allow half a mile an hour.
10	8	5	W. b. N.	SW. b. S.	At 6 P. M. handed fore-top fail.
12	8	5	Leeway	1 point	No observation to day.
2	8	6	South	W.	Variation $1\frac{1}{4}$ point west.
4	9	0	Leeway	0 point	Half a mile an hour, or 12 miles being allowed for the sea setting northward, reduces the diff. lat. 58,7 to 46,7.
6	9	4	E. b. S. $\frac{1}{2}$ E.	S. b. E. $\frac{1}{2}$ E.	
8	8	7	Leeway	2 points	
10	8	8	S. $\frac{3}{4}$ E.	W.	
12	9	3	Leeway	0 point	

TRAVERSE TABLE.					
Courses	D.	N.	S.	E.	W.
S. b. E.	29		28,4	5,7	
ENE.	36	13,8		33,3	
W. b. N. $\frac{1}{4}$ W.	31	4,6			30,7
S. b. E. $\frac{1}{4}$ E.	35		33,9	8,5	
N. E. b. E. $\frac{1}{4}$ E.	36	18,5		30,9	
SSE.	36		33,3	13,8	
		36,9	95,6	92,2	30,7
			36,9	30,7	
			58,7	61,5	

Tried the log-line and glass used this day, and find only $45\frac{1}{2}$ feet on a mean for the length of each knot, and $28\frac{1}{2}$ seconds for the run of the glass.

By this day's dead reckoning, the ship has run S. $52^{\circ} 45'$ E. dist. 77 miles, which is about 3,208 miles an hour on a direct course.

But (by art. 1062) correcting the run, her way will be only 3,065 miles an hour, and her correct. dist. will be 73,6 miles.

Now the diff. lat. 46,7 and dep. 61,5, must be lessened proportionally with the dist. Then to course $52^{\circ} 45'$, and dist. 73,6, the diff. lat. is 44,3, and the departure is 58,4.

Dep. lat. = $40^{\circ} 56'$ N.
 Diff. lat. = $0 44$ S.
 Pref. lat. = $40 12$ N.
 Sum lats. = $81 08$

Mid. lat. = $40^{\circ} 34'$
 Dep. long. = $15 13$ W.
 Diff. long. = $1 17$ E.
 Pref. long. = $13 56$ W.

M. P.

M. P.

Lizard's lat. = $49^{\circ} 57'$ N. 3470
 Ship's lat. = $40 12$ N. 2638
 Diff. lat. = $9 45$ 832
 Or = 585 miles.
 Lizard's long. = $5^{\circ} 14'$ W.
 Ship's long. = $13 56$ W.
 Diff. long. = $8 42$ = 522 m.
 Lizard bears N. $32^{\circ} 15'$ E.
 Distance 694 miles.

Madeira's lat. = $32^{\circ} 38'$ N. 2073
 Ship's lat. = $40 12$ N. 2638
 Diff. lat. = $7 34$ 565
 Or = 454 miles.
 Madeira's lon. = $16^{\circ} 51'$ W.
 Ship's long. = $13 56$ W.
 Diff. long. = $2 55$ = 175 m.
 Madeira bears S. $17^{\circ} 15'$ W.
 Distance 475 miles.

Journal

Journal from England towards Madeira.

LOG-BOARD.				
H.	K.	F.	Courses	Winds
2	9		S. 61° W.	SSE.
4	9		L.	W. 0 pts.
6	9			
8	9			
10	9			
12	9	0		
2	9	4		
4	10	0		
6	10	0		
8	10	0		
10				
12				

Remarks on board.

h. 14th October.
 These 24 hours hard, but steady gales and cloudy weather.
 At 4 P. M. set fore-top fails.
 At 8 A. M. observed the sun's zen. dist. to be S. 72° 55'.
 At 10, saw the sun again, its bearing was S. 25° 15' E., zen. dist. S. 54° 45'.
 Handed both top fails and fore fail, and lay to under main fail.
 At noon made sail.
 Set fore fail and both reef'd-top fails.

TRAVERSE TABLE.

Courses	Dif.	N	S	E	W.
S. 40° W.	202		122,5		152,5

Now having two zenith distances and the time between the observations, the latitude (by art. 1018) will be found to be 38° 20' N.

Also the true azimuth at the second observation, is } N. 142° 45' E.

The observ'd magnetical azimuth, is N. 154 45 E.
 The variation is 12 00 W.

Lat. by observation 38° 20' N.

Departed lat. 40 12 N.

Diff lat. 1 52 = 112 miles.

Dep. lat. = 40° 12' N.

Diff. lat. = 2 03 S.

Pres. lat. by account = 38 09 N.

Lat. by observation = 38 20 N.

Sum lats. = 78 32

Mid. lat. = 39 16

Dep. long. = 13 56 W.

Diff. long. = 3 22 W.

Pres. long. = 17 18 W.

Here, as the lat. by account has out-run the true lat., and the diff. lat. and dep. are nearly equal, the dep. must be corrected by art. 1114.

To d. lat. 122,5 and dep. 152,5, the diff. is 196.

To diff. 196 and d. lat. 112, the dep. is 160,4.

Then the sum of departures is 312,9.

The half sum is the true departure = 156,4.

With which, and co-middle lat., find difference of longitude.

Now a parallel of lat. thro' 38° 20' in the variation chart, cuts the variation line of 12° in the longit. of 17°; and hence I judge my account of longitude is nearly right.

Liz. lat. = 49° 57' N. M. P. 3470

Ship's lat. = 38 20 N. 2494

Diff. lat. = 11 37 976

Or = 697 miles.

Liz. long. = 5° 14' W.

Ship's lon. = 17 18 W.

Diff. lon. = 12 04 W. = 724 m.

Lizard bears N. 37° 30' E.

Distance 880 miles.

M. P. Madeira's lat. = 32° 38' N. 2073

Ship's lat. = 38 20 N. 2494

Diff. lat. = 5 52 421

Or = 352 miles.

Madeira's long. = 16° 51' W.

Ship's long. = 17 18 W.

Diff. long. = 0 27

Madeira bears S. 13° 45' E.

Distance 353 miles.

Journal

Journal from England towards Madeira.

LOG-BOARD.					Remarks on board.
H.	K.	F.	Courses	Winds	☉. 15th October. These 24 hours very hard gales and variable, with clouds and rain. Variation 12 degrees west. Intending next day to take an observation of the moon's culminating, we this day, before and after noon, took equal altitudes of the sun, and set a good second watch to the true noon. Observ'd sun's merid. zen. dist. = $44^{\circ} 44'$ S. Declination = $9^{\circ} 08'$ S.
2	10	0	S.b.W $\frac{1}{2}$ W	Wb.S $\frac{1}{2}$ W	
4	9	6	Leeway	1 point	
6	10	4			
8	10	0	ESE.	South	
10	10	5	Leeway	1 point	
12	9	8			
2	10	3	S.b.W $\frac{1}{2}$ W	ESE.	
4	10	6	Leeway	0 point	
6	11	7			
8	11	8			
10	11	4	ESE.	NE.	
12	11	6	Leeway	1 point	

Lat. by observation = $35^{\circ} 36'$ N.

TRAVERSE TABLE.					
Courses	D.	N.	S.	E.	W.
S $60^{\circ} 30'$ E	60		59,6	6,8	
N $89^{\circ} 15'$ E	51	0,8		61,0	
S. $5^{\circ} 0'$ E	89		88,7		7,8
S. $68^{\circ} 15'$ E	46		17,0	42,7	
		0,8	165,3	110,5	7,8
			0,8	7,8	
	D. 1.		164,5	102,7	Dep.

Dep. lat. = $38^{\circ} 20'$ N.Diff. lat. = $2^{\circ} 44'$ S.Pref. lat. = $35^{\circ} 36'$ N.Sum lats. = $73^{\circ} 56'$ Middle lat. = $36^{\circ} 58'$ Dep. long. = $17^{\circ} 18'$ W.Diff. long. = $2^{\circ} 08'$ E.Pref. long. = $15^{\circ} 10'$ W.

Here the lat. as by account agrees with the lat. by observation.
 Therefore the diff. long. is found with the dep. by account.

Lizard's lat. = $49^{\circ} 57'$ N. M. P. 3470
 Ship's lat. = $35^{\circ} 36'$ N. 2288

Diff. lat. = $14^{\circ} 21'$ 1182

Or = 861 miles.

Lizard's long. = $5^{\circ} 14'$ W.Ship's long. = $15^{\circ} 10'$ W.Diff. long. = $9^{\circ} 56'$ = 596 m.Lizard bears N. $26^{\circ} 45'$ E.

Distance 965 miles.

Madeira's lat. = $32^{\circ} 38'$ N. M. P. 2073
 Ship's lat. = $35^{\circ} 36'$ N. 2288

Diff. lat. = $2^{\circ} 58'$ 215

Or = 178 miles.

Madeira's lon. = $16^{\circ} 51'$ W.Ship's longit. = $15^{\circ} 10'$ W.Diff. longit. = $1^{\circ} 41'$ = 101 m.Madeira bears S. 25° W.

Distance 197 miles.

B b b b

Journal

Journal from England towards Madeira.

LOG-BOARD.				
H.	K.	F.	Courses	Winds
2	8	6	S.b.W $\frac{1}{2}$ W	W.b.S $\frac{1}{2}$ W
4	9	0	Leeway	1 point
6	9	0		
8	8	5		
10	9	0	W. b. S.	North
12	8	6	Leeway	0 point
2	8	5		
4	9	2		
6	8	4	S.b.W $\frac{3}{4}$ W	W. b. S.
8	7	6	Leeway	1 point
10	6	2		
12	5	6		

Remarks on board.

D. 16th October.
 These 24 hours stiff gales, cloudy and variable winds.
 Variation 1 point west.
 Out 1st and 2d reef of both top sails.
 At 7 h. 15 m. 2 $\frac{1}{2}$ f. P. M. the moon's centre passed the meridian.
 At 8 A. M. Isle Porto sancto bore SE., dist. 10 leagues. And Madeira bore SSW. $\frac{3}{4}$ W., dist. 15 leagues.
 Observed sun's mer. zen. dist. 42° 20' S.
 Declination 9 30 S.

TRAVERSE TABLE.					
Courses	D.	N.	S.	E.	W.
S. $\frac{1}{2}$ E.	70		69,7	6,9	
WSW.	71		27,2		65,6
S. $\frac{1}{4}$ E.	55		54,9	2,7	
			151,8	9,6	65,6
					9,6
					56,0

Lat. by observation 32 50 N.
 Départ. lat. 35 36
 Diff. lat. 2 46
 Or 166 miles.

By my Ephemeris, the moon passed the meridian of London at 7 h. 13 m. P. M. which is 2 m. 2 $\frac{1}{2}$ f. before the observed time.

Now by the Ephemeris the moon's daily diff. is 45 min.

Then by art. 1123,

As 45 min. : 2 min. 2 $\frac{1}{2}$ sec. :: 360° : 16° 18', the diff. long.

Dep. lat. = 35° 36' N.
 Diff. lat. = 2 32 S.
 Lat. by account = 33 04 N.
 Lat. by observation = 32 50 N.
 Sum lats. = 68 26
 Mid. lat. = 34 13
 Dep. long. = 15 10 W.
 Diff. long. = 1 15 W.
 Pref. long. = 10 30 W.

M. P.
 Lizard's lat. = 49° 57' N. 3470
 Ship's lat. = 32 50 N. 2088
 Diff. lat. = 17 07 1382
 Or = 1027 miles.
 Lizard's long. = 5° 14' W.
 Ship's long. = 16 18 W.
 Diff. long. = 11 04 W. = 664 m.
 Lizard bears N. 25° 30' E.
 Distance 1140 miles.

The lat. by account falling short of the true lat. by 14 m., and the diff. lat. being above twice the dep., the correction at art. 1112 is to be used.

To diff. lat. 151,8 and dep. 56, the course is 20 deg. 30 min.: With 20 deg. 30 min. and true diff. lat. 166, the true dep. is 62.

With which find the diff. longitude.

The long. by acct. nearly agrees with the long. found by observation.

M. P.
 Fonchal's lat. = 32° 38' N. 2073
 Ship's lat. = 32 50 N. 2088
 Diff. lat. = 0 12 15
 Or = 12 miles.
 Fonchal's lon. = 16° 51' W.
 Ship's long. = 16 18 W.
 Diff. long. = 0 33 miles.
 Fonchal bears S. 65° 15' W.
 Distance 29 miles. SECT.

S E C T I O N X I.

Of the surveying of Coasts and Harbours.

From what has been already said in the latter sections of Plane sailing, an intelligent reader would see how the business of taking the bearings of part of a coast, and of plotting or delineating it, might be done: But as there are some particulars which can be gained only by experience in the art of surveying, it will not be improper to apprize the learner concerning them, and thereby qualify him to go more readily to work.

1137. *To take the draught of a coast in sailing along it.*

1st. Having brought the ship to a convenient place, from whence the principal points of the coast or bay may be seen, either cast anchor if 'tis convenient, or *lye to* as steady as possible; or if the coast is too shoal, let the observations and measures be done in a boat: Then while the vessel is in a stationary situation, take with the azimuth compass the bearings of such points of the coast as form the most material projections or hollows; write down these bearings, and make a rough sketch of the appearance of the coast, observing carefully to mark the points whose bearings were taken.

2d. Then let the ship or boat run in a direct line one, two, or three miles, more or less, until she comes into a situation from whence the same points before observed can be seen again; there let the vessel *lye steady* as at the foregoing station, and again observe the respective bearings of the same noted points, which are also to be wrote down, and a rough sketch of the coast should also be taken from this station: But while the vessel is running the *base line* from station to station, a more accurate drawing of the appearance of the coast should be made.

3d. *To map these observations.* In some convenient part of a sheet of paper describe a circle (the larger the better), on which lay the several bearings taken from the first station, and let them be numbered 1, 2, 3, &c. on the outside of the circle: Also lay down the several bearings taken from the second station, let these be numbered 1, 2, 3, &c. on the inside of the circle, observing that the bearings of the same points are numbered with the same figures.

4th. Draw a line to express the ship's run both in length and course; and from that end of the line expressing the first station, draw lines parallel to the respective bearings taken from that end, and marked on the outside of the circle: Also from the other end, draw lines parallel to the bearings taken at that end, and noted on the inside of the circle; mark the intersection of each pair of lines, directed to the same point, with the number annexed to their bearing; and thro' the intersections so marked, draw by hand a curved line, observing to wave the line in and out, as near as can be like the trending of the coast itself.

5th. Against each part, draw the appearance of the elevated or low ground as marked in the sketches, distinguishing rocks, cliffs, high-land, low-land, sand hills, &c. If there are any currents or eddies, express them in their proper places by darts, the points being turned that way the current sets: Put in the several soundings at low water in small figures, distinguishing whether fathoms or feet; shew the time of high water on the full and change days by roman figures, and tell the rise in feet: Put in a compass corrected by the variation, with a scale of miles or leagues, such as the vessel's run was laid down by.

6th. If there is a shoal or sand on the coast, let it be taken by a boat sailing round it, and keeping an account of the courses, distances and soundings: But to put it in the draught, the boat must from some part of the sand or shoal, take the bearings of two points on the coast, whose bearings have been taken from the ship: Or, the bearing of the boat at some part of the shoal, or of some beacon in that place, must be taken by the ship at the stations where she takes the bearings of the shore; for by either of these means, one point of the sand being obtained, the rest of it can be laid down from the boat's account.

7th. If the coast to be drawn is a bay or harbour, winding in such a manner that all its principal points cannot be seen at two stations, let as many bases or lines be run and exactly measured as may be found necessary; observing that the several distances run should join to one another in the nature of a traverse, that each new set of objects or points observed, should be taken from two stations at the ends of a known distance, and that the objects, whose bearings are taken, do not extend so much beyond the limits of the base, as to make angles with it of less than about $\frac{1}{2}$ or $\frac{3}{4}$ of a point; but rather reserve such objects for the next measured base; for when lines lie very oblique to one another, their intersection is not easily ascertained.

8th. If any particular parts of the harbour cannot be conveniently seen from either of the stations, take the boat into those places, and having well examined them, make sketches thereof, estimating the lengths and breadths of the several inlets either by the rowing or sailing of the boat, taking as many bearings, soundings and other notes, as may be thought necessary; then annex these particular views in their proper places in the general draught.

9th. If there are any dangerous sands or rocks, besides inserting them in their proper places, there should be a double line drawn thro' that point and one or more objects ashore; and for this purpose, chuse a church, mill, house, noted tree, a cliff, or any other remarkable thing that can be distinctly seen at sea, and which can be brought to bear in the same right line with the point to be avoided: But if that point is under water, there must be two land marks brought to bear with the danger, either in the same right line when it can be, or in two lines; and that those land-marks may be put down in their proper places, their bearings must also be taken from two of the ship's stations.

10th. It

10th. It should be remarked in the draught what places, if any, are unfit for anchorage, and what are fit, by writing *rocky ground, foul anchorage, good anchorage*; and in the latter to draw the figure of an anchor: Also, if there is any particular channel more convenient to sail thro' than another, it is to be pointed out by lines drawn to its entrance from two or more noted marks ashore.

The foregoing method of surveying a coast, supposes in general that it is taken by a ship in her passing along, not having an opportunity of going ashore: But when the circumstances will permit the measures and observations to be made on land, the survey can be taken more accurately than on the water.

1138. *To survey a harbour by observations on shore.*

1st. Go round the place to be surveyed, and in the most remarkable points and bends of the shore fix station staves, or strait poles, tall enough to be seen at a considerable distance; but if at any of those places there is a noted tree, house, or any other remarkable thing, that object may serve instead of a station staff; and it will be convenient to black the staves, and tie a piece of white bunting to the top of each.

2d. Chuse a spot of level ground, on which measure, for a base, a length of half a mile, or a mile if convenient, observing that the direction of the measured line be so laid out, that from both ends of it all the station staves before planted, or the objects before remarked, may be seen.

3d. From one end of the base, observe (with any instrument proper to take bearings) the positions or bearings of all the staves or objects, and write them down orderly; do the same from the other end of the base.

4th. Then these measures being plotted or laid down, will give the most conspicuous points of the shore; the intermediate spaces are to be filled up from the sketches of them made on the spot, as in 8th of art. 1137.

5th. But if any of these objects should spread on either hand so far beyond the limits of the base, that at either end thereof, the other end and those objects or staves should appear nearly in the same direction, or to make angles not exceeding about 10 degrees; then it will be proper to chuse another level place, on which a base may be measured like the former, and from the extremities of which, the ends of the first base may be seen, and also the remaining objects or staves which were too oblique for the first base: And proceed in like manner to a 3d, 4th, &c. base if necessary. The rest like the directions in art. 1137.

Sea draughts taken according to these precepts, and neatly drawn, besides the real use they may be of, can't fail to recommend the young mariner, who surveys and constructs them, to the notice of his superiors.

SECTION

S E C T I O N XII.

To estimate distances.

Besides the method of finding the distances of remote objects by trigonometrical computations, there are two other means which may be usefully applied when the former cannot; and these are, the *motion of sound*, and the *curvature of the earth*.

1139.

Of the motion of sound.

It is a well known observation, that the noise or sound arising from any blow or shock made at a distance, is always heard some time after the stroke is seen, or known to be given; and hence it is generally concluded that the seeing of any act done within view is instantaneous, but that sound travels at a perceptible rate.

Some of the most eminent philosophers, judging that the knowledge of the flight of sound might be of use on various occasions, have been at extraordinary pains and expence to measure the rate at which it moved, and the result of their experiments, particularly of those which were best conducted, are as follows.

1st. The velocity of sound is the same whether, by sea or by land, in dry or in rainy weather, by day or by night, in winter or summer.

2d. That sound, whether more or less strong, flies with the same swiftness: For by experiments, a cannon fired with a half pound charge of powder was heard at about the distance of $17\frac{1}{2}$ miles in the same time after the flash was seen, as when fired with a charge of 6 lb.

3d. That the times in which sound is heard is proportional to the distance; that is, at a double distance it is heard in twice the time, at a triple distance in thrice the time, &c.

4th. That sound flies quicker or slower, by just as much as is the velocity of the wind, according as it is with or against the motion of the sound.

5th. That sound travels at the rate of about 1142 feet, or 380 yards in one second of time; so that

Sound moves in 1 second 1142 feet, or 380 yards.

in 1 minute 22840 yards, or about 13 miles.

Which is about a mile in 4,6 seconds, or about a league in 14 seconds.

But sea miles are to land miles, nearly as 7 to 6. (900)

Therefore sound runs a sea league in about 12 seconds of time.

It is a common observation, that persons in good health have about 75 pulsations or beats of the artery at the wrist in a minute.

1140. Therefore in 75 pulsations, sound flies about 13 land miles, and about $11\frac{1}{7}$ sea miles; which is about 1 land mile in 6 pulses, and about 1 sea mile in near 7 pulses, or a league in 20 pulses.

And hence the distance of objects may be found by knowing the time which sound takes to move from those objects to an observer.

EXAMPLE.

EXAMPLE. Upon seeing the flash of a gun at sea, I counted 56 beats of the pulse at the wrist before I heard the report: How far was that gun from me?

Now 56 divided by 20 gives 2,8 leagues, or about 8 miles.

1141.

Of the curvature of the earth.

Most persons know that if they are raised above the surface of the adjacent land or water, they can not only see distant objects that lie on that surface better, but also see those more and more remote as they advance higher. The irregularity of the surface of the land will not be subjected to any one rule that will give the distance to which objects may be seen at different elevations; but at sea, where there is generally an uniform curvature of the surface of the water, upon the supposition of the spherical figure of the earth, those distances may be easily computed.

1142. Let (see fig. 12. Pl. XI.) the eye at E be raised the height ES above the surface s of the water; then will it see an object at f on the surface, in the tangent Ef, which is the distance of that object from the eye.

Now in the triangle Efc, right angled at f, there are known, the side cf = semidiameter of the earth, and the side CE equal to the semidiameter cs increased by SE the height of the eye: To find the tangent Ef.

$$\text{Then by art. 203. } Ef = \sqrt{CE^2 - cf^2} = \sqrt{cs + SE^2 - cf^2} = \sqrt{2cs + SE \times SE}.$$

That is, To the earth's diameter add the height of the eye, multiply the sum by that height; then the square root of the product is the distance at which an object on the surface of the water can be seen by an eye so elevated.

And by this rule was the table at art. 1145 computed, the diameter of the earth being taken at 41798117 feet, according to Sir Isaac Newton's measures.

This table may be usefully applied to estimate the distance of an object at sea, the elevation of that object above its horizon being known.

EXAMPLE I. Sailing towards a headland on which is a lighthouse elevated 600 feet above the surface of the water, we saw the lights at night just appearing in the horizon: How far were we at that time distant from the lighthouse?

Seek in the table at article 1145 for 600 feet in the column signed height in feet, and right against it in the column signed distance in miles, stands 29,994; so that the distance may be reckoned about 30 miles.

EXAMPLE II. Being in company with some merchants walking on a sandy shore on the look out for a vessel which was then expected, whose top-gallant mast was 140 feet above the surface, allowance being made for her immersion in the water; we observed thro' a telescope a ship's vane just appearing in the horizon: How far off is that ship, supposing it the vessel expected?

ANSWER.

ANSWER. Against 140 feet, the height, stands 14,488; that is, her distance is about $14\frac{1}{2}$ miles.

1143. Here is no allowance made for the height of the eye above the horizon; but it is obvious, that the higher the eye, the farther it can see: Now as objects are seen in a strait line, and that line is a tangent to the earth's surface; therefore it follows, that *to find the distance of two elevated objects when the right line joining them touches the surface of the earth between those objects; Seek the distance answering to each height, and their sum is the distance sought.*

Thus in example II. suppose the eye is raised 6 feet above the water's edge, it can see an object on the surface 2,999, or 3 miles off; this distance added to the $14\frac{1}{2}$ miles, makes the distance of the ship to be $17\frac{1}{2}$ miles.

EXAMPLE III. *A man being on the main-top gallant mast of a man of war, 200 feet above the water, sees a 100 gun ship she had engaged the day before, hull to: How far were those ships distant?*

A ship of 100 guns, or a first rate man of war, is about 60 feet from the keel to the rails; from which deduct about 20 feet for the draught of water, leaves 40 feet for the height of her quarter above water: Now a ship is seen *hull to*, when her upper-works just disappear.

Then 200 feet high, gives 17,316 miles.

And against 40 feet stands 7,714 miles.

Their sum, which is 25,030 miles, is their distance.

1144. It might perhaps hereafter be found useful to mariners, could a list of the most considerable lighthouses, and other elevated objects in different parts of the world, be obtained; together with their heights above the water: For as some of these objects are so high as to be seen at the distance of 30 or 40 miles, and even more; and as their distances upon being first seen could be near enough known by the table at art. 1145, were their heights known; therefore such a list might on several occasions not only give much satisfaction to a seaman, but even become of great use to know his distance from those objects, as he could thereby know how to increase or slacken sail in order to save his tide, or other emergency; also he would know better how to lay down the distances of rocks or shoals that lie off a coast, &c. But as such a list is not easy to be come at, unless skilful mariners would be at the trouble of procuring such as occurred to them in their voyages, and on their return communicate them to some public body; therefore the curious in those matters must be contented at present with what is here already said.

Art. 1145. A TABLE for finding the distance of observed terrestrial objects at sea.

Height in feet	Dist. in miles	Height in feet	Dist. in miles	Height in feet	Dist. in miles	Height in feet	Dist. in miles	Height in feet	Dist. in miles
1	1,224	53	8,914	240	18,969	760	33,756	2250	58,083
2	1,732	56	9,163	245	19,165	780	34,197	2300	58,725
3	2,121	59	9,405	250	19,360	800	34,633	2350	59,360
4	2,449	62	9,641	255	19,553	820	35,063	2400	59,988
5	2,718	65	9,872	260	19,743	840	35,488	2450	60,609
6	2,999	68	10,097	265	19,932	860	35,908	2500	61,225
7	3,239	71	10,317	270	20,119	880	36,323	2550	61,834
8	3,463	74	10,533	275	20,305	900	36,734	2600	62,437
9	3,673	77	10,745	280	20,489	920	37,140	2650	63,035
10	3,872	80	10,952	285	20,671	940	37,541	2700	63,626
11	4,061	83	11,156	290	20,851	960	37,938	2750	64,213
12	4,242	86	11,355	295	21,030	980	38,332	2800	64,794
13	4,415	89	11,552	300	21,208	1000	38,721	2850	65,370
14	4,581	92	11,745	310	21,558	1030	39,297	2900	65,941
15	4,742	95	11,931	320	21,903	1060	39,866	2950	66,507
16	4,898	98	12,121	330	22,243	1090	40,426	3000	67,068
17	5,048	101	12,306	340	22,578	1120	40,978	3050	67,625
18	5,195	104	12,487	350	22,907	1150	41,524	3100	68,177
19	5,338	107	12,666	360	23,232	1180	42,062	3150	68,725
20	5,476	110	12,842	370	23,553	1210	42,593	3200	69,268
21	5,611	113	13,016	380	23,869	1240	43,118	3250	69,807
22	5,743	116	13,188	390	24,181	1270	43,636	3300	70,342
23	5,872	119	13,357	400	24,489	1300	44,149	3350	70,873
24	5,999	122	13,525	410	24,793	1330	44,655	3400	71,400
25	6,122	125	13,690	420	25,094	1360	45,156	3450	71,923
26	6,243	128	13,853	430	25,391	1390	45,651	3500	72,443
27	6,362	131	14,015	440	25,684	1420	46,141	3550	72,958
28	6,479	134	14,174	450	25,974	1450	46,629	3600	73,470
29	6,594	137	14,332	460	26,261	1480	47,106	3650	73,979
30	6,709	140	14,488	470	26,545	1510	47,581	3700	74,484
31	6,817	143	14,642	480	26,826	1540	48,052	3750	74,985
32	6,928	146	14,795	490	27,104	1570	48,517	3800	75,484
33	7,034	150	14,995	500	27,379	1600	48,979	3850	75,979
34	7,140	155	15,244	510	27,652	1630	49,436	3900	76,471
35	7,244	160	15,488	520	27,922	1660	49,889	3950	76,959
36	7,347	165	15,728	530	28,189	1690	50,338	4000	77,445
37	7,448	170	15,965	540	28,454	1720	50,782	4050	77,927
38	7,548	175	16,198	550	28,716	1750	51,223	4100	78,407
39	7,647	180	16,427	560	28,976	1780	51,661	4150	78,884
40	7,744	185	16,654	570	29,233	1810	52,094	4200	79,358
41	7,840	190	16,878	580	29,489	1840	52,524	4250	79,829
42	7,935	195	17,098	590	29,742	1870	52,951	4300	80,297
43	8,029	200	17,316	600	29,994	1900	53,374	4350	80,762
44	8,122	205	17,531	620	30,489	1930	53,793	4400	81,225
45	8,214	210	17,744	640	30,976	1960	54,210	4450	81,685
46	8,305	215	17,955	660	31,457	2000	54,761	4500	82,143
47	8,394	220	18,162	680	31,930	2050	55,441	4550	82,598
48	8,483	225	18,366	700	32,396	2100	56,113	4600	83,051
49	8,570	230	18,569	720	32,855	2150	56,777	4650	83,501
50	8,658	235	18,770	740	33,309	2200	57,434	4700	83,949

Cccc

Remarks

Remarks on the use of the following Tables.

1146. Those who require no greater accuracy in the operations than to assign the distances or lengths of lines in whole miles or leagues, and the angles of triangles in points and quarter-points, may be accommodated in the first page only of the following logarithmic tables; wherein are two which will sufficiently answer their purpose. Indeed it is best to use the second of these tables, whenever the angle is given in points and quarters; for the quarter-points, containing degrees minutes and seconds, cannot be accurately found among the common tables of sines, tangents and secants; and therefore was the table at article 1150 added, wherein the log. sines, tangents and secants agree, even to the seconds in the quarter points.

1147. In the table of the logarithms of numbers, article 1151, the indices are omitted, because the logarithms serve as well for fractional numbers as for integers, as shewn in article 84; and the index is easily annexed in their use by the rule at article 82, observing that when whole numbers are used, the index of the logarithm is always one less than the number of places in that whole number; and that the index of a logarithm increased by one, gives the integer places in the corresponding number: So that when these tables are used, the operator must be careful always to annex the index.

1148. In the table, article 1152, there are no secants added, because they are not used in any part of this work; but those who are desirous to compute by them, may easily supply this want, by taking the arithmetical complements (by article 88) of the cosines to the degrees and minutes for which the secant is required: The logarithmic secants and cosines of the same arcs being the arithmetical complements of one another, as shewn in article 367.

Articles 1149, 1150, 1151, 1152.

LOGARITHMIC TABLES.

I. The LOGARITHMS of NUMBERS from 1 to 120.

II. The LOGARITHMIC SINES, TANGENTS and SECANTS, to every Point and Quarter-Point of the COMPASS.

III. The LOGARITHMS of NUMBERS from 100 to 10000.

IV. The LOGARITHMS of the SINES and TANGENTS to every Degree and Minute of the QUADRANT.

1149.

A T A B L E

Of Numbers and their Logarithms from 1 to 120.

N.	Log.	N.	Log.	N.	Log.	N.	Log.	N.	Log.	N.	Log.
1	0,00000	21	1,32222	41	1,61278	61	1,78533	81	1,90848	101	2,00432
2	0,30103	22	1,34242	42	1,62325	62	1,79239	82	1,91381	102	2,00860
3	0,47712	23	1,36173	43	1,63347	63	1,79934	83	1,91908	103	2,01284
4	0,60206	24	1,38021	44	1,64345	64	1,80618	84	1,92428	104	2,01703
5	0,69897	25	1,39794	45	1,65321	65	1,81291	85	1,92942	105	2,02119
6	0,77815	26	1,41497	46	1,66276	66	1,81954	86	1,93450	106	2,02531
7	0,84510	27	1,43136	47	1,67210	67	1,82607	87	1,93952	107	2,02938
8	0,90309	28	1,44716	48	1,68124	68	1,83251	88	1,94448	108	2,03342
9	0,95424	29	1,46240	49	1,69020	69	1,83885	89	1,94939	109	2,03743
10	1,00000	30	1,47712	50	1,69897	70	1,84510	90	1,95424	110	2,04139
11	1,04139	31	1,49136	51	1,70757	71	1,85126	91	1,95904	111	2,04532
12	1,07918	32	1,50515	52	1,71600	72	1,85733	92	1,96379	112	2,04922
13	1,11394	33	1,51851	53	1,72428	73	1,86332	93	1,96848	113	2,05308
14	1,14613	34	1,53148	54	1,73239	74	1,86923	94	1,97313	114	2,05690
15	1,17609	35	1,54407	55	1,74036	75	1,87506	95	1,97772	115	2,06070
16	1,20412	36	1,55630	56	1,74819	76	1,88081	96	1,98227	116	2,06446
17	1,23045	37	1,56820	57	1,75587	77	1,88649	97	1,98677	117	2,06819
18	1,25527	38	1,57978	58	1,76343	78	1,89209	98	1,99123	118	2,07188
19	1,27875	39	1,59106	59	1,77085	79	1,89763	99	1,99563	119	2,07555
20	1,30103	40	1,60206	60	1,77815	80	1,90309	100	2,00000	120	2,07918

1150.

A T A B L E

Of Logarithmic Sines, Tangents and Secants, to every Quarter point of the Compass.

Pts.	Sines	Co-fines	Tangents	Co-tang.	Secants	Co-secants	
0	0,00000	10,00000	0,00000	Infinite	10,00000	Infinite	8
0 $\frac{1}{4}$	8,69079	9,99947	8,69131	11,30867	10,00053	11,30921	7 $\frac{3}{4}$
0 $\frac{1}{2}$	8,99130	9,99790	8,99340	11,00660	10,00210	11,00870	7 $\frac{1}{2}$
0 $\frac{3}{4}$	9,14652	9,99527	9,17124	10,82675	10,00473	10,85348	7 $\frac{1}{4}$
1	9,29024	9,99157	9,29866	10,70134	10,00843	10,70976	7
1 $\frac{1}{4}$	9,38557	9,98678	9,39878	10,60121	10,01322	10,61443	6 $\frac{3}{4}$
1 $\frac{1}{2}$	9,46282	9,98088	9,48194	10,51806	10,01912	10,53717	6 $\frac{1}{2}$
1 $\frac{3}{4}$	9,52749	9,97384	9,55365	10,44635	10,02616	10,47251	6 $\frac{1}{4}$
2	9,58284	9,96562	9,61722	10,38278	10,03438	10,41716	6
2 $\frac{1}{4}$	9,63099	9,95616	9,67482	10,32518	10,04384	10,36901	5 $\frac{3}{4}$
2 $\frac{1}{2}$	9,67339	9,94543	9,72796	10,27204	10,05457	10,32661	5 $\frac{1}{2}$
2 $\frac{3}{4}$	9,71104	9,93335	9,77770	10,22230	10,06665	10,28896	5 $\frac{1}{4}$
3	9,74474	9,91985	9,82489	10,17511	10,08015	10,25526	5
3 $\frac{1}{4}$	9,77503	9,90483	9,87020	10,12980	10,09517	10,22497	4 $\frac{3}{4}$
3 $\frac{1}{2}$	9,80236	9,88819	9,91417	10,08583	10,11181	10,19764	4 $\frac{1}{2}$
3 $\frac{3}{4}$	9,82708	9,86979	9,95729	10,04270	10,13021	10,17292	4 $\frac{1}{4}$
4	9,84948	9,84948	10,00000	10,00000	10,15052	10,15052	4
	Co-fines	Sines	Co-tang.	Tangents	Co-secants	Secants	Pts.

Numbers from 100 to 1600. Logs. from ,00000 to ,20412.

N ^o	0	1	2	3	4	5	6	7	8	9
100	,00000	,00043	,00087	,00130	,00173	,00217	,00261	,00303	,00346	,00389
101	,00432	,00475	,00518	,00561	,00604	,00647	,00689	,00732	,00775	,00817
102	,00860	,00903	,00945	,00988	,01030	,01072	,01115	,01157	,01199	,01242
103	,01284	,01326	,01368	,01410	,01452	,01494	,01536	,01578	,01620	,01661
104	,01703	,01745	,01787	,01828	,01870	,01912	,01953	,01995	,02036	,02077
105	,02119	,02160	,02202	,02243	,02284	,02325	,02366	,02407	,02449	,02490
106	,02531	,02571	,02612	,02653	,02694	,02735	,02776	,02816	,02857	,02898
107	,02938	,02979	,03019	,03060	,03100	,03141	,03181	,03222	,03262	,03302
108	,03342	,03383	,03423	,03463	,03503	,03543	,03583	,03623	,03663	,03703
109	,03743	,03782	,03822	,03862	,03902	,03941	,03981	,04021	,04060	,04100
110	,04139	,04179	,04218	,04257	,04297	,04336	,04375	,04415	,04454	,04493
111	,04532	,04571	,04610	,04649	,04688	,04727	,04766	,04805	,04844	,04883
112	,04922	,04961	,04999	,05038	,05077	,05115	,05154	,05192	,05231	,05269
113	,05308	,05346	,05385	,05423	,05461	,05500	,05538	,05576	,05614	,05652
114	,05690	,05729	,05767	,05805	,05843	,05880	,05918	,05956	,05994	,06032
115	,06070	,06108	,06145	,06183	,06221	,06258	,06296	,06333	,06371	,06408
116	,06446	,06483	,06521	,06558	,06595	,06633	,06670	,06707	,06744	,06781
117	,06819	,06856	,06893	,06930	,06967	,07004	,07041	,07078	,07114	,07151
118	,07188	,07225	,07262	,07298	,07335	,07372	,07408	,07445	,07482	,07518
119	,07555	,07591	,07628	,07664	,07700	,07737	,07773	,07809	,07846	,07882
120	,07918	,07954	,07990	,08027	,08063	,08099	,08134	,08171	,08207	,08243
121	,08278	,08314	,08350	,08386	,08422	,08458	,08493	,08529	,08565	,08600
122	,08636	,08672	,08707	,08743	,08778	,08814	,08849	,08884	,08920	,08955
123	,08990	,09026	,09061	,09096	,09131	,09167	,09202	,09237	,09272	,09307
124	,09342	,09377	,09412	,09447	,09482	,09517	,09552	,09587	,09621	,09656
125	,09691	,09726	,09760	,09795	,09830	,09864	,09899	,09933	,09968	,10003
126	,10037	,10071	,10106	,10140	,10175	,10209	,10243	,10278	,10312	,10346
127	,10380	,10414	,10449	,10483	,10517	,10551	,10585	,10619	,10653	,10687
128	,10721	,10755	,10789	,10823	,10856	,10890	,10924	,10958	,10992	,11025
129	,11059	,11093	,11126	,11160	,11193	,11227	,11260	,11294	,11327	,11361
130	,11394	,11428	,11461	,11494	,11528	,11561	,11594	,11628	,11661	,11694
131	,11727	,11760	,11793	,11826	,11859	,11893	,11926	,11959	,11991	,12024
132	,12057	,12090	,12123	,12156	,12189	,12222	,12254	,12287	,12320	,12352
133	,12385	,12418	,12450	,12483	,12516	,12548	,12581	,12613	,12646	,12678
134	,12710	,12743	,12775	,12808	,12840	,12872	,12904	,12937	,12969	,13001
135	,13033	,13065	,13098	,13130	,13162	,13194	,13226	,13258	,13290	,13322
136	,13354	,13386	,13418	,13450	,13481	,13513	,13545	,13577	,13609	,13640
137	,13672	,13704	,13735	,13767	,13799	,13830	,13862	,13893	,13925	,13956
138	,13988	,14019	,14051	,14082	,14114	,14145	,14176	,14208	,14239	,14270
139	,14301	,14333	,14364	,14395	,14426	,14457	,14488	,14520	,14551	,14582
140	,14613	,14644	,14675	,14706	,14737	,14768	,14798	,14829	,14860	,14891
141	,14922	,14953	,14983	,15014	,15045	,15076	,15106	,15137	,15168	,15198
142	,15229	,15259	,15290	,15320	,15351	,15381	,15412	,15442	,15473	,15503
143	,15534	,15564	,15594	,15625	,15655	,15685	,15715	,15746	,15776	,15806
144	,15836	,15866	,15896	,15927	,15957	,15987	,16017	,16047	,16077	,16107
145	,16137	,16167	,16197	,16227	,16256	,16286	,16316	,16346	,16376	,16405
146	,16435	,16465	,16495	,16524	,16554	,16584	,16613	,16643	,16673	,16702
147	,16732	,16761	,16791	,16820	,16850	,16879	,16909	,16938	,16967	,16997
148	,17026	,17055	,17085	,17114	,17143	,17172	,17202	,17231	,17260	,17289
149	,17319	,17348	,17377	,17406	,17435	,17464	,17493	,17522	,17551	,17580
150	,17609	,17638	,17667	,17696	,17725	,17754	,17782	,17811	,17840	,17869
151	,17898	,17926	,17955	,17984	,18013	,18041	,18070	,18099	,18127	,18156
152	,18184	,18213	,18241	,18270	,18298	,18327	,18355	,18384	,18412	,18441
153	,18469	,18497	,18526	,18554	,18583	,18611	,18639	,18667	,18696	,18724
154	,18752	,18780	,18808	,18837	,18865	,18893	,18921	,18949	,18977	,19005
155	,19033	,19061	,19089	,19117	,19145	,19173	,19201	,19229	,19257	,19285
156	,19312	,19340	,19368	,19396	,19424	,19451	,19479	,19507	,19535	,19562
157	,19590	,19618	,19645	,19673	,19700	,19728	,19756	,19783	,19811	,19838
158	,19866	,19893	,19921	,19948	,19975	,20003	,20030	,20058	,20085	,20112
159	,20140	,20167	,20194	,20222	,20249	,20276	,20303	,20330	,20358	,20385
N ^o	0	1	2	3	4	5	6	7	8	9

Numbers from 1600 to 2200. Logs. from ,20412 to ,34242.

N ^o	0	1	2	3	4	5	6	7	8	9
160	,20412	,20439	,20466	,20493	,20520	,20547	,20574	,20602	,20629	,20656
161	,20683	,20709	,20736	,20763	,20790	,20817	,20844	,20871	,20898	,20925
162	,20951	,20978	,21005	,21032	,21059	,21085	,21112	,21139	,21165	,21192
163	,21219	,21245	,21272	,21299	,21325	,21352	,21378	,21405	,21431	,21458
164	,21484	,21511	,21537	,21564	,21590	,21617	,21643	,21669	,21696	,21722
165	,21748	,21775	,21801	,21827	,21853	,21880	,21906	,21932	,21958	,21985
166	,22011	,22037	,22063	,22089	,22115	,22141	,22167	,22194	,22220	,22246
167	,22272	,22298	,22324	,22350	,22375	,22401	,22427	,22453	,22479	,22505
168	,22531	,22557	,22583	,22608	,22634	,22660	,22686	,22711	,22737	,22763
169	,22789	,22814	,22840	,22866	,22891	,22917	,22943	,22968	,22994	,23019
170	,23045	,23070	,23096	,23121	,23147	,23172	,23198	,23223	,23249	,23274
171	,23300	,23325	,23350	,23376	,23401	,23426	,23452	,23477	,23502	,23528
172	,23553	,23578	,23603	,23628	,23654	,23679	,23704	,23729	,23754	,23779
173	,23805	,23830	,23855	,23880	,23905	,23930	,23955	,23980	,24005	,24030
174	,24055	,24080	,24105	,24130	,24155	,24179	,24204	,24229	,24254	,24279
175	,24304	,24329	,24353	,24378	,24403	,24428	,24452	,24477	,24502	,24527
176	,24551	,24576	,24601	,24625	,24650	,24674	,24699	,24724	,24748	,24773
177	,24797	,24822	,24846	,24871	,24895	,24920	,24944	,24969	,24993	,25018
178	,25042	,25066	,25091	,25115	,25139	,25164	,25188	,25212	,25237	,25261
179	,25285	,25310	,25334	,25358	,25382	,25406	,25431	,25455	,25479	,25503
180	,25527	,25551	,25575	,25600	,25624	,25648	,25672	,25696	,25720	,25744
181	,25768	,25792	,25816	,25840	,25864	,25888	,25912	,25935	,25959	,25983
182	,26007	,26031	,26055	,26079	,26102	,26126	,26150	,26174	,26198	,26221
183	,26245	,26269	,26292	,26316	,26340	,26364	,26387	,26411	,26434	,26458
184	,26482	,26505	,26529	,26552	,26576	,26600	,26623	,26647	,26670	,26694
185	,26717	,26741	,26764	,26787	,26811	,26834	,26858	,26881	,26905	,26928
186	,26951	,26975	,26998	,27021	,27045	,27068	,27091	,27114	,27138	,27161
187	,27184	,27207	,27231	,27254	,27277	,27300	,27323	,27346	,27370	,27393
188	,27416	,27439	,27462	,27485	,27508	,27531	,27554	,27577	,27600	,27623
189	,27646	,27669	,27692	,27715	,27738	,27761	,27784	,27807	,27830	,27852
190	,27875	,27898	,27921	,27944	,27967	,27989	,28012	,28035	,28058	,28081
191	,28103	,28126	,28149	,28171	,28194	,28217	,28239	,28262	,28285	,28307
192	,28330	,28353	,28375	,28398	,28420	,28443	,28466	,28488	,28511	,28533
193	,28556	,28578	,28601	,28623	,28646	,28668	,28690	,28713	,28735	,28758
194	,28780	,28802	,28825	,28847	,28870	,28892	,28914	,28937	,28959	,28981
195	,29003	,29026	,29048	,29070	,29092	,29115	,29137	,29159	,29181	,29203
196	,29226	,29248	,29270	,29292	,29314	,29336	,29358	,29380	,29402	,29425
197	,29447	,29469	,29491	,29513	,29535	,29557	,29579	,29601	,29623	,29645
198	,29666	,29688	,29710	,29732	,29754	,29776	,29798	,29820	,29842	,29863
199	,29885	,29907	,29929	,29951	,29972	,29994	,30016	,30038	,30059	,30081
200	,30103	,30125	,30146	,30168	,30190	,30211	,30233	,30255	,30276	,30298
201	,30320	,30341	,30363	,30384	,30406	,30427	,30449	,30471	,30492	,30514
202	,30535	,30557	,30578	,30600	,30621	,30642	,30664	,30685	,30707	,30728
203	,30750	,30771	,30792	,30814	,30835	,30856	,30878	,30899	,30920	,30942
204	,30963	,30984	,31006	,31027	,31048	,31069	,31091	,31112	,31133	,31154
205	,31175	,31197	,31218	,31239	,31260	,31281	,31302	,31323	,31344	,31366
206	,31387	,31408	,31429	,31450	,31471	,31492	,31513	,31534	,31555	,31576
207	,31597	,31618	,31639	,31660	,31681	,31702	,31723	,31744	,31764	,31785
208	,31806	,31827	,31848	,31869	,31890	,31911	,31931	,31952	,31973	,31994
209	,32015	,32035	,32056	,32077	,32098	,32118	,32139	,32160	,32180	,32201
210	,32222	,32243	,32263	,32284	,32305	,32325	,32346	,32366	,32387	,32408
211	,32428	,32449	,32469	,32490	,32510	,32531	,32552	,32572	,32593	,32613
212	,32634	,32654	,32674	,32695	,32715	,32736	,32756	,32777	,32797	,32818
213	,32838	,32858	,32879	,32899	,32919	,32940	,32960	,32980	,33001	,33021
214	,33041	,33062	,33082	,33102	,33122	,33143	,33163	,33183	,33203	,33224
215	,33244	,33264	,33284	,33304	,33326	,33345	,33365	,33385	,33405	,33425
216	,33445	,33465	,33486	,33506	,33526	,33546	,33566	,33586	,33606	,33626
217	,33646	,33666	,33686	,33706	,33726	,33746	,33766	,33786	,33806	,33826
218	,33846	,33865	,33885	,33905	,33925	,33945	,33965	,33985	,34005	,34025
219	,34044	,34064	,34084	,34104	,34123	,34143	,34163	,34183	,34203	,34222
N ^o	0	1	2	3	4	5	6	7	8	9

Numbers from 2200 to 2800. Logs. from ,34242 to ,44716.

N ^o	0	1	2	3	4	5	6	7	8	9
220	,34242	,34262	,34282	,34301	,34321	,34341	,34360	,34380	,34400	,34420
221	,34439	,34459	,34478	,34498	,34518	,34537	,34557	,34577	,34596	,34616
222	,34635	,34655	,34674	,34694	,34713	,34733	,34752	,34772	,34791	,34811
223	,34830	,34850	,34869	,34889	,34908	,34928	,34947	,34966	,34986	,35005
224	,35025	,35044	,35064	,35083	,35102	,35122	,35141	,35160	,35180	,35199
225	,35218	,35237	,35257	,35276	,35295	,35315	,35334	,35353	,35372	,35392
226	,35411	,35430	,35449	,35468	,35488	,35507	,35526	,35545	,35564	,35583
227	,35603	,35622	,35641	,35660	,35679	,35698	,35717	,35736	,35755	,35774
228	,35793	,35812	,35832	,35851	,35870	,35889	,35908	,35927	,35946	,35965
229	,35983	,36002	,36021	,36040	,36059	,36078	,36097	,36116	,36135	,36154
230	,36173	,36192	,36210	,36229	,36248	,36267	,36286	,36305	,36324	,36342
231	,36361	,36380	,36399	,36418	,36436	,36455	,36474	,36492	,36511	,36530
232	,36549	,36567	,36586	,36605	,36624	,36642	,36661	,36680	,36698	,36717
233	,36736	,36754	,36773	,36791	,36810	,36829	,36847	,36866	,36884	,36903
234	,36922	,36940	,36959	,36977	,36996	,37014	,37033	,37051	,37070	,37088
235	,37107	,37125	,37144	,37162	,37181	,37199	,37217	,37236	,37254	,37273
236	,37291	,37320	,37328	,37346	,37365	,37383	,37401	,37420	,37438	,37456
237	,37475	,37493	,37511	,37530	,37548	,37566	,37585	,37603	,37621	,37639
238	,37658	,37676	,37694	,37712	,37731	,37749	,37767	,37785	,37803	,37822
239	,37840	,37858	,37876	,37894	,37912	,37930	,37949	,37967	,37985	,38003
240	,38021	,38039	,38057	,38075	,38093	,38111	,38130	,38148	,38166	,38184
241	,38202	,38220	,38238	,38256	,38274	,38292	,38310	,38328	,38346	,38364
242	,38381	,38399	,38417	,38435	,38453	,38471	,38489	,38507	,38525	,38543
243	,38561	,38578	,38596	,38614	,38632	,38650	,38668	,38685	,38703	,38721
244	,38739	,38757	,38775	,38792	,38810	,38828	,38846	,38863	,38881	,38899
245	,38917	,38934	,38952	,38970	,38987	,39005	,39023	,39040	,39058	,39076
246	,39093	,39111	,39129	,39146	,39164	,39182	,39199	,39217	,39234	,39252
247	,39270	,39287	,39305	,39322	,39340	,39357	,39375	,39392	,39410	,39428
248	,39445	,39463	,39480	,39498	,39515	,39533	,39550	,39568	,39585	,39602
249	,39620	,39637	,39655	,39672	,39690	,39707	,39724	,39742	,39759	,39777
250	,39794	,39811	,39829	,39846	,39863	,39881	,39898	,39915	,39932	,39950
251	,39967	,39985	,40002	,40019	,40036	,40054	,40071	,40088	,40106	,40123
252	,40140	,40157	,40174	,40192	,40209	,40226	,40243	,40260	,40278	,40295
253	,40312	,40329	,40346	,40363	,40381	,40398	,40415	,40432	,40449	,40466
254	,40483	,40500	,40517	,40535	,40552	,40569	,40586	,40603	,40620	,40637
255	,40654	,40671	,40688	,40705	,40722	,40739	,40756	,40773	,40790	,40807
256	,40824	,40841	,40858	,40875	,40892	,40909	,40926	,40943	,40959	,40976
257	,40993	,41010	,41027	,41044	,41061	,41078	,41095	,41111	,41128	,41145
258	,41162	,41179	,41196	,41212	,41229	,41246	,41263	,41280	,41296	,41313
259	,41330	,41347	,41363	,41380	,41397	,41414	,41430	,41447	,41464	,41481
260	,41497	,41514	,41531	,41547	,41564	,41581	,41597	,41614	,41631	,41647
261	,41664	,41681	,41697	,41714	,41731	,41747	,41764	,41780	,41797	,41813
262	,41830	,41847	,41863	,41880	,41896	,41913	,41929	,41946	,41962	,41979
263	,41996	,42012	,42029	,42045	,42062	,42078	,42094	,42111	,42127	,42144
264	,42160	,42177	,42193	,42220	,42226	,42243	,42259	,42275	,42292	,42308
265	,42325	,42341	,42357	,42374	,42390	,42406	,42423	,42439	,42455	,42472
266	,42488	,42504	,42521	,42537	,42553	,42570	,42586	,42602	,42619	,42635
267	,42651	,42667	,42684	,42700	,42716	,42732	,42749	,42765	,42781	,42797
268	,42813	,42830	,42846	,42862	,42878	,42894	,42911	,42927	,42943	,42959
269	,42975	,42991	,43007	,43024	,43040	,43056	,43072	,43088	,43104	,43120
270	,43136	,43152	,43168	,43185	,43201	,43217	,43233	,43249	,43265	,43281
271	,43296	,43313	,43329	,43345	,43361	,43377	,43393	,43409	,43425	,43441
272	,43457	,43473	,43489	,43505	,43521	,43537	,43553	,43568	,43584	,43600
273	,43616	,43632	,43648	,43664	,43680	,43696	,43712	,43727	,43743	,43759
274	,43775	,43791	,43807	,43823	,43838	,43854	,43870	,43886	,43902	,43917
275	,43933	,43949	,43965	,43981	,43996	,44012	,44028	,44044	,44059	,44075
276	,44091	,44107	,44122	,44138	,44154	,44169	,44185	,44201	,44217	,44232
277	,44248	,44264	,44279	,44295	,44311	,44326	,44342	,44358	,44373	,44389
278	,44404	,44420	,44436	,44451	,44467	,44482	,44498	,44514	,44529	,44545
279	,44560	,44576	,44591	,44607	,44623	,44638	,44654	,44669	,44685	,44700
N ^o	0	1	2	3	4	5	6	7	8	9

Numbers from 2800 to 3400. Logs. from ,44716 to ,53148.

N ^o	0	1	2	3	4	5	6	7	8	9
280	,4716	,44731	,44747	,44762	,44778	,44793	,44809	,44824	,44840	,44855
281	,4871	,44886	,44901	,44917	,44932	,44948	,44963	,44979	,44994	,45009
282	,45025	,45040	,45056	,45071	,45086	,45102	,45117	,45132	,45148	,45163
283	,45179	,45194	,45209	,45225	,45240	,45255	,45271	,45286	,45301	,45316
284	,45332	,45347	,45362	,45378	,45393	,45408	,45423	,45439	,45454	,45469
285	,45484	,45500	,45515	,45530	,45545	,45561	,45576	,45591	,45606	,45621
286	,45637	,45652	,45667	,45682	,45697	,45712	,45728	,45743	,45758	,45773
287	,45788	,45803	,45818	,45834	,45849	,45864	,45879	,45894	,45909	,45924
288	,45939	,45954	,45969	,45984	,45999	,46015	,46030	,46045	,46060	,46075
289	,46090	,46105	,46120	,46135	,46150	,46165	,46180	,46195	,46210	,46225
290	,4624	,46255	,46270	,46285	,46300	,46315	,46329	,46344	,46359	,46374
291	,46389	,46404	,46419	,46434	,46449	,46464	,46479	,46494	,46508	,46523
292	,46538	,46553	,46568	,46583	,46598	,46612	,46627	,46642	,46657	,46672
293	,46687	,46702	,46716	,46731	,46746	,46761	,46776	,46790	,46805	,46820
294	,46835	,46849	,46864	,46879	,46894	,46908	,46923	,46938	,46953	,46967
295	,46982	,46997	,47012	,47026	,47041	,47056	,47070	,47085	,47100	,47114
296	,47129	,47144	,47158	,47173	,47188	,47202	,47217	,47232	,47246	,47261
297	,47276	,47290	,47305	,47319	,47334	,47349	,47363	,47378	,47392	,47407
298	,47422	,47436	,47451	,47465	,47480	,47494	,47508	,47523	,47538	,47553
299	,47567	,47582	,47596	,47611	,47625	,47640	,47654	,47669	,47683	,47698
300	,47712	,47727	,47741	,47755	,47770	,47784	,47799	,47813	,47828	,47842
301	,47857	,47871	,47885	,47900	,47914	,47929	,47943	,47957	,47972	,47986
302	,48001	,48015	,48029	,48044	,48058	,48072	,48087	,48101	,48116	,48130
303	,48144	,48159	,48173	,48187	,48202	,48216	,48230	,48244	,48259	,48273
304	,48287	,48302	,48316	,48330	,48344	,48359	,48372	,48387	,48401	,48416
305	,48430	,48444	,48458	,48473	,48487	,48501	,48515	,48529	,48544	,48558
306	,48572	,48586	,48600	,48615	,48629	,48643	,48657	,48671	,48685	,48700
307	,48714	,48728	,48742	,48756	,48770	,48784	,48799	,48813	,48827	,48841
308	,48855	,48869	,48883	,48897	,48911	,48925	,48940	,48954	,48968	,48982
309	,48996	,49010	,49024	,49038	,49052	,49066	,49080	,49094	,49108	,49122
310	,49136	,49150	,49164	,49178	,49192	,49206	,49220	,49234	,49248	,49262
311	,49276	,49290	,49304	,49318	,49332	,49346	,49360	,49374	,49388	,49402
312	,49415	,49429	,49443	,49457	,49471	,49485	,49499	,49513	,49527	,49541
313	,49554	,49568	,49582	,49596	,49610	,49624	,49638	,49651	,49665	,49679
314	,49693	,49707	,49721	,49734	,49748	,49762	,49776	,49790	,49803	,49817
315	,49831	,49845	,49859	,49872	,49886	,49900	,49914	,49927	,49941	,49955
316	,49969	,49982	,49996	,50010	,50024	,50037	,50051	,50065	,50078	,50092
317	,50106	,50120	,50133	,50147	,50161	,50174	,50188	,50202	,50215	,50229
318	,50243	,50256	,50270	,50284	,50297	,50311	,50325	,50338	,50352	,50365
319	,50379	,50393	,50406	,50420	,50433	,50447	,50461	,50474	,50488	,50501
320	,50515	,50529	,50542	,50556	,50569	,50583	,50596	,50610	,50623	,50637
321	,50650	,50664	,50677	,50691	,50705	,50718	,50732	,50745	,50759	,50772
322	,50786	,50799	,50812	,50826	,50839	,50851	,50866	,50880	,50893	,50907
323	,50920	,50934	,50947	,50961	,50974	,50987	,51001	,51014	,51028	,51041
324	,51054	,51068	,51081	,51095	,51108	,51121	,51135	,51148	,51161	,51175
325	,51188	,51202	,51215	,51228	,51242	,51255	,51268	,51282	,51295	,51308
326	,51322	,51335	,51348	,51362	,51375	,51388	,51402	,51415	,51428	,51441
327	,51455	,51468	,51481	,51495	,51508	,51521	,51534	,51548	,51561	,51574
328	,51587	,51601	,51614	,51627	,51640	,51653	,51667	,51680	,51693	,51706
329	,51720	,51733	,51746	,51759	,51772	,51785	,51799	,51812	,51825	,51838
330	,51851	,51864	,51878	,51891	,51904	,51917	,51930	,51943	,51956	,51970
331	,51983	,51996	,52009	,52022	,52035	,52048	,52061	,52074	,52088	,52101
332	,52114	,52127	,52140	,52153	,52166	,52179	,52192	,52205	,52218	,52231
333	,52244	,52257	,52270	,52283	,52297	,52310	,52323	,52336	,52349	,52362
334	,52375	,52388	,52401	,52414	,52427	,52440	,52453	,52466	,52478	,52491
335	,52504	,52517	,52530	,52543	,52556	,52569	,52582	,52595	,52608	,52621
336	,52634	,52647	,52660	,52673	,52686	,52698	,52711	,52724	,52737	,52750
337	,52763	,52776	,52789	,52802	,52814	,52827	,52840	,52853	,52866	,52879
338	,52892	,52904	,52917	,52930	,52943	,52956	,52969	,52981	,52994	,53007
339	,53020	,53033	,53042	,53058	,53071	,53084	,53097	,53109	,53122	,53135
N ^o	0	1	2	3	4	5	6	7	8	9

Numbers from 3400 to 4000. Logs. from ,53148 to ,60206.

N ^o	0	1	2	3	4	5	6	7	8	9
340	,53148	,53161	,53173	,53186	,53199	,53212	,53224	,53237	,53250	,53263
341	,53275	,53288	,53301	,53314	,53326	,53339	,53352	,53364	,53377	,53390
342	,53403	,53415	,53428	,53441	,53453	,53466	,53479	,53491	,53504	,53517
343	,53529	,53542	,53555	,53567	,53580	,53593	,53605	,53618	,53631	,53643
344	,53656	,53668	,53681	,53694	,53706	,53719	,53731	,53744	,53757	,53769
345	,53782	,53794	,53807	,53820	,53832	,53845	,53857	,53870	,53882	,53895
346	,53908	,53920	,53933	,53945	,53958	,53970	,53983	,53995	,54008	,54020
347	,54033	,54045	,54058	,54070	,54083	,54095	,54108	,54120	,54133	,54145
348	,54158	,54170	,54183	,54195	,54208	,54220	,54233	,54245	,54258	,54270
349	,54282	,54295	,54307	,54320	,54332	,54345	,54357	,54370	,54382	,54394
350	,54407	,54419	,54432	,54444	,54456	,54469	,54481	,54494	,54506	,54518
351	,54531	,54543	,54555	,54568	,54580	,54592	,54605	,54617	,54630	,54642
352	,54654	,54667	,54679	,54691	,54704	,54716	,54728	,54740	,54753	,54765
353	,54777	,54790	,54802	,54814	,54827	,54839	,54851	,54863	,54876	,54888
354	,54900	,54913	,54925	,54937	,54949	,54962	,54974	,54986	,54998	,55011
355	,55023	,55035	,55047	,55059	,55072	,55084	,55096	,55108	,55121	,55133
356	,55145	,55157	,55169	,55182	,55194	,55206	,55218	,55230	,55242	,55255
357	,55267	,55279	,55291	,55303	,55315	,55328	,55340	,55352	,55364	,55376
358	,55388	,55400	,55413	,55425	,55437	,55449	,55461	,55473	,55485	,55497
359	,55509	,55521	,55534	,55546	,55558	,55570	,55582	,55594	,55606	,55618
360	,5563	,55642	,55654	,55666	,55678	,55690	,55703	,55715	,55727	,55739
361	,55751	,55763	,55775	,55787	,55799	,55811	,55823	,55835	,55847	,55859
362	,55871	,55883	,55895	,55907	,55919	,55931	,55943	,55955	,55967	,55979
363	,55991	,56003	,56015	,56026	,56038	,56050	,56062	,56074	,56086	,56098
364	,56110	,56122	,56134	,56146	,56158	,56170	,56182	,56194	,56205	,56217
365	,56229	,56241	,56253	,56265	,56277	,56289	,56301	,56312	,56324	,56336
366	,56348	,56360	,56372	,56384	,56395	,56407	,56419	,56431	,56443	,56455
367	,56467	,56478	,56490	,56502	,56514	,56526	,56537	,56549	,56561	,56573
368	,56585	,56597	,56608	,56620	,56632	,56644	,56655	,56667	,56679	,56691
369	,56703	,56714	,56726	,56738	,56750	,56761	,56773	,56785	,56797	,56808
370	,56820	,56832	,56844	,56855	,56867	,56879	,56890	,56902	,56914	,56926
371	,56937	,56949	,56961	,56972	,56984	,56996	,57008	,57019	,57031	,57043
372	,57054	,57066	,57078	,57089	,57101	,57113	,57124	,57136	,57148	,57159
373	,57171	,57182	,57194	,57206	,57217	,57229	,57241	,57252	,57264	,57275
374	,57287	,57299	,57310	,57322	,57334	,57345	,57357	,57368	,57380	,57391
375	,57403	,57415	,57426	,57438	,57449	,57461	,57473	,57484	,57496	,57507
376	,57519	,57530	,57542	,57553	,57565	,57576	,57588	,57600	,57611	,57623
377	,57634	,57646	,57657	,57669	,57680	,57692	,57703	,57715	,57726	,57738
378	,57749	,57761	,57772	,57784	,57795	,57807	,57818	,57829	,57841	,57852
379	,57864	,57875	,57887	,57898	,57910	,57921	,57933	,57944	,57955	,57967
380	,57978	,57990	,58001	,58013	,58024	,58035	,58047	,58058	,58070	,58081
381	,58092	,58104	,58115	,58127	,58138	,58149	,58161	,58172	,58184	,58195
382	,58206	,58218	,58229	,58240	,58252	,58263	,58274	,58286	,58297	,58308
383	,58320	,58331	,58342	,58354	,58365	,58376	,58388	,58399	,58410	,58422
384	,58433	,58444	,58456	,58467	,58478	,58490	,58501	,58512	,58523	,58535
385	,58546	,58557	,58569	,58580	,58591	,58602	,58614	,58625	,58636	,58647
386	,58659	,58670	,58681	,58692	,58704	,58715	,58726	,58737	,58749	,58760
387	,58771	,58782	,58793	,58805	,58816	,58827	,58838	,58850	,58861	,58872
388	,58883	,58894	,58905	,58917	,58928	,58939	,58950	,58961	,58973	,58984
389	,58995	,59006	,59017	,59028	,59040	,59051	,59062	,59073	,59084	,59095
390	,59106	,59118	,59129	,59140	,59151	,59162	,59173	,59184	,59195	,59207
391	,59218	,59229	,59240	,59251	,59262	,59273	,59284	,59295	,59306	,59317
392	,59329	,59340	,59351	,59362	,59373	,59384	,59395	,59406	,59417	,59428
393	,59439	,59450	,59461	,59472	,59483	,59494	,59505	,59516	,59527	,59539
394	,59550	,59561	,59572	,59583	,59594	,59605	,59616	,59627	,59638	,59649
395	,59660	,59671	,59682	,59693	,59704	,59714	,59726	,59737	,59748	,59758
396	,59769	,59780	,59791	,59802	,59813	,59824	,59835	,59846	,59857	,59868
397	,59879	,59890	,59901	,59912	,59923	,59934	,59945	,59956	,59966	,59977
398	,59988	,59999	,60010	,60021	,60032	,60043	,60053	,60065	,60076	,60086
399	,60097	,60108	,60119	,60130	,60141	,60152	,60162	,60173	,60184	,60195
N ^o	0	1	2	3	4	5	6	7	8	9

Numbers from 4000 to 4600. Logs. from ,60206 to ,66276.

N ^o	0	1	2	3	4	5	6	7	8	9
400	,60206	,60217	,60228	,60239	,60249	,60260	,60271	,60282	,60293	,60304
401	,60314	,60325	,60336	,60347	,60358	,60368	,60379	,60390	,60401	,60412
402	,60423	,60433	,60444	,60455	,60466	,60477	,60487	,60498	,60509	,60520
403	,60530	,60541	,60552	,60563	,60574	,60584	,60595	,60606	,60617	,60627
404	,60638	,60649	,60660	,60670	,60681	,60692	,60703	,60713	,60724	,60735
405	,60745	,60756	,60767	,60778	,60788	,60799	,60810	,60820	,60831	,60842
406	,60853	,60863	,60874	,60885	,60895	,60906	,60917	,60927	,60938	,60949
407	,60959	,60970	,60981	,60991	,61002	,61013	,61023	,61034	,61045	,61055
408	,61066	,61077	,61087	,61098	,61109	,61119	,61130	,61140	,61151	,61162
409	,61172	,61183	,61194	,61204	,61215	,61225	,61236	,61247	,61257	,61268
410	,61278	,61289	,61300	,61310	,61321	,61331	,61342	,61352	,61363	,61374
411	,61384	,61395	,61405	,61416	,61426	,61437	,61447	,61458	,61469	,61479
412	,61490	,61500	,61511	,61521	,61532	,61542	,61553	,61563	,61574	,61584
413	,61590	,61605	,61616	,61626	,61637	,61647	,61658	,61668	,61679	,61689
414	,61700	,61710	,61721	,61731	,61742	,61752	,61763	,61773	,61784	,61794
415	,61805	,61815	,61826	,61836	,61847	,61857	,61867	,61878	,61888	,61899
416	,61909	,61920	,61930	,61941	,61951	,61961	,61972	,61982	,61993	,62003
417	,62014	,62024	,62034	,62045	,62055	,62066	,62076	,62086	,62097	,62107
418	,62118	,62128	,62138	,62149	,62159	,62169	,62180	,62190	,62201	,62211
419	,62221	,62232	,62242	,62252	,62263	,62273	,62283	,62294	,62304	,62315
420	,62325	,62335	,62346	,62356	,62366	,62377	,62387	,62397	,62408	,62418
421	,62428	,62438	,62449	,62459	,62469	,62480	,62490	,62500	,62511	,62521
422	,62531	,62541	,62552	,62562	,62572	,62583	,62593	,62603	,62613	,62624
423	,62634	,62644	,62655	,62665	,62675	,62685	,62696	,62706	,62716	,62726
424	,62737	,62747	,62757	,62767	,62777	,62788	,62798	,62808	,62818	,62829
425	,62839	,62849	,62859	,62869	,62880	,62890	,62900	,62910	,62921	,62931
426	,62941	,62951	,62961	,62971	,62982	,62992	,63002	,63012	,63022	,63033
427	,63043	,63053	,63063	,63073	,63083	,63094	,63104	,63114	,63124	,63134
428	,63144	,63154	,63165	,63175	,63185	,63195	,63205	,63215	,63225	,63235
429	,63246	,63256	,63266	,63276	,63286	,63296	,63306	,63316	,63327	,63337
430	,63347	,63357	,63367	,63377	,63387	,63397	,63407	,63417	,63428	,63438
431	,63448	,63458	,63468	,63478	,63488	,63498	,63508	,63518	,63528	,63538
432	,63548	,63558	,63568	,63578	,63589	,63599	,63609	,63619	,63629	,63639
433	,63649	,63659	,63669	,63679	,63689	,63699	,63709	,63719	,63729	,63739
434	,63749	,63759	,63769	,63779	,63789	,63800	,63809	,63819	,63829	,63839
435	,63849	,63859	,63869	,63879	,63889	,63899	,63909	,63919	,63929	,63939
436	,63949	,63959	,63969	,63978	,63988	,63998	,64008	,64018	,64028	,64038
437	,64048	,64058	,64068	,64078	,64088	,64098	,64108	,64118	,64128	,64137
438	,64147	,64157	,64167	,64177	,64187	,64197	,64207	,64217	,64227	,64237
439	,64246	,64256	,64266	,64276	,64286	,64296	,64306	,64316	,64325	,64335
440	,64345	,64355	,64365	,64375	,64385	,64395	,64404	,64414	,64424	,64434
441	,64444	,64454	,64463	,64473	,64483	,64493	,64503	,64513	,64523	,64532
442	,64542	,64552	,64562	,64572	,64581	,64591	,64601	,64611	,64621	,64631
443	,64640	,64650	,64660	,64670	,64680	,64689	,64699	,64709	,64719	,64728
444	,64738	,64748	,64758	,64768	,64777	,64787	,64797	,64807	,64816	,64826
445	,64836	,64846	,64855	,64865	,64875	,64885	,64894	,64904	,64914	,64924
446	,64933	,64943	,64953	,64963	,64972	,64982	,64992	,65002	,65011	,65021
447	,65031	,65040	,65050	,65060	,65070	,65079	,65089	,65099	,65108	,65118
448	,65128	,65137	,65147	,65157	,65167	,65176	,65186	,65196	,65205	,65215
449	,65225	,65234	,65244	,65254	,65263	,65273	,65283	,65292	,65302	,65312
450	,65321	,65331	,65340	,65350	,65360	,65369	,65379	,65389	,65398	,65408
451	,65418	,65427	,65437	,65446	,65456	,65466	,65475	,65485	,65495	,65504
452	,65514	,65523	,65533	,65543	,65552	,65562	,65571	,65581	,65591	,65600
453	,65610	,65619	,65629	,65639	,65648	,65658	,65667	,65677	,65686	,65696
454	,65706	,65715	,65725	,65734	,65744	,65753	,65763	,65772	,65782	,65792
455	,65801	,65811	,65820	,65830	,65839	,65849	,65858	,65868	,65877	,65887
456	,65896	,65906	,65915	,65925	,65935	,65944	,65954	,65963	,65973	,65982
457	,65992	,66001	,66011	,66020	,66030	,66039	,66049	,66058	,66068	,66077
458	,66086	,66096	,66105	,66115	,66124	,66134	,66143	,66153	,66162	,66172
459	,66181	,66191	,66200	,66210	,66219	,66228	,66238	,66247	,66257	,66266
N ^o	0	1	2	3	4	5	6	7	8	9

Numbers from 4600 to 5200. Logs. from ,66276 to ,71600.

N ^o	0	1	2	3	4	5	6	7	8	9
460	,66276	,66285	,66295	,66304	,66313	,66323	,66332	,66342	,66351	,66361
461	,66370	,66379	,66389	,66398	,66408	,66417	,66427	,66436	,66445	,66455
462	,66464	,66474	,66483	,66492	,66502	,66511	,66521	,66530	,66539	,66549
463	,66558	,66567	,66577	,66586	,66596	,66605	,66614	,66624	,66633	,66642
464	,66652	,66661	,66670	,66680	,66689	,66699	,66708	,66717	,66727	,66736
465	,66745	,66755	,66764	,66773	,66783	,66792	,66801	,66810	,66820	,66829
466	,66839	,66848	,66857	,66866	,66876	,66885	,66894	,66904	,66913	,66922
467	,66932	,66941	,66950	,66960	,66969	,66978	,66987	,66997	,67006	,67015
468	,67025	,67034	,67043	,67052	,67062	,67071	,67080	,67089	,67099	,67108
469	,67117	,67126	,67136	,67145	,67154	,67164	,67173	,67182	,67191	,67200
470	,67210	,67219	,67228	,67237	,67247	,67256	,67265	,67274	,67284	,67293
471	,67302	,67311	,67320	,67330	,67339	,67348	,67357	,67367	,67376	,67385
472	,67394	,67403	,67413	,67422	,67431	,67440	,67449	,67459	,67468	,67477
473	,67486	,67495	,67504	,67514	,67523	,67532	,67541	,67550	,67559	,67569
474	,67578	,67587	,67596	,67605	,67614	,67624	,67633	,67642	,67651	,67660
475	,67669	,67678	,67688	,67697	,67706	,67715	,67724	,67733	,67742	,67752
476	,67761	,67770	,67779	,67788	,67797	,67806	,67815	,67824	,67834	,67843
477	,67852	,67861	,67870	,67879	,67888	,67897	,67906	,67915	,67925	,67934
478	,67943	,67952	,67961	,67970	,67979	,67988	,67997	,68006	,68015	,68024
479	,68033	,68043	,68052	,68061	,68070	,68079	,68088	,68097	,68106	,68115
480	,68124	,68133	,68142	,68151	,68160	,68169	,68178	,68187	,68196	,68205
481	,68214	,68223	,68233	,68242	,68251	,68260	,68269	,68278	,68287	,68296
482	,68305	,68314	,68323	,68332	,68341	,68350	,68359	,68368	,68377	,68386
483	,68395	,68404	,68413	,68422	,68431	,68440	,68449	,68458	,68467	,68476
484	,68484	,68493	,68502	,68511	,68520	,68529	,68538	,68547	,68556	,68565
485	,68574	,68583	,68592	,68601	,68610	,68619	,68628	,68637	,68646	,68655
486	,68664	,68673	,68681	,68690	,68699	,68708	,68717	,68726	,68735	,68744
487	,68753	,68762	,68771	,68780	,68788	,68797	,68806	,68815	,68824	,68833
488	,68842	,68851	,68860	,68869	,68878	,68886	,68895	,68904	,68913	,68922
489	,68931	,68940	,68948	,68957	,68966	,68975	,68984	,68993	,69002	,69011
490	,69020	,69028	,69037	,69046	,69055	,69064	,69073	,69082	,69090	,69099
491	,69108	,69117	,69126	,69135	,69143	,69152	,69161	,69170	,69179	,69188
492	,69196	,69205	,69214	,69223	,69232	,69241	,69249	,69258	,69267	,69276
493	,69285	,69293	,69302	,69311	,69320	,69329	,69337	,69346	,69355	,69364
494	,69373	,69381	,69390	,69399	,69408	,69417	,69425	,69434	,69443	,69452
495	,69460	,69469	,69478	,69487	,69496	,69504	,69513	,69522	,69531	,69539
496	,69548	,69557	,69566	,69574	,69583	,69592	,69601	,69609	,69618	,69627
497	,69636	,69644	,69653	,69662	,69671	,69679	,69688	,69697	,69705	,69714
498	,69723	,69732	,69740	,69749	,69758	,69766	,69775	,69784	,69793	,69801
499	,69810	,69819	,69827	,69836	,69845	,69853	,69862	,69871	,69880	,69888
500	,69897	,69906	,69914	,69923	,69932	,69940	,69949	,69958	,69966	,69975
501	,69984	,69992	,70001	,70010	,70018	,70027	,70036	,70044	,70053	,70062
502	,70070	,70079	,70088	,70096	,70105	,70114	,70122	,70131	,70139	,70148
503	,70157	,70165	,70174	,70183	,70191	,70200	,70209	,70217	,70226	,70234
504	,70243	,70252	,70260	,70269	,70277	,70286	,70295	,70303	,70312	,70320
505	,70329	,70338	,70346	,70355	,70363	,70372	,70381	,70389	,70398	,70406
506	,70415	,70424	,70432	,70441	,70449	,70458	,70466	,70475	,70484	,70492
507	,70501	,70509	,70518	,70526	,70535	,70544	,70552	,70561	,70569	,70578
508	,70586	,70595	,70603	,70612	,70620	,70629	,70638	,70646	,70655	,70663
509	,70672	,70680	,70689	,70697	,70706	,70714	,70723	,70731	,70740	,70748
510	,70757	,70765	,70774	,70783	,70791	,70800	,70808	,70817	,70825	,70834
511	,70842	,70851	,70859	,70868	,70876	,70885	,70893	,70901	,70910	,70918
512	,70927	,70935	,70944	,70952	,70961	,70969	,70978	,70986	,70995	,71003
513	,71012	,71020	,71029	,71037	,71045	,71054	,71062	,71071	,71079	,71088
514	,71096	,71105	,71113	,71122	,71130	,71138	,71147	,71155	,71164	,71172
515	,71181	,71189	,71198	,71206	,71214	,71223	,71231	,71240	,71248	,71256
516	,71265	,71273	,71282	,71290	,71299	,71307	,71315	,71324	,71332	,71341
517	,71349	,71357	,71366	,71374	,71383	,71391	,71399	,71408	,71416	,71425
518	,71433	,71441	,71450	,71458	,71466	,71475	,71483	,71492	,71500	,71508
519	,71517	,71525	,71533	,71542	,71550	,71559	,71567	,71575	,71584	,71592
N ^o	0	1	2	3	4	5	6	7	8	9

Numbers from 5200 to 5800. Logs. from ,71600 to ,76343.

N ^o	0	1	2	3	4	5	6	7	8	9
520	,71600	,71609	,71617	,71625	,71634	,71642	,71650	,71659	,71667	,71675
521	,71684	,71692	,71700	,71709	,71717	,71725	,71734	,71742	,71750	,71759
522	,71767	,71775	,71784	,71792	,71800	,71809	,71817	,71825	,71834	,71842
523	,71850	,71858	,71867	,71875	,71883	,71892	,71900	,71908	,71916	,71925
524	,71933	,71941	,71950	,71958	,71966	,71974	,71983	,71991	,71999	,72008
525	,72016	,72024	,72032	,72041	,72049	,72057	,72065	,72074	,72082	,72090
526	,72099	,72107	,72115	,72123	,72132	,72140	,72148	,72156	,72165	,72173
527	,72181	,72189	,72197	,72206	,72214	,72222	,72230	,72239	,72247	,72255
528	,72263	,72272	,72280	,72288	,72296	,72304	,72313	,72321	,72329	,72337
529	,72346	,72354	,72362	,72370	,72378	,72387	,72395	,72403	,72411	,72419
530	,72428	,72436	,72444	,72452	,72460	,72468	,72477	,72485	,72493	,72501
531	,72509	,72518	,72526	,72534	,72542	,72550	,72558	,72567	,72575	,72583
532	,72591	,72599	,72607	,72616	,72624	,72632	,72640	,72648	,72656	,72665
533	,72673	,72681	,72689	,72697	,72705	,72713	,72722	,72730	,72738	,72746
534	,72754	,72762	,72770	,72778	,72787	,72795	,72803	,72811	,72819	,72827
535	,72835	,72843	,72852	,72860	,72868	,72876	,72884	,72892	,72900	,72908
536	,72916	,72925	,72933	,72941	,72949	,72957	,72965	,72973	,72981	,72989
537	,72997	,73005	,73014	,73022	,73030	,73038	,73046	,73054	,73062	,73070
538	,73078	,73086	,73094	,73102	,73110	,73119	,73127	,73135	,73143	,73151
539	,73159	,73167	,73175	,73183	,73191	,73199	,73207	,73215	,73223	,73231
540	,73239	,73247	,73255	,73263	,73271	,73280	,73288	,73296	,73304	,73311
541	,73320	,73328	,73336	,73344	,73352	,73360	,73368	,73376	,73384	,73392
542	,73400	,73408	,73416	,73424	,73432	,73440	,73448	,73456	,73464	,73472
543	,73480	,73488	,73496	,73504	,73512	,73520	,73528	,73536	,73544	,73552
544	,73560	,73568	,73576	,73584	,73592	,73600	,73608	,73616	,73624	,73632
545	,73640	,73648	,73656	,73663	,73671	,73679	,73687	,73695	,73703	,73711
546	,73719	,73727	,73735	,73743	,73751	,73759	,73767	,73775	,73783	,73791
547	,73799	,73807	,73815	,73822	,73830	,73838	,73846	,73854	,73862	,73870
548	,73878	,73886	,73894	,73902	,73910	,73918	,73926	,73933	,73941	,73949
549	,73957	,73965	,73973	,73981	,73989	,73997	,74005	,74013	,74020	,74028
550	,74036	,74044	,74052	,74060	,74068	,74076	,74084	,74091	,74099	,74107
551	,74115	,74123	,74131	,74139	,74147	,74155	,74162	,74170	,74178	,74186
552	,74194	,74202	,74210	,74217	,74225	,74233	,74241	,74249	,74257	,74265
553	,74272	,74280	,74288	,74296	,74304	,74312	,74320	,74327	,74335	,74343
554	,74351	,74359	,74367	,74374	,74382	,74390	,74398	,74406	,74414	,74421
555	,74429	,74437	,74445	,74453	,74461	,74468	,74476	,74484	,74492	,74500
556	,74507	,74515	,74523	,74531	,74539	,74546	,74554	,74562	,74570	,74578
557	,74585	,74593	,74601	,74609	,74617	,74624	,74632	,74640	,74648	,74656
558	,74663	,74671	,74679	,74687	,74694	,74702	,74710	,74718	,74726	,74733
559	,74741	,74749	,74757	,74764	,74772	,74780	,74788	,74795	,74803	,74811
560	,74819	,74827	,74834	,74842	,74850	,74858	,74865	,74873	,74881	,74888
561	,74896	,74904	,74912	,74919	,74927	,74935	,74943	,74950	,74958	,74966
562	,74974	,74981	,74989	,74997	,75004	,75012	,75020	,75028	,75035	,75043
563	,75051	,75059	,75066	,75074	,75082	,75089	,75097	,75105	,75112	,75120
564	,75128	,75136	,75143	,75151	,75159	,75166	,75174	,75182	,75189	,75197
565	,75205	,75212	,75220	,75228	,75236	,75243	,75251	,75259	,75266	,75274
566	,75282	,75289	,75297	,75305	,75312	,75320	,75328	,75335	,75343	,75351
567	,75358	,75366	,75374	,75381	,75389	,75397	,75404	,75412	,75419	,75427
568	,75435	,75442	,75450	,75458	,75465	,75473	,75481	,75488	,75496	,75503
569	,75511	,75519	,75526	,75534	,75542	,75549	,75557	,75565	,75572	,75580
570	,75587	,75595	,75603	,75610	,75618	,75626	,75633	,75641	,75648	,75656
571	,75664	,75671	,75679	,75686	,75694	,75702	,75709	,75717	,75724	,75732
572	,75740	,75747	,75755	,75762	,75771	,75777	,75785	,75793	,75800	,75808
573	,75815	,75823	,75831	,75838	,75846	,75854	,75861	,75868	,75876	,75884
574	,75891	,75899	,75906	,75914	,75921	,75929	,75937	,75944	,75952	,75959
575	,75967	,75974	,75982	,75989	,75997	,76004	,76012	,76020	,76027	,76035
576	,76042	,76050	,76057	,76065	,76072	,76080	,76087	,76095	,76102	,76110
577	,76118	,76125	,76133	,76140	,76148	,76155	,76162	,76170	,76178	,76185
578	,76193	,76200	,76208	,76215	,76223	,76230	,76238	,76245	,76253	,76260
579	,76268	,76275	,76283	,76290	,76298	,76305	,76313	,76320	,76328	,76335
N ^o	0	1	2	3	4	5	6	7	8	9

Numbers from 5800 to 6400. Logs. from ,76343 to ,80618.

N ^o	0	1	2	3	4	5	6	7	8	9
580	,76343	,76350	,76358	,76365	,76373	,76380	,76388	,76395	,76403	,76410
581	,76418	,76425	,76433	,76440	,76447	,76455	,76462	,76470	,76477	,76485
582	,76492	,76500	,76507	,76515	,76522	,76530	,76537	,76544	,76552	,76559
583	,76567	,76574	,76582	,76589	,76597	,76604	,76611	,76619	,76626	,76634
584	,76641	,76649	,76656	,76664	,76671	,76678	,76686	,76693	,76701	,76708
585	,76716	,76723	,76730	,76738	,76745	,76753	,76760	,76767	,76775	,76782
586	,76790	,76797	,76804	,76812	,76819	,76827	,76834	,76842	,76849	,76856
587	,76864	,76871	,76879	,76886	,76893	,76901	,76908	,76916	,76923	,76930
588	,76938	,76945	,76952	,76960	,76967	,76975	,76982	,76989	,76997	,77004
589	,77011	,77019	,77026	,77034	,77041	,77048	,77056	,77063	,77070	,77078
590	,77085	,77093	,77100	,77107	,77115	,77122	,77129	,77137	,77144	,77151
591	,77159	,77166	,77173	,77181	,77188	,77195	,77203	,77210	,77217	,77225
592	,77232	,77239	,77247	,77254	,77261	,77269	,77276	,77283	,77291	,77298
593	,77305	,77313	,77320	,77327	,77335	,77342	,77349	,77357	,77364	,77371
594	,77379	,77386	,77393	,77401	,77408	,77415	,77422	,77430	,77437	,77444
595	,77452	,77459	,77466	,77474	,77481	,77488	,77495	,77503	,77510	,77517
596	,77525	,77532	,77539	,77546	,77554	,77561	,77568	,77576	,77583	,77590
597	,77597	,77605	,77612	,77619	,77626	,77634	,77641	,77648	,77656	,77663
598	,77670	,77677	,77685	,77692	,77699	,77706	,77714	,77721	,77728	,77735
599	,77743	,77750	,77757	,77764	,77772	,77779	,77786	,77793	,77801	,77808
600	,77815	,77822	,77810	,77837	,77844	,77851	,77858	,77866	,77873	,77880
601	,77887	,77895	,77902	,77909	,77916	,77924	,77931	,77938	,77945	,77952
602	,77960	,77967	,77974	,77981	,77988	,77996	,78003	,78010	,78017	,78024
603	,78032	,78039	,78046	,78053	,78060	,78068	,78075	,78082	,78089	,78096
604	,78104	,78111	,78118	,78125	,78132	,78140	,78147	,78154	,78161	,78168
605	,78175	,78183	,78190	,78197	,78204	,78211	,78219	,78226	,78233	,78240
606	,78247	,78254	,78262	,78269	,78276	,78283	,78290	,78297	,78305	,78312
607	,78319	,78326	,78333	,78340	,78347	,78355	,78362	,78369	,78376	,78383
608	,78390	,78397	,78405	,78412	,78419	,78426	,78433	,78440	,78447	,78455
609	,78462	,78469	,78476	,78483	,78490	,78497	,78504	,78512	,78519	,78526
610	,78533	,78540	,78547	,78554	,78561	,78569	,78576	,78583	,78590	,78597
611	,78604	,78611	,78618	,78625	,78632	,78640	,78647	,78654	,78661	,78668
612	,78675	,78682	,78689	,78696	,78703	,78711	,78718	,78725	,78732	,78739
613	,78746	,78753	,78760	,78767	,78774	,78781	,78788	,78796	,78803	,78810
614	,78817	,78824	,78831	,78838	,78845	,78852	,78859	,78866	,78873	,78880
615	,78887	,78895	,78902	,78909	,78915	,78923	,78930	,78937	,78944	,78951
616	,78958	,78965	,78972	,78979	,78986	,78993	,79000	,79007	,79014	,79021
617	,79028	,79036	,79043	,79050	,79057	,79064	,79071	,79078	,79085	,79092
618	,79099	,79106	,79113	,79120	,79127	,79134	,79141	,79148	,79155	,79162
619	,79169	,79176	,79183	,79190	,79197	,79204	,79211	,79218	,79225	,79232
620	,79239	,79246	,79253	,79260	,79267	,79274	,79281	,79288	,79295	,79302
621	,79309	,79316	,79323	,79330	,79337	,79344	,79351	,79358	,79365	,79372
622	,79379	,79386	,79393	,79400	,79407	,79414	,79421	,79428	,79435	,79442
623	,79449	,79456	,79463	,79470	,79477	,79484	,79491	,79498	,79504	,79511
624	,79518	,79525	,79532	,79539	,79546	,79553	,79560	,79567	,79574	,79581
625	,79588	,79595	,79602	,79609	,79616	,79623	,79630	,79637	,79644	,79650
626	,79657	,79664	,79671	,79678	,79685	,79692	,79699	,79706	,79713	,79720
627	,79727	,79734	,79741	,79747	,79754	,79761	,79768	,79775	,79782	,79789
628	,79796	,79803	,79810	,79817	,79824	,79830	,79837	,79844	,79851	,79858
629	,79865	,79872	,79879	,79886	,79893	,79900	,79906	,79913	,79920	,79927
630	,79934	,79941	,79948	,79955	,79962	,79968	,79975	,79982	,79989	,79996
631	,80003	,80010	,80017	,80024	,80030	,80037	,80044	,80051	,80058	,80065
632	,80072	,80079	,80085	,80092	,80099	,80106	,80113	,80120	,80127	,80133
633	,80140	,80147	,80154	,80161	,80168	,80175	,80181	,80188	,80195	,80202
634	,80209	,80216	,80223	,80229	,80237	,80243	,80250	,80257	,80264	,80270
635	,80277	,80284	,80291	,80298	,80305	,80312	,80318	,80325	,80332	,80339
636	,80346	,80352	,80359	,80366	,80373	,80380	,80387	,80393	,80400	,80407
637	,80414	,80421	,80428	,80434	,80441	,80448	,80455	,80462	,80468	,80475
638	,80482	,80489	,80496	,80502	,80509	,80516	,80523	,80530	,80536	,80543
639	,80550	,80557	,80564	,80570	,80577	,80584	,80591	,80598	,80604	,80611
N ^o	0	1	2	3	4	5	6	7	8	9

Numbers from 6400 to 7000. Logs. from ,80618 to ,84510.

N ^o	0	1	2	3	4	5	6	7	8	9
640	,80618	,80625	,80632	,80638	,80645	,80652	,80659	,80665	,80672	,80679
641	,80686	,80693	,80699	,80706	,80713	,80720	,80726	,80733	,80740	,80747
642	,80753	,80760	,80767	,80774	,80781	,80787	,80794	,80801	,80807	,80814
643	,80821	,80828	,80835	,80841	,80848	,80855	,80862	,80868	,80875	,80882
644	,80889	,80895	,80902	,80909	,80915	,80922	,80929	,80936	,80942	,80949
645	,80956	,80963	,80969	,80976	,80983	,80990	,80996	,81003	,81010	,81016
646	,81023	,81030	,81037	,81043	,81050	,81057	,81064	,81070	,81077	,81084
647	,81090	,81097	,81104	,81111	,81117	,81124	,81131	,81137	,81144	,81151
648	,81157	,81164	,81171	,81178	,81184	,81191	,81198	,81204	,81211	,81218
649	,81224	,81231	,81238	,81244	,81251	,81258	,81265	,81271	,81278	,81285
650	,81291	,81298	,81305	,81311	,81318	,81325	,81331	,81338	,81345	,81351
651	,81358	,81365	,81371	,81378	,81385	,81391	,81398	,81405	,81411	,81418
652	,81425	,81431	,81438	,81445	,81451	,81458	,81465	,81471	,81478	,81485
653	,81491	,81498	,81505	,81511	,81518	,81525	,81531	,81538	,81544	,81551
654	,81558	,81564	,81571	,81578	,81584	,81591	,81598	,81604	,81611	,81617
655	,81624	,81631	,81637	,81644	,81651	,81657	,81664	,81670	,81677	,81684
656	,81690	,81697	,81704	,81710	,81717	,81723	,81730	,81737	,81743	,81750
657	,81756	,81763	,81770	,81776	,81783	,81790	,81796	,81803	,81809	,81816
658	,81823	,81829	,81836	,81842	,81849	,81856	,81862	,81869	,81875	,81882
659	,81888	,81895	,81902	,81908	,81915	,81921	,81928	,81935	,81941	,81948
660	,81954	,81961	,81967	,81974	,81981	,81987	,81994	,82000	,82007	,82014
661	,82020	,82027	,82033	,82040	,82046	,82053	,82059	,82066	,82073	,82079
662	,82086	,82092	,82099	,82105	,82112	,82119	,82125	,82132	,82138	,82145
663	,82151	,82158	,82164	,82171	,82177	,82184	,82191	,82197	,82204	,82210
664	,82217	,82223	,82230	,82236	,82243	,82249	,82256	,82263	,82269	,82276
665	,82282	,82289	,82295	,82302	,82308	,82315	,82321	,82328	,82334	,82341
666	,82347	,82354	,82360	,82367	,82373	,82380	,82386	,82393	,82400	,82406
667	,82413	,82419	,82426	,82432	,82439	,82445	,82452	,82458	,82464	,82471
668	,82478	,82484	,82491	,82497	,82504	,82510	,82517	,82523	,82529	,82536
669	,82543	,82549	,82556	,82562	,82569	,82575	,82581	,82588	,82594	,82601
670	,82607	,82614	,82620	,82627	,82633	,82640	,82646	,82653	,82659	,82666
671	,82672	,82679	,82685	,82692	,82698	,82704	,82711	,82717	,82724	,82730
672	,82737	,82743	,82750	,82756	,82763	,82769	,82776	,82782	,82789	,82795
673	,82801	,82808	,82814	,82821	,82827	,82834	,82840	,82847	,82853	,82859
674	,82866	,82872	,82879	,82885	,82892	,82898	,82905	,82911	,82917	,82924
675	,82930	,82937	,82943	,82950	,82956	,82962	,82969	,82975	,82982	,82988
676	,82995	,83001	,83007	,83014	,83020	,83027	,83033	,83040	,83046	,83052
677	,83059	,83065	,83072	,83078	,83084	,83091	,83097	,83104	,83110	,83117
678	,83123	,83129	,83136	,83142	,83149	,83155	,83161	,83168	,83174	,83181
679	,83187	,83193	,83200	,83206	,83212	,83219	,83225	,83232	,83238	,83244
680	,83251	,83257	,83264	,83270	,83276	,83283	,83289	,83296	,83302	,83308
681	,83315	,83321	,83327	,83334	,83340	,83347	,83353	,83359	,83366	,83372
682	,83378	,83385	,83391	,83397	,83404	,83410	,83417	,83423	,83429	,83436
683	,83442	,83448	,83455	,83461	,83467	,83474	,83480	,83487	,83493	,83499
684	,83506	,83512	,83518	,83525	,83531	,83537	,83544	,83550	,83556	,83563
685	,83569	,83575	,83582	,83588	,83594	,83601	,83607	,83613	,83620	,83626
686	,83632	,83639	,83645	,83651	,83658	,83664	,83670	,83677	,83683	,83689
687	,83696	,83702	,83708	,83715	,83721	,83727	,83734	,83740	,83746	,83752
688	,83759	,83765	,83771	,83778	,83784	,83790	,83797	,83803	,83809	,83816
689	,83822	,83828	,83834	,83841	,83847	,83853	,83860	,83866	,83872	,83879
690	,83885	,83891	,83897	,83904	,83910	,83916	,83923	,83929	,83935	,83941
691	,83948	,83954	,83960	,83967	,83973	,83979	,83985	,83992	,83998	,84004
692	,84011	,84017	,84023	,84029	,84036	,84042	,84048	,84054	,84061	,84067
693	,84073	,84080	,84086	,84092	,84098	,84105	,84111	,84117	,84123	,84130
694	,84136	,84142	,84148	,84155	,84161	,84167	,84173	,84180	,84186	,84192
695	,84198	,84205	,84211	,84217	,84223	,84230	,84236	,84242	,84248	,84254
696	,84261	,84267	,84273	,84280	,84286	,84292	,84298	,84305	,84311	,84317
697	,84323	,84329	,84336	,84342	,84348	,84354	,84361	,84367	,84373	,84379
698	,84385	,84392	,84398	,84404	,84410	,84417	,84423	,84429	,84435	,84441
699	,84448	,84454	,84460	,84466	,84473	,84479	,84485	,84491	,84497	,84504
N ^o	0	1	2	3	4	5	6	7	8	9

Numbers from 7000 to 7600. Logs. from ,84510 to ,88081.

N ^o	0	1	2	3	4	5	6	7	8	9
700	,84510	,84516	,84522	,84528	,84535	,84541	,84547	,84553	,84559	,84566
701	,84572	,84578	,84584	,84590	,84596	,84602	,84609	,84615	,84621	,84627
702	,84634	,84640	,84646	,84652	,84658	,84665	,84671	,84677	,84683	,84689
703	,84695	,84701	,84708	,84714	,84720	,84726	,84733	,84739	,84745	,84751
704	,84757	,84763	,84770	,84776	,84782	,84788	,84794	,84800	,84807	,84813
705	,84819	,84825	,84831	,84837	,84843	,84850	,84856	,84862	,84868	,84874
706	,84880	,84887	,84893	,84899	,84905	,84911	,84917	,84923	,84930	,84936
707	,84942	,84948	,84954	,84960	,84966	,84973	,84979	,84985	,84991	,84997
708	,85003	,85009	,85016	,85022	,85028	,85034	,85040	,85046	,85052	,85058
709	,85065	,85071	,85077	,85083	,85089	,85095	,85101	,85107	,85114	,85120
710	,85126	,85132	,85138	,85144	,85150	,85156	,85162	,85169	,85175	,85181
711	,85187	,85193	,85199	,85205	,85211	,85217	,85224	,85230	,85236	,85242
712	,85248	,85254	,85260	,85266	,85272	,85278	,85285	,85291	,85297	,85303
713	,85309	,85315	,85321	,85327	,85333	,85339	,85345	,85352	,85358	,85364
714	,85370	,85376	,85382	,85388	,85394	,85400	,85406	,85412	,85418	,85424
715	,85431	,85437	,85443	,85449	,85455	,85461	,85467	,85473	,85479	,85485
716	,85491	,85497	,85503	,85509	,85516	,85522	,85528	,85534	,85540	,85546
717	,85552	,85558	,85564	,85570	,85576	,85582	,85588	,85594	,85600	,85606
718	,85612	,85618	,85624	,85631	,85637	,85643	,85649	,85655	,85661	,85667
719	,85673	,85679	,85685	,85691	,85697	,85703	,85709	,85715	,85721	,85727
720	,85733	,85739	,85745	,85751	,85757	,85763	,85769	,85775	,85781	,85787
721	,85793	,85799	,85806	,85812	,85818	,85824	,85830	,85836	,85842	,85848
722	,85854	,85860	,85866	,85872	,85878	,85884	,85890	,85896	,85902	,85908
723	,85914	,85920	,85926	,85932	,85938	,85944	,85950	,85956	,85962	,85968
724	,85974	,85980	,85986	,85992	,85998	,86004	,86010	,86016	,86022	,86028
725	,86034	,86040	,86046	,86052	,86058	,86064	,86070	,86076	,86082	,86088
726	,86094	,86100	,86106	,86112	,86118	,86124	,86130	,86135	,86141	,86147
727	,86153	,86159	,86165	,86171	,86177	,86183	,86189	,86195	,86201	,86207
728	,86213	,86219	,86225	,86231	,86237	,86243	,86249	,86255	,86261	,86267
729	,86273	,86279	,86285	,86291	,86297	,86302	,86308	,86314	,86320	,86326
730	,86332	,86338	,86344	,86350	,86357	,86362	,86368	,86374	,86380	,86386
731	,86392	,86398	,86404	,86410	,86415	,86421	,86427	,86433	,86439	,86445
732	,86451	,86457	,86463	,86469	,86475	,86481	,86487	,86493	,86498	,86504
733	,86510	,86516	,86522	,86528	,86534	,86540	,86546	,86552	,86558	,86564
734	,86570	,86575	,86581	,86587	,86593	,86599	,86605	,86611	,86617	,86623
735	,86629	,86635	,86640	,86646	,86652	,86658	,86664	,86670	,86676	,86682
736	,86688	,86694	,86700	,86705	,86711	,86717	,86723	,86729	,86735	,86741
737	,86747	,86753	,86758	,86764	,86770	,86776	,86782	,86788	,86794	,86800
738	,86806	,86811	,86817	,86823	,86829	,86835	,86841	,86847	,86853	,86859
739	,86864	,86870	,86876	,86882	,86888	,86894	,86900	,86906	,86911	,86917
740	,86923	,86929	,86935	,86941	,86947	,86952	,86958	,86964	,86970	,86976
741	,86982	,86988	,86993	,86999	,87005	,87011	,87017	,87023	,87029	,87034
742	,87040	,87046	,87052	,87058	,87064	,87070	,87075	,87081	,87087	,87093
743	,87099	,87105	,87111	,87116	,87122	,87128	,87134	,87140	,87146	,87151
744	,87157	,87163	,87169	,87175	,87181	,87186	,87192	,87198	,87204	,87210
745	,87216	,87221	,87227	,87233	,87239	,87245	,87251	,87256	,87262	,87268
746	,87274	,87280	,87285	,87291	,87297	,87303	,87309	,87315	,87320	,87326
747	,87332	,87338	,87344	,87349	,87355	,87361	,87367	,87373	,87378	,87384
748	,87390	,87396	,87402	,87408	,87413	,87419	,87425	,87431	,87437	,87442
749	,87448	,87454	,87460	,87466	,87471	,87477	,87483	,87489	,87494	,87500
750	,87506	,87512	,87518	,87523	,87529	,87535	,87541	,87547	,87552	,87558
751	,87564	,87570	,87576	,87581	,87587	,87593	,87599	,87604	,87610	,87616
752	,87622	,87628	,87633	,87639	,87645	,87651	,87656	,87662	,87668	,87674
753	,87679	,87685	,87691	,87697	,87702	,87708	,87714	,87720	,87726	,87731
754	,87737	,87743	,87749	,87754	,87760	,87766	,87772	,87777	,87783	,87789
755	,87794	,87800	,87806	,87812	,87818	,87823	,87829	,87835	,87841	,87846
756	,87852	,87858	,87864	,87869	,87875	,87881	,87887	,87892	,87898	,87904
757	,87910	,87915	,87921	,87927	,87932	,87938	,87944	,87950	,87955	,87961
758	,87967	,87973	,87978	,87984	,87990	,87996	,88001	,88007	,88013	,88018
759	,88024	,88030	,88036	,88041	,88047	,88052	,88058	,88064	,88070	,88076
N ^o	0	1	2	3	4	5	6	7	8	9

Numbers from 7600 to 8200. Logs. from ,88081 to ,91381.

N ^o	0	1	2	3	4	5	6	7	8	9
760	,88081	,88087	,88093	,88098	,88104	,88110	,88116	,88121	,88127	,88133
761	,88138	,88144	,88150	,88156	,88161	,88167	,88173	,88178	,88184	,88190
762	,88195	,88201	,88207	,88213	,88218	,88224	,88230	,88235	,88241	,88247
763	,88252	,88258	,88264	,88269	,88275	,88281	,88287	,88292	,88298	,88304
764	,88309	,88315	,88321	,88326	,88332	,88338	,88343	,88349	,88355	,88360
765	,88366	,88372	,88377	,88383	,88389	,88394	,88400	,88406	,88411	,88417
766	,88423	,88428	,88434	,88440	,88445	,88451	,88457	,88462	,88468	,88474
767	,88479	,88485	,88491	,88496	,88502	,88508	,88513	,88519	,88525	,88530
768	,88536	,88542	,88547	,88553	,88559	,88564	,88570	,88576	,88581	,88587
769	,88593	,88598	,88604	,88610	,88615	,88621	,88626	,88632	,88638	,88643
770	,88649	,88655	,88660	,88666	,88672	,88677	,88683	,88688	,88694	,88700
771	,88705	,88711	,88717	,88722	,88728	,88734	,88739	,88745	,88750	,88756
772	,88762	,88767	,88773	,88779	,88784	,88790	,88795	,88801	,88807	,88812
773	,88818	,88824	,88829	,88835	,88840	,88846	,88852	,88857	,88863	,88868
774	,88874	,88880	,88885	,88891	,88896	,88902	,88908	,88913	,88919	,88925
775	,88930	,88936	,88941	,88947	,88954	,88958	,88964	,88969	,88975	,88981
776	,88986	,88992	,88997	,89003	,89009	,89014	,89020	,89025	,89031	,89036
777	,89042	,89048	,89053	,89059	,89064	,89070	,89075	,89081	,89087	,89092
778	,89098	,89103	,89109	,89115	,89120	,89126	,89131	,89137	,89143	,89148
779	,89154	,89159	,89165	,89170	,89176	,89182	,89187	,89193	,89198	,89204
780	,89209	,89215	,89221	,89226	,89232	,89237	,89243	,89248	,89254	,89259
781	,89265	,89271	,89276	,89282	,89287	,89293	,89298	,89304	,89310	,89315
782	,89321	,89326	,89332	,89337	,89343	,89348	,89354	,89359	,89365	,89371
783	,89376	,89382	,89387	,89393	,89398	,89404	,89409	,89415	,89420	,89426
784	,89432	,89437	,89443	,89448	,89454	,89459	,89465	,89470	,89476	,89481
785	,89487	,89492	,89498	,89504	,89509	,89515	,89520	,89526	,89531	,89537
786	,89542	,89548	,89553	,89559	,89564	,89570	,89575	,89581	,89586	,89592
787	,89597	,89603	,89608	,89614	,89619	,89625	,89631	,89636	,89642	,89647
788	,89652	,89658	,89664	,89669	,89675	,89680	,89686	,89691	,89697	,89702
789	,89708	,89713	,89719	,89724	,89730	,89735	,89741	,89746	,89752	,89757
790	,89762	,89768	,89774	,89779	,89785	,89790	,89796	,89801	,89807	,89812
791	,89818	,89823	,89829	,89834	,89840	,89845	,89851	,89856	,89861	,89867
792	,89872	,89878	,89883	,89889	,89894	,89900	,89905	,89911	,89916	,89922
793	,89927	,89933	,89938	,89944	,89949	,89955	,89960	,89966	,89971	,89977
794	,89982	,89987	,89993	,89998	,90004	,90009	,90015	,90020	,90026	,90031
795	,90037	,90042	,90048	,90053	,90059	,90064	,90069	,90075	,90080	,90085
796	,90091	,90097	,90102	,90108	,90113	,90119	,90124	,90129	,90135	,90141
797	,90146	,90151	,90157	,90162	,90168	,90173	,90178	,90184	,90189	,90195
798	,90200	,90206	,90211	,90217	,90222	,90227	,90233	,90238	,90244	,90249
799	,90255	,90260	,90265	,90271	,90276	,90282	,90287	,90293	,90298	,90304
800	,90309	,90314	,90320	,90325	,90331	,90336	,90342	,90347	,90352	,90358
801	,90363	,90369	,90374	,90379	,90385	,90390	,90396	,90401	,90407	,90412
802	,90417	,90423	,90428	,90434	,90439	,90444	,90450	,90455	,90461	,90466
803	,90471	,90477	,90482	,90488	,90493	,90499	,90504	,90509	,90515	,90520
804	,90526	,90531	,90536	,90542	,90547	,90553	,90558	,90563	,90569	,90574
805	,90580	,90585	,90590	,90596	,90601	,90606	,90612	,90617	,90622	,90628
806	,90633	,90639	,90644	,90650	,90655	,90660	,90666	,90671	,90677	,90682
807	,90687	,90693	,90698	,90703	,90709	,90714	,90720	,90725	,90730	,90736
808	,90741	,90746	,90752	,90757	,90763	,90768	,90773	,90779	,90784	,90789
809	,90795	,90800	,90806	,90811	,90816	,90822	,90827	,90832	,90838	,90843
810	,90848	,90854	,90859	,90865	,90870	,90875	,90881	,90886	,90891	,90897
811	,90902	,90907	,90913	,90918	,90923	,90929	,90934	,90939	,90945	,90950
812	,90956	,90961	,90966	,90972	,90977	,90982	,90988	,90993	,90998	,91004
813	,91009	,91014	,91020	,91025	,91030	,91036	,91041	,91046	,91052	,91057
814	,91062	,91068	,91073	,91078	,91084	,91089	,91094	,91100	,91105	,91110
815	,91116	,91121	,91126	,91132	,91137	,91142	,91148	,91153	,91158	,91164
816	,91169	,91174	,91180	,91185	,91190	,91196	,91201	,91206	,91212	,91217
817	,91222	,91227	,91233	,91238	,91243	,91249	,91254	,91259	,91265	,91270
818	,91275	,91281	,91286	,91291	,91297	,91301	,91307	,91312	,91318	,91323
819	,91328	,91334	,91339	,91344	,91350	,91355	,91360	,91365	,91371	,91376
N ^o	0	1	2	3	4	5	6	7	8	9

Numbers from 8200 to 8800. Logs. from ,91381 to ,94448.

N ^o	0	1	2	3	4	5	6	7	8	9
820	,91381	,91387	,91392	,91397	,91403	,91408	,91413	,91418	,91424	,91429
821	,91434	,91440	,91445	,91450	,91455	,91461	,91466	,91471	,91477	,91482
822	,91487	,91492	,91498	,91503	,91508	,91514	,91519	,91524	,91529	,91535
823	,91540	,91545	,91550	,91556	,91561	,91566	,91572	,91577	,91582	,91587
824	,91593	,91598	,91603	,91608	,91614	,91619	,91624	,91630	,91635	,91640
825	,91645	,91650	,91656	,91661	,91666	,91672	,91677	,91682	,91687	,91693
826	,91698	,91703	,91708	,91714	,91719	,91724	,91729	,91735	,91740	,91745
827	,91750	,91756	,91761	,91766	,91771	,91777	,91782	,91787	,91792	,91798
828	,91803	,91808	,91813	,91819	,91824	,91829	,91834	,91840	,91845	,91850
829	,91855	,91861	,91866	,91871	,91876	,91882	,91887	,91892	,91897	,91903
830	,91908	,91913	,91918	,91923	,91929	,91934	,91939	,91944	,91950	,91955
831	,91960	,91965	,91970	,91976	,91981	,91986	,91991	,91997	,92002	,92007
832	,92012	,92017	,92023	,92028	,92033	,92038	,92044	,92049	,92054	,92059
833	,92064	,92070	,92075	,92080	,92085	,92091	,92096	,92101	,92106	,92111
834	,92117	,92122	,92127	,92132	,92137	,92143	,92148	,92153	,92158	,92163
835	,92169	,92174	,92179	,92184	,92189	,92195	,92200	,92205	,92210	,92215
836	,92221	,92226	,92231	,92236	,92241	,92247	,92252	,92257	,92262	,92267
837	,92272	,92278	,92283	,92288	,92293	,92298	,92304	,92309	,92314	,92319
838	,92324	,92330	,92335	,92340	,92345	,92350	,92355	,92361	,92366	,92371
839	,92376	,92381	,92386	,92392	,92397	,92402	,92407	,92412	,92418	,92423
840	,92428	,92433	,92438	,92443	,92449	,92454	,92459	,92464	,92469	,92474
841	,92480	,92485	,92490	,92495	,92500	,92505	,92511	,92516	,92521	,92526
842	,92531	,92536	,92541	,92547	,92552	,92557	,92562	,92567	,92572	,92578
843	,92583	,92588	,92593	,92598	,92603	,92608	,92614	,92619	,92624	,92629
844	,92634	,92639	,92644	,92650	,92655	,92660	,92665	,92670	,92675	,92680
845	,92686	,92691	,92696	,92701	,92706	,92711	,92716	,92722	,92727	,92732
846	,92737	,92742	,92747	,92752	,92758	,92763	,92768	,92773	,92778	,92783
847	,92788	,92793	,92799	,92804	,92809	,92814	,92819	,92824	,92829	,92834
848	,92840	,92845	,92850	,92855	,92860	,92865	,92870	,92875	,92880	,92886
849	,92891	,92896	,92901	,92906	,92911	,92916	,92921	,92927	,92932	,92937
850	,92942	,92947	,92952	,92957	,92962	,92967	,92972	,92978	,92983	,92988
851	,92993	,92998	,93003	,93008	,93013	,93018	,93024	,93029	,93034	,93039
852	,93044	,93049	,93054	,93059	,93064	,93069	,93074	,93080	,93085	,93090
853	,93095	,93100	,93105	,93110	,93115	,93120	,93125	,93130	,93136	,93141
854	,93146	,93151	,93156	,93161	,93166	,93171	,93176	,93181	,93186	,93191
855	,93197	,93202	,93207	,93212	,93217	,93222	,93227	,93232	,93237	,93242
856	,93247	,93252	,93257	,93263	,93268	,93273	,93278	,93283	,93288	,93293
857	,93298	,93303	,93308	,93313	,93318	,93323	,93328	,93333	,93339	,93344
858	,93349	,93354	,93359	,93364	,93369	,93374	,93379	,93384	,93389	,93394
859	,93399	,93404	,93409	,93414	,93419	,93425	,93430	,93435	,93440	,93445
860	,93450	,93455	,93460	,93465	,93470	,93475	,93480	,93485	,93490	,93495
861	,93500	,93505	,93510	,93515	,93520	,93525	,93531	,93536	,93541	,93546
862	,93551	,93556	,93561	,93566	,93571	,93576	,93581	,93586	,93591	,93596
863	,93601	,93606	,93611	,93616	,93621	,93626	,93631	,93636	,93641	,93646
864	,93651	,93656	,93661	,93666	,93671	,93676	,93681	,93686	,93692	,93697
865	,93702	,93707	,93712	,93717	,93722	,93727	,93732	,93737	,93742	,93747
866	,93752	,93757	,93762	,93767	,93772	,93777	,93782	,93787	,93792	,93797
867	,93802	,93807	,93812	,93817	,93822	,93827	,93832	,93837	,93842	,93847
868	,93852	,93857	,93862	,93867	,93872	,93877	,93882	,93887	,93892	,93897
869	,93902	,93907	,93912	,93917	,93922	,93927	,93932	,93937	,93942	,93947
870	,93952	,93957	,93962	,93967	,93972	,93977	,93982	,93987	,93992	,93997
871	,94002	,94007	,94012	,94017	,94022	,94027	,94032	,94037	,94042	,94047
872	,94052	,94057	,94062	,94067	,94072	,94076	,94081	,94086	,94091	,94096
873	,94101	,94106	,94111	,94116	,94121	,94126	,94131	,94136	,94141	,94146
874	,94151	,94156	,94161	,94166	,94171	,94176	,94181	,94186	,94191	,94196
875	,94201	,94206	,94211	,94216	,94221	,94226	,94230	,94235	,94240	,94245
876	,94250	,94255	,94260	,94265	,94270	,94275	,94280	,94285	,94290	,94295
877	,94300	,94305	,94310	,94315	,94320	,94325	,94330	,94335	,94340	,94344
878	,94349	,94354	,94359	,94364	,94369	,94374	,94379	,94384	,94389	,94394
879	,94399	,94404	,94409	,94414	,94419	,94424	,94428	,94433	,94438	,94443
N ^o	0	1	2	3	4	5	6	7	8	9

Numbers from 8800 to 9400. Logs. from ,94448 to ,97313.

N ^o	0	1	2	3	4	5	6	7	8	9
880	,94448	,94453	,94458	,94463	,94468	,94473	,94478	,94483	,94488	,94493
881	,94498	,94502	,94507	,94512	,94517	,94522	,94527	,94532	,94537	,94542
882	,94547	,94552	,94557	,94562	,94566	,94571	,94576	,94581	,94586	,94591
883	,94596	,94601	,94609	,94611	,94616	,94621	,94626	,94630	,94635	,94640
884	,94645	,94650	,94655	,94660	,94665	,94670	,94675	,94680	,94684	,94689
885	,94694	,94699	,94704	,94709	,94714	,94719	,94724	,94729	,94734	,94738
886	,94743	,94748	,94753	,94758	,94763	,94768	,94773	,94778	,94783	,94787
887	,94792	,94797	,94802	,94807	,94812	,94817	,94822	,94827	,94831	,94836
888	,94841	,94846	,94851	,94856	,94861	,94866	,94871	,94875	,94880	,94885
889	,94890	,94895	,94900	,94905	,94910	,94915	,94919	,94924	,94929	,94934
890	,94939	,94944	,94949	,94953	,94958	,94963	,94968	,94973	,94978	,94983
891	,94988	,94993	,94997	,95002	,95007	,95012	,95017	,95022	,95027	,95031
892	,95036	,95041	,95046	,95051	,95056	,95061	,95066	,95070	,95075	,95080
893	,95085	,95090	,95095	,95100	,95105	,95109	,95114	,95119	,95124	,95129
894	,95134	,95139	,95143	,95148	,95153	,95158	,95163	,95168	,95173	,95177
895	,95182	,95187	,95192	,95197	,95202	,95207	,95211	,95216	,95221	,95226
896	,95231	,95236	,95240	,95245	,95250	,95255	,95260	,95265	,95270	,95274
897	,95279	,95284	,95289	,95294	,95299	,95303	,95308	,95313	,95318	,95323
898	,95328	,95332	,95337	,95342	,95347	,95352	,95357	,95361	,95366	,95371
899	,95376	,95381	,95386	,95390	,95395	,95400	,95405	,95410	,95425	,95419
900	,95424	,95429	,95434	,95439	,95443	,95448	,95453	,95458	,95463	,95468
901	,95472	,95477	,95482	,95487	,95492	,95497	,95501	,95506	,95511	,95516
902	,95521	,95525	,95530	,95535	,95540	,95545	,95549	,95554	,95559	,95564
903	,95569	,95574	,95578	,95583	,95588	,95593	,95598	,95602	,95607	,95612
904	,95617	,95622	,95626	,95631	,95636	,95641	,95646	,95650	,95655	,95660
905	,95665	,95670	,95674	,95679	,95684	,95689	,95694	,95698	,95703	,95708
906	,95713	,95718	,95722	,95727	,95732	,95737	,95742	,95746	,95751	,95756
907	,95761	,95765	,95771	,95775	,95780	,95785	,95789	,95794	,95799	,95804
908	,95809	,95813	,95818	,95823	,95828	,95832	,95837	,95842	,95847	,95852
909	,95856	,95861	,95866	,95871	,95875	,95880	,95885	,95890	,95895	,95899
910	,95904	,95909	,95914	,95918	,95923	,95928	,95933	,95937	,95942	,95947
911	,95952	,95957	,95961	,95966	,95971	,95976	,95980	,95985	,95990	,95995
912	,95999	,96004	,96009	,96014	,96018	,96023	,96028	,96033	,96038	,96042
913	,96047	,96052	,96057	,96061	,96066	,96071	,96076	,96080	,96085	,96090
914	,96095	,96099	,96104	,96109	,96114	,96118	,96123	,96128	,96133	,96137
915	,96142	,96147	,96152	,96156	,96161	,96165	,96171	,96175	,96180	,96185
916	,96189	,96194	,96199	,96204	,96208	,96213	,96218	,96223	,96227	,96232
917	,96237	,96242	,96246	,96251	,96256	,96261	,96265	,96270	,96275	,96279
918	,96284	,96289	,96293	,96298	,96303	,96308	,96313	,96317	,96322	,96327
919	,96331	,96336	,96341	,96346	,96350	,96355	,96360	,96365	,96369	,96374
920	,96379	,96383	,96388	,96393	,96398	,96402	,96407	,96412	,96416	,96421
921	,96426	,96431	,96435	,96440	,96445	,96449	,96454	,96459	,96464	,96468
922	,96473	,96478	,96482	,96487	,96492	,96497	,96501	,96506	,96511	,96515
923	,96520	,96525	,96530	,96534	,96539	,96544	,96548	,96553	,96558	,96562
924	,96567	,96572	,96577	,96581	,96586	,96591	,96595	,96600	,96605	,96609
925	,96614	,96619	,96624	,96628	,96633	,96638	,96642	,96647	,96651	,96656
926	,96661	,96666	,96670	,96675	,96680	,96684	,96689	,96694	,96699	,96703
927	,96708	,96713	,96717	,96722	,96727	,96731	,96736	,96741	,96745	,96750
928	,96755	,96759	,96764	,96769	,96773	,96778	,96783	,96787	,96792	,96797
929	,96802	,96806	,96811	,96816	,96820	,96825	,96830	,96834	,96839	,96844
930	,96848	,96853	,96858	,96862	,96867	,96872	,96876	,96881	,96886	,96890
931	,96895	,96900	,96904	,96909	,96914	,96918	,96923	,96928	,96932	,96937
932	,96942	,96946	,96951	,96956	,96960	,96965	,96969	,96974	,96979	,96983
933	,96988	,96993	,96997	,97002	,97007	,97011	,97016	,97021	,97025	,97030
934	,97035	,97039	,97044	,97049	,97053	,97058	,97062	,97067	,97072	,97076
935	,97081	,97086	,97090	,97095	,97100	,97104	,97109	,97114	,97118	,97123
936	,97128	,97132	,97137	,97141	,97146	,97151	,97155	,97160	,97165	,97169
937	,97174	,97179	,97183	,97188	,97192	,97197	,97202	,97206	,97211	,97216
938	,97220	,97225	,97229	,97234	,97239	,97243	,97248	,97253	,97257	,97262
939	,97267	,97271	,97276	,97280	,97285	,97290	,97294	,97299	,97303	,97308
N ^o	0	1	2	3	4	5	6	7	8	9

Numbers from 9400 to 10000. Logs. from ,97313 to ,99996.

N ^o	0	1	2	3	4	5	6	7	8	9
940	,97313	,97317	,97322	,97326	,97331	,97336	,97340	,97345	,97350	,97354
941	,97359	,97364	,97368	,97373	,97377	,97382	,97387	,97391	,97396	,97400
942	,97405	,97410	,97414	,97419	,97423	,97428	,97433	,97437	,97442	,97447
943	,97451	,97456	,97460	,97465	,97470	,97474	,97479	,97483	,97488	,97492
944	,97507	,97502	,97506	,97511	,97516	,97520	,97525	,97529	,97534	,97539
945	,97543	,97548	,97552	,97557	,97562	,97566	,97571	,97575	,97580	,97584
946	,97589	,97594	,97598	,97603	,97607	,97612	,97617	,97621	,97626	,97630
947	,97635	,97640	,97644	,97649	,97653	,97658	,97662	,97667	,97672	,97676
948	,97681	,97685	,97690	,97694	,97699	,97704	,97708	,97713	,97717	,97722
949	,97727	,97731	,97736	,97740	,97745	,97749	,97754	,97759	,97763	,97768
950	,97772	,97777	,97781	,97786	,97791	,97795	,97800	,97804	,97809	,97813
951	,97818	,97823	,97827	,97832	,97836	,97841	,97845	,97850	,97855	,97859
952	,97864	,97868	,97873	,97877	,97882	,97886	,97891	,97896	,97900	,97905
953	,97909	,97914	,97918	,97923	,97927	,97932	,97937	,97941	,97946	,97950
954	,97955	,97959	,97964	,97968	,97973	,97978	,97982	,97987	,97991	,97996
955	,98000	,98005	,98009	,98014	,98018	,98023	,98028	,98032	,98037	,98041
956	,98046	,98050	,98055	,98059	,98064	,98068	,98073	,98078	,98082	,98087
957	,98091	,98096	,98100	,98105	,98109	,98114	,98118	,98123	,98127	,98132
958	,98136	,98141	,98146	,98150	,98155	,98159	,98164	,98168	,98173	,98177
959	,98182	,98186	,98191	,98195	,98200	,98204	,98209	,98213	,98218	,98223
960	,98227	,98232	,98236	,98241	,98245	,98250	,98254	,98259	,98263	,98268
961	,98272	,98277	,98281	,98286	,98290	,98295	,98299	,98304	,98308	,98313
962	,98317	,98322	,98326	,98331	,98336	,98340	,98345	,98349	,98354	,98358
963	,98363	,98367	,98372	,98376	,98381	,98385	,98390	,98394	,98399	,98403
964	,98408	,98412	,98417	,98421	,98426	,98430	,98435	,98439	,98444	,98448
965	,98453	,98457	,98462	,98466	,98471	,98475	,98480	,98484	,98489	,98493
966	,98498	,98502	,98507	,98511	,98516	,98520	,98525	,98529	,98534	,98538
967	,98543	,98547	,98552	,98556	,98561	,98565	,98570	,98574	,98579	,98583
968	,98587	,98592	,98596	,98601	,98605	,98610	,98614	,98619	,98623	,98628
969	,98632	,98637	,98641	,98646	,98650	,98655	,98659	,98664	,98668	,98673
970	,98677	,98682	,98686	,98691	,98695	,98699	,98704	,98708	,98713	,98717
971	,98722	,98726	,98731	,98735	,98740	,98744	,98749	,98753	,98758	,98762
972	,98767	,98771	,98776	,98780	,98784	,98789	,98793	,98798	,98802	,98807
973	,98811	,98816	,98820	,98825	,98829	,98834	,98838	,98842	,98847	,98851
974	,98856	,98860	,98865	,98869	,98874	,98878	,98883	,98887	,98891	,98896
975	,98900	,98905	,98909	,98914	,98918	,98923	,98927	,98932	,98936	,98940
976	,98945	,98949	,98954	,98958	,98963	,98967	,98972	,98976	,98981	,98985
977	,98989	,98994	,98998	,99003	,99007	,99012	,99016	,99021	,99025	,99029
978	,99034	,99038	,99043	,99047	,99052	,99056	,99060	,99065	,99069	,99074
979	,99078	,99083	,99087	,99092	,99096	,99100	,99105	,99109	,99114	,99118
980	,99123	,99127	,99131	,99136	,99140	,99145	,99149	,99154	,99158	,99162
981	,99167	,99171	,99176	,99180	,99185	,99189	,99193	,99198	,99202	,99207
982	,99211	,99216	,99220	,99224	,99229	,99233	,99238	,99242	,99246	,99251
983	,99255	,99259	,99264	,99269	,99273	,99277	,99282	,99286	,99291	,99295
984	,99299	,99304	,99308	,99313	,99317	,99322	,99326	,99330	,99335	,99339
985	,99344	,99348	,99352	,99357	,99361	,99366	,99370	,99374	,99379	,99383
986	,99388	,99392	,99396	,99401	,99405	,99410	,99414	,99418	,99423	,99427
987	,99432	,99436	,99440	,99445	,99449	,99454	,99458	,99462	,99467	,99471
988	,99476	,99480	,99484	,99489	,99493	,99498	,99502	,99506	,99511	,99515
989	,99520	,99524	,99528	,99533	,99537	,99542	,99546	,99550	,99555	,99559
990	,99563	,99568	,99572	,99577	,99581	,99585	,99590	,99594	,99599	,99603
991	,99607	,99612	,99616	,99620	,99625	,99629	,99634	,99638	,99642	,99647
992	,99651	,99655	,99660	,99664	,99669	,99673	,99677	,99682	,99686	,99690
993	,99695	,99699	,99704	,99708	,99712	,99717	,99721	,99725	,99730	,99734
994	,99739	,99743	,99747	,99752	,99756	,99760	,99765	,99769	,99774	,99778
995	,99782	,99787	,99791	,99795	,99800	,99804	,99808	,99813	,99817	,99822
996	,99826	,99830	,99835	,99839	,99843	,99848	,99852	,99856	,99861	,99865
997	,99869	,99874	,99878	,99883	,99887	,99891	,99896	,99900	,99904	,99909
998	,99913	,99917	,99922	,99926	,99930	,99935	,99939	,99943	,99948	,99952
999	,99956	,99961	,99965	,99971	,99974	,99978	,99983	,99987	,99991	,99996
N ^o	0	1	2	3	4	5	6	7	8	9

0 Degree					1 Degree				
M	Sine	Co-sine	Tangent	Co-tang.	M	Sine	Co-sine	Tangent	Co-tang.
0	0,00000	1,00000	0,00000	Infinite	60	0,824185	9,99993	8,24192	11,75808
1	6,46373	9,99999	6,46373	13,53627	59	1	8,24903	9,99993	8,24910
2	6,76476	9,99999	6,76476	13,23524	58	2	8,25609	9,99993	8,25616
3	6,94085	9,99999	6,94085	13,0915	57	3	8,26304	9,99993	8,26311
4	7,00579	9,99999	7,00579	12,93421	56	4	8,26988	9,99992	8,26996
5	7,16270	9,99999	7,16270	12,83730	55	5	8,27661	9,99992	8,27669
6	7,24188	9,99999	7,24188	12,75812	54	6	8,28324	9,99992	8,28332
7	7,30882	9,99999	7,30882	12,69117	53	7	8,28977	9,99992	8,28986
8	7,3682	9,99999	7,36682	12,63318	52	8	8,29621	9,99991	8,29629
9	7,41797	9,99999	7,41797	12,58203	51	9	8,30255	9,99991	8,30263
10	7,46372	9,99999	7,46373	12,53627	50	10	8,30879	9,99991	8,30888
11	7,5052	9,99999	7,50512	12,49488	49	11	8,31495	9,99990	8,31504
12	7,54291	9,99999	7,54291	12,45709	48	12	8,32103	9,99990	8,32112
13	7,57767	9,99999	7,57767	12,42233	47	13	8,32702	9,99990	8,32711
14	7,60985	9,99999	7,60986	12,39014	46	14	8,33292	9,99990	8,33302
15	7,63982	9,99999	7,63982	12,36018	45	15	8,33875	9,99989	8,33886
16	7,66784	9,99999	7,66785	12,33215	44	16	8,34450	9,99989	8,34461
17	7,69417	9,99999	7,69418	12,30582	43	17	8,35018	9,99989	8,35029
18	7,71900	9,99999	7,71900	12,28100	42	18	8,35578	9,99989	8,35589
19	7,74248	9,99999	7,74248	12,25752	41	19	8,36131	9,99988	8,36143
20	7,76475	9,99999	7,76476	12,23524	40	20	8,36678	9,99988	8,36689
21	7,78594	9,99999	7,78595	12,21405	39	21	8,37217	9,99988	8,37229
22	7,80615	9,99999	7,80615	12,19384	38	22	8,37750	9,99988	8,37762
23	7,82545	9,99999	7,82546	12,17454	37	23	8,38276	9,99987	8,38289
24	7,84393	9,99999	7,84394	12,15606	36	24	8,38796	9,99987	8,38809
25	7,86166	9,99999	7,86167	12,13833	35	25	8,39310	9,99987	8,39323
26	7,87869	9,99999	7,87871	12,12129	34	26	8,39818	9,99986	8,39831
27	7,89508	9,99999	7,89510	12,10490	33	27	8,40320	9,99986	8,40334
28	7,91088	9,99999	7,91089	12,08911	32	28	8,40816	9,99986	8,40830
29	7,92612	9,99998	7,92613	12,07387	31	29	8,41307	9,99985	8,41321
30	7,94084	9,99998	7,94086	12,05914	30	30	8,41792	9,99985	8,41807
31	7,95508	9,99998	7,95510	12,04490	29	31	8,42272	9,99985	8,42287
32	7,96887	9,99998	7,96889	12,03111	28	32	8,42746	9,99984	8,42762
33	7,98223	9,99998	7,98225	12,01775	27	33	8,43216	9,99984	8,43231
34	7,99520	9,99998	7,99522	12,00478	26	34	8,43680	9,99984	8,43696
35	8,00779	9,99998	8,00781	11,99219	25	35	8,44139	9,99983	8,44156
36	8,02002	9,99998	8,02004	11,97995	24	36	8,44594	9,99983	8,44611
37	8,03192	9,99997	8,03194	11,96805	23	37	8,45044	9,99983	8,45061
38	8,04350	9,99997	8,04353	11,95647	22	38	8,45489	9,99982	8,45507
39	8,05478	9,99997	8,05481	11,94519	21	39	8,45930	9,99982	8,45948
40	8,06578	9,99997	8,06581	11,93419	20	40	8,46366	9,99982	8,46385
41	8,07650	9,99997	8,07653	11,92347	19	41	8,46798	9,99981	8,46817
42	8,08696	9,99997	8,08700	11,91300	18	42	8,47226	9,99981	8,47245
43	8,09718	9,99997	8,09722	11,90278	17	43	8,47650	9,99980	8,47669
44	8,10717	9,99996	8,10720	11,89280	16	44	8,48069	9,99980	8,48089
45	8,11693	9,99996	8,11696	11,88304	15	45	8,48485	9,99980	8,48505
46	8,12647	9,99996	8,12651	11,87349	14	46	8,48896	9,99979	8,48917
47	8,13581	9,99996	8,13585	11,86415	13	47	8,49304	9,99979	8,49325
48	8,14495	9,99996	8,14500	11,85500	12	48	8,49708	9,99979	8,49729
49	8,15391	9,99996	8,15395	11,84605	11	49	8,50108	9,99978	8,50130
50	8,16268	9,99995	8,16273	11,83727	10	50	8,50504	9,99978	8,50527
51	8,17128	9,99995	8,17133	11,82867	9	51	8,50897	9,99977	8,50920
52	8,17971	9,99995	8,17976	11,82024	8	52	8,51287	9,99977	8,51310
53	8,18798	9,99995	8,18804	11,81196	7	53	8,51673	9,99976	8,51696
54	8,19610	9,99995	8,19616	11,80384	6	54	8,52055	9,99976	8,52079
55	8,20407	9,99994	8,20413	11,79587	5	55	8,52434	9,99976	8,52459
56	8,21189	9,99994	8,21195	11,78805	4	56	8,52810	9,99975	8,52835
57	8,21958	9,99994	8,21964	11,78036	3	57	8,53183	9,99975	8,53208
58	8,22713	9,99994	8,22719	11,77280	2	58	8,53552	9,99974	8,53578
59	8,23456	9,99994	8,23462	11,76538	1	59	8,53919	9,99974	8,53945
60	8,24185	9,99993	8,24192	11,75808	0	60	8,54282	9,99973	8,54308
Co-sine Sine Co-tan. Tangent M					Co-sine Sine Co-tan. Tangent M				

89 Degrees

88 Degrees

2 Degrees

3 Degrees

M	Sine	Co-sine	Tangent	Co-tang.	M	Sine	Co-sine	Tangent	Co-tang.
0	8,54282	9,99973	8,54308	11,45692	60	8,71880	9,99940	8,71940	11,28060
1	8,54642	9,99973	8,54669	11,45331	59	8,72120	9,99940	8,72181	11,27819
2	8,54999	9,99973	8,55027	11,44973	58	8,72359	9,99939	8,72420	11,27580
3	8,55354	9,99972	8,55382	11,44618	57	8,72597	9,99938	8,72659	11,27341
4	8,55705	9,99972	8,55734	11,44266	56	8,72834	9,99938	8,72896	11,27104
5	8,56054	9,99971	8,56083	11,43917	55	8,73069	9,99937	8,73132	11,26868
6	8,56400	9,99970	8,56429	11,43571	54	8,73303	9,99936	8,73366	11,26634
7	8,56743	9,99970	8,56773	11,43227	53	8,73535	9,99936	8,73600	11,26400
8	8,57084	9,99970	8,57114	11,42886	52	8,73767	9,99935	8,73832	11,26168
9	8,57421	9,99969	8,57452	11,42548	51	8,73997	9,99934	8,74063	11,25937
10	8,57757	9,99969	8,57788	11,42212	50	8,74226	9,99934	8,74292	11,25708
11	8,58089	9,99968	8,58121	11,41879	49	8,74454	9,99933	8,74521	11,25479
12	8,58419	9,99968	8,58451	11,41549	48	8,74680	9,99932	8,74748	11,25252
13	8,58747	9,99967	8,58779	11,41221	47	8,74905	9,99931	8,74974	11,25026
14	8,59072	9,99967	8,59105	11,40895	46	8,75130	9,99931	8,75199	11,24801
15	8,59395	9,99966	8,59428	11,40572	45	8,75353	9,99930	8,75423	11,24577
16	8,59715	9,99966	8,59749	11,40251	44	8,75575	9,99929	8,75645	11,24355
17	8,60033	9,99965	8,60068	11,39932	43	8,75795	9,99929	8,75867	11,24133
18	8,60349	9,99965	8,60384	11,39616	42	8,76015	9,99928	8,76087	11,23913
19	8,60662	9,99964	8,60698	11,39302	41	8,76234	9,99927	8,76306	11,23693
20	8,60973	9,99964	8,61009	11,38991	40	8,76451	9,99926	8,76525	11,23475
21	8,61282	9,99963	8,61319	11,38681	39	8,76667	9,99926	8,76742	11,23258
22	8,61589	9,99963	8,61626	11,38374	38	8,76883	9,99925	8,76958	11,23042
23	8,61894	9,99962	8,61931	11,38069	37	8,77097	9,99924	8,77172	11,22827
24	8,62196	9,99962	8,62234	11,37766	36	8,77310	9,99923	8,77387	11,22613
25	8,62496	9,99961	8,62535	11,37465	35	8,77522	9,99923	8,77599	11,22400
26	8,62795	9,99961	8,62834	11,37166	34	8,77733	9,99922	8,77811	11,22189
27	8,63091	9,99960	8,63131	11,36869	33	8,77943	9,99921	8,78022	11,21978
28	8,63385	9,99960	8,63426	11,36574	32	8,78152	9,99920	8,78232	11,21768
29	8,63678	9,99959	8,63718	11,36281	31	8,78360	9,99920	8,78441	11,21559
30	8,63968	9,99959	8,64009	11,35991	30	8,78567	9,99919	8,78649	11,21351
31	8,64256	9,99958	8,64298	11,35702	29	8,78774	9,99918	8,78855	11,21145
32	8,64543	9,99957	8,64585	11,35415	28	8,78979	9,99917	8,79061	11,20939
33	8,64827	9,99957	8,64870	11,35130	27	8,79183	9,99917	8,79266	11,20734
34	8,65110	9,99956	8,65154	11,34846	26	8,79386	9,99915	8,79470	11,20530
35	8,65391	9,99956	8,65435	11,34565	25	8,79588	9,99915	8,79673	11,20327
36	8,65670	9,99955	8,65715	11,34285	24	8,79789	9,99914	8,79855	11,20125
37	8,65947	9,99955	8,65993	11,34007	23	8,79990	9,99913	8,80076	11,19924
38	8,66223	9,99954	8,66269	11,33731	22	8,80189	9,99913	8,80276	11,19723
39	8,66497	9,99953	8,66543	11,33457	21	8,80388	9,99912	8,80476	11,19524
40	8,66769	9,99953	8,66816	11,33184	20	8,80585	9,99911	8,80674	11,19326
41	8,67039	9,99952	8,67087	11,32913	19	8,80782	9,99910	8,80872	11,19128
42	8,67308	9,99952	8,67356	11,32644	18	8,80978	9,99909	8,81068	11,18932
43	8,67575	9,99951	8,67624	11,32376	17	8,81173	9,99909	8,81264	11,18736
44	8,67840	9,99951	8,67890	11,32110	16	8,81367	9,99908	8,81459	11,18541
45	8,68104	9,99950	8,68154	11,31846	15	8,81560	9,99907	8,81653	11,18347
46	8,68366	9,99949	8,68417	11,31583	14	8,81752	9,99906	8,81846	11,18154
47	8,68627	9,99949	8,68678	11,31322	13	8,81944	9,99905	8,82038	11,17962
48	8,68886	9,99948	8,68938	11,31062	12	8,82134	9,99904	8,82230	11,17770
49	8,69144	9,99947	8,69196	11,30804	11	8,82324	9,99904	8,82420	11,17579
50	8,69400	9,99947	8,69453	11,30547	10	8,82513	9,99903	8,82610	11,17390
51	8,69654	9,99946	8,69708	11,30292	9	8,82701	9,99902	8,82799	11,17201
52	8,69907	9,99946	8,69962	11,30038	8	8,82888	9,99901	8,82987	11,17013
53	8,70159	9,99945	8,70214	11,29786	7	8,83075	9,99900	8,83175	11,16825
54	8,70409	9,99944	8,70465	11,29535	6	8,83261	9,99899	8,83361	11,16639
55	8,70658	9,99944	8,70714	11,29286	5	8,83446	9,99898	8,83547	11,16453
56	8,70905	9,99943	8,70962	11,29038	4	8,83630	9,99898	8,83732	11,16268
57	8,71151	9,99942	8,71208	11,28792	3	8,83813	9,99897	8,83916	11,16084
58	8,71395	9,99942	8,71453	11,28546	2	8,83996	9,99896	8,84100	11,15900
59	8,71638	9,99941	8,71697	11,28303	1	8,84177	9,99895	8,84282	11,15717
60	8,71880	9,99940	8,71940	11,28060	0	8,84358	9,99894	8,84464	11,15536
Co fine	Sine	Co-tan.	Tangent	M	Co fine	Sine	Co-tan.	Tangent	M

87 Degrees

80 Degrees

4 Degrees					5 Degrees					
M	Sine	Co-sine	Tangent	Co-tang.	M	Sine	Co-sine	Tangent	Co-tang.	
0	8,84358	9,99894	8,84464	11,15536	60	8,94030	9,99834	8,94195	11,05805	
1	8,84539	9,99893	8,84645	11,15354	59	8,94174	9,99833	8,94340	11,05660	
2	8,84718	9,99892	8,84826	11,15174	58	8,94317	9,99832	8,94485	11,05515	
3	8,84897	9,99891	8,85006	11,14994	57	8,94461	9,99831	8,94629	11,05370	
4	8,85075	9,99890	8,85185	11,14815	56	8,94603	9,99830	8,94773	11,05227	
5	8,85252	9,99890	8,85363	11,14637	55	8,94746	9,99829	8,94917	11,05083	
6	8,85429	9,99889	8,85540	11,14460	54	8,94887	9,99828	8,95060	11,04940	
7	8,85605	9,99888	8,85717	11,14283	53	8,95029	9,99827	8,95202	11,04798	
8	8,85780	9,99887	8,85893	11,14107	52	8,95170	9,99825	8,95344	11,04656	
9	8,85955	9,99886	8,86069	11,13931	51	8,95310	9,99824	8,95486	11,04514	
10	8,86128	9,99885	8,86243	11,13757	50	8,95450	9,99823	8,95627	11,04373	
11	8,86301	9,99884	8,86417	11,13583	49	8,95590	9,99823	8,95767	11,04233	
12	8,86474	9,99883	8,86590	11,13409	48	8,95728	9,99822	8,95907	11,04092	
13	8,86645	9,99882	8,86763	11,13237	47	8,95867	9,99820	8,96047	11,03953	
14	8,86816	9,99881	8,86935	11,13065	46	8,96005	9,99819	8,96187	11,03813	
15	8,86987	9,99880	8,87106	11,12894	45	8,96143	9,99817	8,96325	11,03674	
16	8,87156	9,99879	8,87277	11,12723	44	8,96280	9,99816	8,96464	11,03536	
17	8,87325	9,99878	8,87447	11,12553	43	8,96417	9,99815	8,96602	11,03398	
18	8,87494	9,99878	8,87616	11,12384	42	8,96553	9,99814	8,96739	11,03261	
19	8,87661	9,99877	8,87785	11,12215	41	8,96689	9,99813	8,96877	11,03123	
20	8,87828	9,99876	8,87953	11,12047	40	8,96825	9,99812	8,97013	11,02987	
21	8,87995	9,99875	8,88120	11,11880	39	8,96960	9,99810	8,97150	11,02850	
22	8,88161	9,99874	8,88287	11,11713	38	8,97095	9,99809	8,97285	11,02714	
23	8,88326	9,99873	8,88453	11,11547	37	8,97229	9,99808	8,97421	11,02579	
24	8,88490	9,99872	8,88618	11,11381	36	8,97363	9,99807	8,97556	11,02444	
25	8,88654	9,99871	8,88783	11,11217	35	8,97496	9,99806	8,97691	11,02309	
26	8,88817	9,99870	8,88948	11,11052	34	8,97629	9,99804	8,97825	11,02175	
27	8,88980	9,99869	8,89111	11,10889	33	8,97762	9,99803	8,97959	11,02041	
28	8,89142	9,99868	8,89274	11,10726	32	8,97894	9,99802	8,98092	11,01908	
29	8,89303	9,99867	8,89437	11,10563	31	8,98026	9,99801	8,98225	11,01775	
30	8,89464	9,99866	8,89598	11,10402	30	8,98157	9,99800	8,98358	11,01642	
31	8,89624	9,99865	8,89760	11,10240	29	8,98288	9,99798	8,98490	11,01510	
32	8,89784	9,99864	8,89920	11,10080	28	8,98419	9,99797	8,98622	11,01378	
33	8,89943	9,99863	8,90080	11,09920	27	8,98559	9,99796	8,98753	11,01247	
34	8,90102	9,99862	8,90240	11,09760	26	8,98688	9,99795	8,98884	11,01116	
35	8,90260	9,99861	8,90399	11,09601	25	8,98818	9,99793	8,99015	11,00985	
36	8,90417	9,99860	8,90557	11,09443	24	8,98947	9,99792	8,99145	11,00855	
37	8,90574	9,99859	8,90715	11,09285	23	8,99066	9,99791	8,99275	11,00725	
38	8,90730	9,99858	8,90872	11,09128	22	8,99194	9,99790	8,99404	11,00595	
39	8,90885	9,99857	8,91028	11,08971	21	8,99322	9,99788	8,99534	11,00466	
40	8,91040	9,99856	8,91185	11,08815	20	8,99450	9,99787	8,99662	11,00338	
41	8,91195	9,99855	8,91340	11,08660	19	8,99577	9,99786	8,99791	11,00209	
42	8,91349	9,99854	8,91495	11,08505	18	8,99704	9,99785	8,99919	11,00081	
43	8,91502	9,99853	8,91649	11,08350	17	8,99830	9,99783	9,00046	10,99953	
44	8,91655	9,99852	8,91803	11,08197	16	8,99956	9,99782	9,00174	10,99826	
45	8,91807	9,99851	8,91957	11,08043	15	9,00082	9,99781	9,00301	10,99699	
46	8,91959	9,99849	8,92110	11,07890	14	9,00207	9,99780	9,00427	10,99573	
47	8,92110	9,99848	8,92262	11,07738	13	9,00332	9,99778	9,00553	10,99447	
48	8,92261	9,99847	8,92414	11,07586	12	9,00456	9,99777	9,00679	10,99321	
49	8,92411	9,99846	8,92565	11,07435	11	9,00580	9,99776	9,00805	10,99195	
50	8,92561	9,99845	8,92716	11,07285	10	9,00704	9,99774	9,00930	10,99070	
51	8,92710	9,99844	8,92866	11,07134	9	9,00828	9,99773	9,01055	10,98945	
52	8,92859	9,99843	8,93015	11,06984	8	9,00951	9,99772	9,01179	10,98821	
53	8,93007	9,99842	8,93165	11,06835	7	9,01074	9,99771	9,01303	10,98697	
54	8,93154	9,99841	8,93313	11,06687	6	9,01196	9,99769	9,01427	10,98573	
55	8,93301	9,99840	8,93462	11,06538	5	9,01318	9,99768	9,01550	10,98450	
56	8,93448	9,99839	8,93609	11,06391	4	9,01440	9,99767	9,01673	10,98327	
57	8,93594	9,99838	8,93756	11,06243	3	9,01561	9,99765	9,01796	10,98204	
58	8,93740	9,99837	8,93903	11,06097	2	9,01682	9,99764	9,01918	10,98082	
59	8,93885	9,99835	8,94049	11,05951	1	9,01803	9,99763	9,02040	10,97960	
60	8,94030	9,99834	8,94195	11,05805	0	9,01923	9,99761	9,02162	10,97838	
	Co-sine	Sine	Co-tan.	Tangent	M	Co-sine	Sine	Co-tan.	Tangent	M

85 Degrees					84 Degrees				
M	Sine	Co-sine	Tangent	Co-tang.	M	Sine	Co-sine	Tangent	Co-tang.
0	8,94030	9,99834	8,94195	11,05805	60	8,94030	9,99834	8,94195	11,05805
1	8,94174	9,99833	8,94340	11,05660	59	8,94174	9,99833	8,94340	11,05660
2	8,94317	9,99832	8,94485	11,05515	58	8,94317	9,99832	8,94485	11,05515
3	8,94461	9,99831	8,94629	11,05370	57	8,94461	9,99831	8,94629	11,05370
4	8,94603	9,99830	8,94773	11,05227	56	8,94603	9,99830	8,94773	11,05227
5	8,94746	9,99829	8,94917	11,05083	55	8,94746	9,99829	8,94917	11,05083
6	8,94887	9,99828	8,95060	11,04940	54	8,94887	9,99828	8,95060	11,04940
7	8,95029	9,99827	8,95202	11,04798	53	8,95029	9,99827	8,95202	11,04798
8	8,95170	9,99825	8,95344	11,04656	52	8,95170	9,99825	8,95344	11,04656
9	8,95310	9,99824	8,95486	11,04514	51	8,95310	9,99824	8,95486	11,04514
10	8,95450	9,99823	8,95627	11,04373	50	8,95450	9,99823	8,95627	11,04373
11	8,95590	9,99823	8,95767	11,04233	49	8,95590	9,99823	8,95767	11,04233
12	8,95728	9,99822	8,95907	11,04092	48	8,95728	9,99822	8,95907	11,04092
13	8,95867	9,99820	8,96047	11,03953	47	8,95867	9,99820	8,96047	11,03953
14	8,96005	9,99819	8,96187	11,03813	46	8,96005	9,99819	8,96187	11,03813
15	8,96143	9,99817	8,96325	11,03674	45	8,96143	9,99817	8,96325	11,03674
16	8,96280	9,99816	8,96464	11,03536	44	8,96280	9,99816	8,96464	11,03536
17	8,96417	9,99815	8,96602	11,03398	43	8,96417	9,99815	8,96602	11,03398
18	8,96553	9,99814	8,96739	11,03261	42	8,96553	9,99814	8,96739	11,03261
19	8,96689	9,99813	8,96877	11,03123	41	8,96689	9,99813	8,96877	11,03123
20	8,96825	9,99812	8,97013	11,02987	40	8,96825	9,99812	8,97013	11,02987
21	8,96960	9,99810	8,97150	11,02850	39	8,96960	9,99810	8,97150	11,02850
22	8,97095	9,99809	8,97285	11,02714	38	8,97095	9,99809	8,97285	11,02714
23	8,97229	9,99808	8,97421	11,02579	37	8,97229	9,99808	8,97421	11,02579
24	8,97363	9,99807	8,97556	11,02444	36	8,97363	9,99807	8,97556	11,02444
25	8,97496	9,99806	8,97691	11,02309	35	8,97496	9,99806	8,97691	11,02309
26	8,97629	9,99804	8,97825	11,02175	34	8,97629	9,99804	8,97825	11,02175
27	8,97762	9,99803	8,97959	11,02041	33	8,97762	9,99803	8,97959	11,02041
28	8,97894	9,99802	8,98092	11,01908	32	8,97894	9,99802	8,98092	11,01908
29	8,98026	9,99801	8,98225	11,01775	31	8,98026	9,99801	8,98225	11,01775
30	8,98157	9,99800	8,98358	11,01642	30	8,98157	9,99800	8,98358	11,01642
31	8,98288	9,99798	8,98490	11,01510	29	8,98288	9,99798	8,98490	11,01510
32	8,98419	9,99797	8,98622	11,01378	28	8,98419	9,99797	8,98622	11,01378
33	8,98559	9,99796	8,98753	11,01247	27	8,98559	9,99796	8,98753	11,01247
34	8,98688	9,99795	8,98884	11,01116	26	8,98688	9,99795	8,98884	11,01116
35	8,98818	9,99793	8,99015	11,00985	25	8,98818	9,99793	8,99015	11,00985
36	8,98947	9,99792	8,99145	11,00855	24	8,98947	9,99792	8,99145	11,00855
37	8,99066	9,99791	8,99275	11,00725	23	8,99066	9,99791	8,99275	11,00725
38	8,99194	9,99790	8,99404	11,00595	22	8,99194	9,99790	8,99404	11,00595
39	8,99322	9,99788	8,99534	11,00466	21	8,99322	9,99788	8,99534	11,00466
40	8,99450	9,99787	8,99662	11,00338	20	8,99450	9,99787	8,99662	11,00338
41	8,99577	9,99786	8,99791	11,00209	19	8,99577	9,99786	8,99791	11,00209
42	8,99704	9,99785	8,99919	11,00081	18	8,99704	9,99785	8,99919	11,00081
43	8,99830	9,99783	9,00046	10,99953	17	8,99830	9		

85 Degrees

84 Degrees

6 Degrees

7 Degrees

M	Sine	Co-sine	Tangent	Co-tang.		M	Sine	Co-sine	Tangent	Co-tang.	
0	9,01923	9,99761	9,02162	10,97838	60	0	9,08589	9,99675	9,08914	10,91086	60
1	9,02043	9,99760	9,02283	10,97717	59	1	9,08692	9,99673	9,09019	10,90981	59
2	9,02163	9,99759	9,02404	10,97596	58	2	9,08795	9,99672	9,09123	10,90877	58
3	9,02282	9,99757	9,02525	10,97475	57	3	9,08897	9,99670	9,09227	10,90773	57
4	9,02402	9,99756	9,02645	10,97354	56	4	9,08999	9,99669	9,09330	10,90670	56
5	9,02520	9,99755	9,02765	10,97234	55	5	9,09101	9,99667	9,09433	10,90566	55
6	9,02639	9,99753	9,02885	10,97115	54	6	9,09202	9,99666	9,09537	10,90463	54
7	9,02757	9,99752	9,03005	10,96995	53	7	9,09304	9,99664	9,09640	10,90360	53
8	9,02874	9,99751	9,03124	10,96876	52	8	9,09405	9,99662	9,09742	10,90258	52
9	9,02992	9,99749	9,03242	10,96757	51	9	9,09506	9,99661	9,09845	10,90155	51
10	9,03109	9,99748	9,03361	10,96639	50	10	9,09606	9,99659	9,09947	10,90053	50
11	9,03226	9,99747	9,03479	10,96521	49	11	9,09706	9,99658	9,10049	10,89951	49
12	9,03342	9,99745	9,03597	10,96403	48	12	9,09807	9,99656	9,10150	10,89849	48
13	9,03458	9,99744	9,03714	10,96286	47	13	9,09906	9,99655	9,10252	10,89748	47
14	9,03574	9,99742	9,03832	10,96168	46	14	9,10006	9,99653	9,10353	10,89647	46
15	9,03690	9,99741	9,03948	10,96051	45	15	9,10106	9,99651	9,10454	10,89546	45
16	9,03805	9,99740	9,04065	10,95935	44	16	9,10205	9,99650	9,10555	10,89445	44
17	9,03920	9,99738	9,04181	10,95819	43	17	9,10304	9,99648	9,10656	10,89344	43
18	9,04034	9,99737	9,04297	10,95703	42	18	9,10402	9,99646	9,10756	10,89244	42
19	9,04148	9,99735	9,04413	10,95587	41	19	9,10501	9,99645	9,10856	10,89144	41
20	9,04262	9,99734	9,04528	10,95472	40	20	9,10599	9,99643	9,10956	10,89044	40
21	9,04376	9,99733	9,04643	10,95357	39	21	9,10697	9,99642	9,11056	10,88944	39
22	9,04489	9,99731	9,04758	10,95242	38	22	9,10795	9,99640	9,11155	10,88845	38
23	9,04603	9,99730	9,04873	10,95127	37	23	9,10893	9,99638	9,11254	10,88746	37
24	9,04715	9,99728	9,04987	10,95013	36	24	9,10990	9,99637	9,11353	10,88647	36
25	9,04828	9,99727	9,05101	10,94899	35	25	9,11087	9,99635	9,11452	10,88548	35
26	9,04940	9,99726	9,05214	10,94876	34	26	9,11184	9,99633	9,11551	10,88449	34
27	9,05052	9,99724	9,05328	10,94672	33	27	9,11121	9,99632	9,11649	10,88351	33
28	9,05163	9,99723	9,05441	10,94559	32	28	9,11377	9,99630	9,11747	10,88253	32
29	9,05275	9,99721	9,05553	10,94447	31	29	9,11474	9,99628	9,11845	10,88155	31
30	9,05386	9,99720	9,05666	10,94334	30	30	9,11570	9,99627	9,11943	10,88057	30
31	9,05497	9,99718	9,05778	10,94222	29	31	9,11666	9,99625	9,12040	10,87960	29
32	9,05607	9,99717	9,05890	10,94110	28	32	9,11761	9,99623	9,12138	10,87862	28
33	9,05717	9,99716	9,06002	10,93998	27	33	9,11857	9,99622	9,12235	10,87765	27
34	9,05827	9,99714	9,06113	10,93887	26	34	9,11952	9,99620	9,12332	10,87668	26
35	9,05937	9,99713	9,06224	10,93776	25	35	9,12047	9,99618	9,12428	10,87572	25
36	9,06046	9,99711	9,06335	10,93665	24	36	9,12142	9,99617	9,12525	10,87475	24
37	9,06155	9,99710	9,06445	10,93555	23	37	9,12236	9,99615	9,12621	10,87379	23
38	9,06264	9,99708	9,06556	10,93444	22	38	9,12331	9,99613	9,12717	10,87283	22
39	9,06372	9,99707	9,06665	10,93334	21	39	9,12425	9,99612	9,12813	10,87187	21
40	9,06481	9,99705	9,06775	10,93225	20	40	9,12519	9,99610	9,12909	10,87091	20
41	9,06588	9,99704	9,06885	10,93115	19	41	9,12612	9,99608	9,13004	10,86996	19
42	9,06696	9,99702	9,06994	10,93006	18	42	9,12706	9,99607	9,13099	10,86901	18
43	9,06804	9,99701	9,07103	10,92897	17	43	9,12799	9,99605	9,13194	10,86806	17
44	9,06911	9,99699	9,07211	10,92789	16	44	9,12892	9,99603	9,13289	10,86711	16
45	9,07018	9,99698	9,07320	10,92680	15	45	9,12985	9,99601	9,13384	10,86616	15
46	9,07124	9,99696	9,07428	10,92572	14	46	9,13078	9,99600	9,13478	10,86522	14
47	9,07230	9,99695	9,07536	10,92464	13	47	9,13171	9,99598	9,13573	10,86427	13
48	9,07337	9,99693	9,07643	10,92357	12	48	9,13263	9,99596	9,13667	10,86333	12
49	9,07442	9,99692	9,07750	10,92249	11	49	9,13355	9,99595	9,13760	10,86239	11
50	9,07548	9,99690	9,07858	10,92142	10	50	9,13447	9,99593	9,13854	10,86146	10
51	9,07653	9,99689	9,07964	10,92036	9	51	9,13539	9,99591	9,13948	10,86052	9
52	9,07758	9,99687	9,08071	10,91929	8	52	9,13630	9,99589	9,14041	10,85959	8
53	9,07863	9,99686	9,08177	10,91823	7	53	9,13722	9,99588	9,14134	10,85866	7
54	9,07968	9,99684	9,08283	10,91717	6	54	9,13813	9,99586	9,14227	10,85773	6
55	9,08072	9,99683	9,08389	10,91611	5	55	9,13904	9,99584	9,14320	10,85680	5
56	9,08176	9,99681	9,08495	10,91505	4	56	9,13994	9,99582	9,14412	10,85588	4
57	9,08280	9,99680	9,08601	10,91400	3	57	9,14085	9,99581	9,14504	10,85496	3
58	9,08383	9,99678	9,08705	10,91295	2	58	9,14175	9,99579	9,14596	10,85403	2
59	9,08486	9,99677	9,08810	10,91190	1	59	9,14265	9,99577	9,14688	10,85311	1
60	9,08589	9,99675	9,08914	10,91086	0	60	9,14355	9,99575	9,14780	10,85220	0
	Co-sine	Sine	Co-tan.	Tangent	M		Co-sine	Sine	Co-tan.	Tangent	M

83 Degrees

82 Degrees

8 Degrees

M	Sine	Co-sine	Tangent	Co-tang.	M
0	9,14355	9,99575	9,14780	10,85220	60
1	9,14445	9,99573	9,14872	10,85128	59
2	9,14535	9,99572	9,14963	10,85037	58
3	9,14624	9,99570	9,15054	10,84946	57
4	9,14714	9,99568	9,15145	10,84855	56
5	9,14803	9,99566	9,15236	10,84764	55
6	9,14891	9,99565	9,15327	10,84673	54
7	9,14980	9,99563	9,15417	10,84583	53
8	9,15069	9,99561	9,15508	10,84492	52
9	9,15157	9,99559	9,15598	10,84402	51
10	9,15245	9,99557	9,15688	10,84312	50
11	9,15333	9,99555	9,15777	10,84222	49
12	9,15421	9,99554	9,15867	10,84133	48
13	9,15508	9,99552	9,15956	10,84043	47
14	9,15596	9,99550	9,16046	10,83954	46
15	9,15683	9,99548	9,16135	10,83865	45
16	9,15770	9,99546	9,16224	10,83776	44
17	9,15857	9,99544	9,16312	10,83688	43
18	9,15943	9,99543	9,16401	10,83599	42
19	9,16030	9,99541	9,16489	10,83511	41
20	9,16116	9,99539	9,16577	10,83423	40
21	9,16202	9,99537	9,16665	10,83335	39
22	9,16288	9,99535	9,16753	10,83247	38
23	9,16374	9,99533	9,16841	10,83159	37
24	9,16460	9,99532	9,16928	10,83072	36
25	9,16545	9,99530	9,17016	10,82984	35
26	9,16631	9,99528	9,17103	10,82897	34
27	9,16716	9,99526	9,17189	10,82810	33
28	9,16801	9,99524	9,17277	10,82723	32
29	9,16886	9,99522	9,17363	10,82637	31
30	9,16970	9,99520	9,17450	10,82550	30
31	9,17055	9,99518	9,17536	10,82464	29
32	9,17139	9,99516	9,17622	10,82378	28
33	9,17223	9,99515	9,17708	10,82292	27
34	9,17307	9,99513	9,17794	10,82206	26
35	9,17391	9,99511	9,17880	10,82120	25
36	9,17474	9,99509	9,17965	10,82034	24
37	9,17558	9,99507	9,18051	10,81949	23
38	9,17641	9,99505	9,18136	10,81864	22
39	9,17724	9,99503	9,18221	10,81779	21
40	9,17807	9,99501	9,18306	10,81694	20
41	9,17890	9,99499	9,18391	10,81609	19
42	9,17973	9,99497	9,18475	10,81525	18
43	9,18055	9,99495	9,18560	10,81440	17
44	9,18137	9,99493	9,18644	10,81356	16
45	9,18220	9,99492	9,18728	10,81272	15
46	9,18302	9,99490	9,18812	10,81188	14
47	9,18383	9,99488	9,18896	10,81104	13
48	9,18465	9,99486	9,18979	10,81021	12
49	9,18547	9,99484	9,19063	10,80937	11
50	9,18628	9,99482	9,19146	10,80854	10
51	9,18709	9,99480	9,19229	10,80771	9
52	9,18790	9,99478	9,19312	10,80688	8
53	9,18871	9,99476	9,19395	10,80605	7
54	9,18952	9,99474	9,19478	10,80522	6
55	9,19032	9,99473	9,19561	10,80439	5
56	9,19113	9,99470	9,19643	10,80357	4
57	9,19193	9,99468	9,19725	10,80275	3
58	9,19273	9,99466	9,19807	10,80193	2
59	9,19353	9,99464	9,19889	10,80111	1
60	9,19433	9,99462	9,19971	10,80029	0
	Co-sine	Sine	Co-tan.	Tangent	M

81 Degrees

9 Degrees

M	Sine	Co-sine	Tangent	Co-tang.	M
0	9,19433	9,99462	9,19971	10,80029	60
1	9,19513	9,99460	9,20053	10,79947	59
2	9,19592	9,99458	9,20134	10,79865	58
3	9,19672	9,99456	9,20216	10,79784	57
4	9,19751	9,99454	9,20297	10,79703	56
5	9,19830	9,99452	9,20378	10,79621	55
6	9,19909	9,99450	9,20459	10,79541	54
7	9,19988	9,99448	9,20540	10,79460	53
8	9,20067	9,99446	9,20621	10,79379	52
9	9,20145	9,99444	9,20701	10,79298	51
10	9,20223	9,99442	9,20782	10,79218	50
11	9,20302	9,99440	9,20862	10,79138	49
12	9,20380	9,99438	9,20942	10,79058	48
13	9,20458	9,99436	9,21022	10,78978	47
14	9,20535	9,99434	9,21102	10,78898	46
15	9,20613	9,99432	9,21181	10,78818	45
16	9,20691	9,99429	9,21261	10,78739	44
17	9,20768	9,99427	9,21340	10,78659	43
18	9,20845	9,99425	9,21420	10,78580	42
19	9,20922	9,99423	9,21499	10,78501	41
20	9,20999	9,99421	9,21578	10,78422	40
21	9,21076	9,99419	9,21657	10,78343	39
22	9,21153	9,99417	9,21736	10,78264	38
23	9,21229	9,99415	9,21814	10,78186	37
24	9,21305	9,99413	9,21893	10,78107	36
25	9,21382	9,99410	9,21971	10,78029	35
26	9,21458	9,99409	9,22049	10,77951	34
27	9,21534	9,99407	9,22127	10,77873	33
28	9,21610	9,99404	9,22205	10,77795	32
29	9,21685	9,99402	9,22283	10,77717	31
30	9,21761	9,99400	9,22361	10,77639	30
31	9,21836	9,99398	9,22438	10,77562	29
32	9,21912	9,99396	9,22516	10,77484	28
33	9,21987	9,99394	9,22593	10,77407	27
34	9,22062	9,99392	9,22670	10,77330	26
35	9,22137	9,99390	9,22747	10,77253	25
36	9,22211	9,99387	9,22824	10,77176	24
37	9,22286	9,99385	9,22901	10,77099	23
38	9,22361	9,99383	9,22977	10,77022	22
39	9,22435	9,99381	9,23054	10,76946	21
40	9,22509	9,99379	9,23131	10,76870	20
41	9,22583	9,99377	9,23206	10,76793	19
42	9,22657	9,99375	9,23283	10,76717	18
43	9,22731	9,99372	9,23359	10,76641	17
44	9,22805	9,99370	9,23434	10,76565	16
45	9,22878	9,99368	9,23510	10,76490	15
46	9,22952	9,99366	9,23586	10,76414	14
47	9,23025	9,99364	9,23661	10,76339	13
48	9,23098	9,99362	9,23737	10,76263	12
49	9,23171	9,99359	9,23812	10,76188	11
50	9,23244	9,99357	9,23887	10,76113	10
51	9,23317	9,99355	9,23962	10,76038	9
52	9,23390	9,99353	9,24037	10,75963	8
53	9,23462	9,99351	9,24112	10,75888	7
54	9,23535	9,99348	9,24186	10,75813	6
55	9,23607	9,99346	9,24261	10,75739	5
56	9,23679	9,99344	9,24335	10,75665	4
57	9,23751	9,99342	9,24410	10,75590	3
58	9,23823	9,99340	9,24484	10,75516	2
59	9,23895	9,99337	9,24558	10,75442	1
60	9,23967	9,99335	9,24632	10,75368	0
	Co-sine	Sine	Co-tan.	Tangent	M

80 Degrees

TABLE of SINES and TANGENTS, 1152.

585

10 Degree

11 Degrees

M	Sine	Co-sine	Tangent	Co-tang.	M	Sine	Co-sine	Tangent	Co-tang.
0	9,23967	9,99335	9,24632	10,75368	60	9,28060	9,99195	9,28865	10,71135
1	9,24039	9,99333	9,24706	10,75294	59	9,28125	9,99192	9,28933	10,71067
2	9,24110	9,99331	9,24779	10,75221	58	9,28190	9,99190	9,29000	10,71000
3	9,24181	9,99328	9,24853	10,75147	57	9,28254	9,99187	9,29067	10,70933
4	9,24253	9,99326	9,24926	10,75074	56	9,28319	9,99185	9,29134	10,70866
5	9,24324	9,99324	9,25000	10,75000	55	9,28384	9,99182	9,29201	10,70799
6	9,24395	9,99322	9,25073	10,74927	54	9,28448	9,99180	9,29268	10,70732
7	9,24466	9,99319	9,25146	10,74854	53	9,28512	9,99177	9,29335	10,70665
8	9,24536	9,99317	9,25219	10,74781	52	9,28577	9,99175	9,29402	10,70598
9	9,24607	9,99315	9,25292	10,74708	51	9,28641	9,99172	9,29468	10,70532
10	9,24677	9,99313	9,25365	10,74635	50	9,28705	9,99170	9,29535	10,70465
11	9,24748	9,99310	9,25437	10,74563	49	9,28769	9,99167	9,29601	10,70399
12	9,24818	9,99308	9,25510	10,74490	48	9,28833	9,99165	9,29668	10,70332
13	9,24888	9,99306	9,25582	10,74418	47	9,28896	9,99162	9,29734	10,70266
14	9,24958	9,99304	9,25655	10,74345	46	9,28960	9,99160	9,29800	10,70200
15	9,25028	9,99301	9,25727	10,74273	45	9,29024	9,99157	9,29866	10,70134
16	9,25098	9,99299	9,25799	10,74201	44	9,29087	9,99155	9,29932	10,70068
17	9,25168	9,99297	9,25871	10,74129	43	9,29150	9,99152	9,29998	10,70002
18	9,25237	9,99294	9,25943	10,74057	42	9,29214	9,99150	9,30064	10,69936
19	9,25307	9,99292	9,26015	10,73985	41	9,29277	9,99147	9,30129	10,69870
20	9,25376	9,99290	9,26086	10,73914	40	9,29340	9,99145	9,30195	10,69805
21	9,25445	9,99287	9,26158	10,73842	39	9,29403	9,99142	9,30261	10,69739
22	9,25514	9,99285	9,26229	10,73771	38	9,29466	9,99140	9,30326	10,69674
23	9,25583	9,99283	9,26300	10,73699	37	9,29529	9,99137	9,30391	10,69609
24	9,25652	9,99281	9,26372	10,73628	36	9,29591	9,99135	9,30457	10,69543
25	9,25721	9,99278	9,26443	10,73557	35	9,29654	9,99132	9,30522	10,69478
26	9,25790	9,99276	9,26514	10,73486	34	9,29716	9,99129	9,30587	10,69413
27	9,25858	9,99274	9,26585	10,73415	33	9,29779	9,99127	9,30652	10,69348
28	9,25927	9,99271	9,26655	10,73344	32	9,29841	9,99124	9,30717	10,69283
29	9,25995	9,99269	9,26726	10,73274	31	9,29903	9,99122	9,30781	10,69218
30	9,26063	9,99267	9,26797	10,73203	30	9,29965	9,99119	9,30846	10,69154
31	9,26131	9,99264	9,26867	10,73133	29	9,30028	9,99117	9,30911	10,69089
32	9,26199	9,99262	9,26937	10,73063	28	9,30089	9,99114	9,30975	10,69025
33	9,26267	9,99260	9,27008	10,72992	27	9,30151	9,99111	9,31040	10,68960
34	9,26335	9,99257	9,27078	10,72922	26	9,30213	9,99109	9,31104	10,68896
35	9,26403	9,99255	9,27148	10,72852	25	9,30275	9,99106	9,31168	10,68831
36	9,26470	9,99252	9,27218	10,72782	24	9,30336	9,99104	9,31233	10,68767
37	9,26538	9,99250	9,27288	10,72712	23	9,30398	9,99101	9,31297	10,68703
38	9,26605	9,99248	9,27357	10,72643	22	9,30459	9,99099	9,31361	10,68639
39	9,26672	9,99245	9,27427	10,72573	21	9,30521	9,99096	9,31425	10,68575
40	9,26739	9,99243	9,27496	10,72504	20	9,30582	9,99093	9,31488	10,68511
41	9,26806	9,99241	9,27566	10,72434	19	9,30643	9,99091	9,31552	10,68448
42	9,26873	9,99238	9,27635	10,72365	18	9,30704	9,99088	9,31616	10,68384
43	9,26940	9,99236	9,27704	10,72296	17	9,30765	9,99085	9,31679	10,68320
44	9,27007	9,99233	9,27773	10,72227	16	9,30826	9,99083	9,31743	10,68257
45	9,27073	9,99231	9,27842	10,72158	15	9,30887	9,99080	9,31806	10,68194
46	9,27140	9,99229	9,27911	10,72089	14	9,30947	9,99078	9,31870	10,68130
47	9,27206	9,99226	9,27980	10,72020	13	9,31008	9,99075	9,31933	10,68067
48	9,27273	9,99224	9,28049	10,71951	12	9,31068	9,99072	9,31996	10,68004
49	9,27339	9,99221	9,28117	10,71883	11	9,31129	9,99070	9,32059	10,67941
50	9,27405	9,99219	9,28186	10,71814	10	9,31189	9,99067	9,32122	10,67878
51	9,27471	9,99217	9,28254	10,71746	9	9,31249	9,99064	9,32185	10,67815
52	9,27537	9,99214	9,28322	10,71677	8	9,31310	9,99062	9,32248	10,67752
53	9,27602	9,99212	9,28391	10,71609	7	9,31370	9,99059	9,32311	10,67689
54	9,27668	9,99209	9,28459	10,71541	6	9,31430	9,99056	9,32373	10,67627
55	9,27734	9,99207	9,28527	10,71473	5	9,31490	9,99054	9,32436	10,67564
56	9,27799	9,99204	9,28595	10,71405	4	9,31549	9,99051	9,32498	10,67502
57	9,27864	9,99202	9,28662	10,71337	3	9,31609	9,99048	9,32561	10,67439
58	9,27930	9,99200	9,28730	10,71270	2	9,31669	9,99045	9,32623	10,67377
59	9,27995	9,99197	9,28798	10,71202	1	9,31728	9,99043	9,32685	10,67315
60	9,28060	9,99195	9,28865	10,71135	0	9,31788	9,99040	9,32747	10,67252
Co-sine	Sine	Co-tan.	Tangent	M	Co-sine	Sine	Co-tan.	Tangent	M

79 Degrees

78 Degrees

F f f f

12 Degrees					13 Degrees				
M	Sine	Co-sine	Tangent	Co-tang.	M	Sine	Co-sine	Tangent	Co-tang.
0	9,31788	9,99040	9,32747	10,67252	60	9,35209	9,98872	9,36336	10,63664
1	9,31847	9,99038	9,32809	10,67190	59	9,35263	9,98869	9,36394	10,63606
2	9,31907	9,99035	9,32871	10,67128	58	9,35318	9,98866	9,36451	10,63548
3	9,31966	9,99032	9,32933	10,67066	57	9,35373	9,98864	9,36509	10,63491
4	9,32025	9,99030	9,32995	10,67005	56	9,35427	9,98861	9,36566	10,63434
5	9,32084	9,99027	9,33057	10,66994	55	9,35481	9,98858	9,36624	10,63376
6	9,32143	9,99024	9,33119	10,66881	54	9,35536	9,98855	9,36681	10,63319
7	9,32202	9,99021	9,33180	10,66820	53	9,35590	9,98852	9,36738	10,63262
8	9,32261	9,99019	9,33242	10,66758	52	9,35644	9,98849	9,36795	10,63205
9	9,32319	9,99016	9,33303	10,66697	51	9,35698	9,98846	9,36852	10,63148
10	9,32378	9,99013	9,33365	10,66635	50	9,35752	9,98843	9,36909	10,63091
11	9,32437	9,99010	9,33426	10,66574	49	9,35806	9,98840	9,36966	10,63034
12	9,32495	9,99008	9,33487	10,66513	48	9,35860	9,98837	9,37023	10,62977
13	9,32553	9,99005	9,33548	10,66452	47	9,35914	9,98834	9,37080	10,62920
14	9,32612	9,99002	9,33609	10,66391	46	9,35968	9,98831	9,37137	10,62863
15	9,32670	9,99000	9,33670	10,66330	45	9,36021	9,98828	9,37193	10,62807
16	9,32728	9,98997	9,33731	10,66269	44	9,36075	9,98825	9,37250	10,62750
17	9,32786	9,98994	9,33792	10,66208	43	9,36129	9,98822	9,37306	10,62693
18	9,32844	9,98991	9,33853	10,66147	42	9,36182	9,98819	9,37363	10,62637
19	9,32902	9,98989	9,33913	10,66087	41	9,36236	9,98816	9,37419	10,62581
20	9,32960	9,98986	9,33974	10,66026	40	9,36289	9,98813	9,37476	10,62524
21	9,33018	9,98983	9,34034	10,65966	39	9,36342	9,98810	9,37532	10,62468
22	9,33075	9,98980	9,34095	10,65905	38	9,36395	9,98807	9,37588	10,62412
23	9,33133	9,98978	9,34155	10,65845	37	9,36448	9,98804	9,37644	10,62356
24	9,33190	9,98975	9,34215	10,65784	36	9,36502	9,98801	9,37700	10,62300
25	9,33248	9,98972	9,34276	10,65724	35	9,36555	9,98798	9,37756	10,62244
26	9,33305	9,98969	9,34336	10,65664	34	9,36607	9,98795	9,37812	10,62188
27	9,33362	9,98966	9,34396	10,65604	33	9,36660	9,98792	9,37868	10,62132
28	9,33419	9,98964	9,34456	10,65544	32	9,36713	9,98789	9,37924	10,62076
29	9,33477	9,98961	9,34516	10,65484	31	9,36766	9,98786	9,37980	10,62020
30	9,33534	9,98958	9,34575	10,65424	30	9,36818	9,98783	9,38035	10,61965
31	9,33591	9,98955	9,34635	10,65365	29	9,36871	9,98780	9,38091	10,61909
32	9,33647	9,98952	9,34695	10,65305	28	9,36924	9,98777	9,38146	10,61853
33	9,33704	9,98950	9,34754	10,65245	27	9,36976	9,98774	9,38202	10,61798
34	9,33761	9,98947	9,34814	10,65186	26	9,37038	9,98771	9,38257	10,61742
35	9,33818	9,98944	9,34873	10,65126	25	9,37081	9,98768	9,38312	10,61687
36	9,33874	9,98941	9,34933	10,65067	24	9,37133	9,98765	9,38368	10,61632
37	9,33931	9,98938	9,34992	10,65008	23	9,37185	9,98762	9,38423	10,61577
38	9,33987	9,98936	9,35051	10,64949	22	9,37237	9,98759	9,38479	10,61521
39	9,34043	9,98933	9,35111	10,64889	21	9,37289	9,98756	9,38534	10,61466
40	9,34100	9,98930	9,35170	10,64830	20	9,37341	9,98753	9,38589	10,61411
41	9,34156	9,98927	9,35229	10,64771	19	9,37393	9,98749	9,38644	10,61356
42	9,34212	9,98924	9,35288	10,64712	18	9,37445	9,98746	9,38699	10,61301
43	9,34268	9,98921	9,35346	10,64653	17	9,37497	9,98743	9,38754	10,61246
44	9,34324	9,98919	9,35405	10,64595	16	9,37549	9,98740	9,38808	10,61192
45	9,34380	9,98916	9,35464	10,64536	15	9,37600	9,98737	9,38863	10,61137
46	9,34435	9,98913	9,35523	10,64478	14	9,37652	9,98734	9,38918	10,61082
47	9,34491	9,98910	9,35581	10,64419	13	9,37704	9,98731	9,38972	10,61028
48	9,34547	9,98907	9,35640	10,64360	12	9,37755	9,98728	9,39028	10,60973
49	9,34602	9,98904	9,35698	10,64302	11	9,37806	9,98725	9,39081	10,60918
50	9,34658	9,98901	9,35757	10,64243	10	9,37858	9,98722	9,39136	10,60864
51	9,34713	9,98898	9,35815	10,64185	9	9,37908	9,98719	9,39190	10,60810
52	9,34769	9,98896	9,35873	10,64127	8	9,37960	9,98715	9,39245	10,60755
53	9,34824	9,98893	9,35931	10,64069	7	9,38011	9,98712	9,39299	10,60701
54	9,34879	9,98890	9,35989	10,64011	6	9,38062	9,98709	9,39353	10,60645
55	9,34934	9,98887	9,36047	10,63953	5	9,38113	9,98706	9,39407	10,60593
56	9,34989	9,98884	9,36105	10,63895	4	9,38164	9,98703	9,39461	10,60539
57	9,35044	9,98881	9,36163	10,63837	3	9,38215	9,98700	9,39515	10,60484
58	9,35099	9,98878	9,36221	10,63779	2	9,38266	9,98697	9,39569	10,60431
59	9,35154	9,98875	9,36279	10,63721	1	9,38317	9,98694	9,39623	10,60377
60	9,35209	9,98872	9,36336	10,63664	0	9,38367	9,98690	9,39677	10,60323
Co-sine	Sine	Co-tan.	Tangent	M	Co-sine	Sine	Co-tan.	Tangent	M
77 Degrees					76 Degrees				

14 Degrees

M	Sine	Co-fine	Tangent	Co-tang.	M
0	9,38367	9,98690	9,39677	10,60323	60
1	9,38418	9,98687	9,39731	10,60269	59
2	9,38469	9,98684	9,39785	10,60215	58
3	9,38519	9,98681	9,39838	10,60162	57
4	9,38570	9,98678	9,39892	10,60108	56
5	9,38620	9,98675	9,39945	10,60054	55
6	9,38670	9,98671	9,39999	10,60001	54
7	9,38721	9,98668	9,40052	10,59948	53
8	9,38771	9,98665	9,40106	10,59894	52
9	9,38821	9,98662	9,40159	10,59840	51
10	9,38871	9,98659	9,40212	10,59788	50
11	9,38921	9,98655	9,40266	10,59734	49
12	9,38971	9,98652	9,40319	10,59681	48
13	9,39021	9,98649	9,40372	10,59628	47
14	9,39071	9,98646	9,40425	10,59575	46
15	9,39121	9,98643	9,40478	10,59522	45
16	9,39170	9,98639	9,40531	10,59469	44
17	9,39220	9,98636	9,40584	10,59416	43
18	9,39279	9,98633	9,40636	10,59363	42
19	9,39319	9,98630	9,40689	10,59311	41
20	9,39368	9,98627	9,40742	10,59258	40
21	9,39418	9,98623	9,40794	10,59205	39
22	9,39467	9,98620	9,40847	10,59153	38
23	9,39517	9,98617	9,40900	10,59100	37
24	9,39566	9,98614	9,40952	10,59048	36
25	9,39615	9,98610	9,41004	10,58995	35
26	9,39664	9,98607	9,41057	10,58943	34
27	9,39713	9,98604	9,41109	10,58891	33
28	9,39762	9,98601	9,41161	10,58838	32
29	9,39811	9,98597	9,41214	10,58786	31
30	9,39860	9,98594	9,41266	10,58734	30
31	9,39909	9,98591	9,41318	10,58682	29
32	9,39957	9,98588	9,41370	10,58630	28
33	9,40006	9,98584	9,41422	10,58578	27
34	9,40055	9,98581	9,41474	10,58526	26
35	9,40103	9,98578	9,41526	10,58474	25
36	9,40152	9,98574	9,41577	10,58422	24
37	9,40200	9,98571	9,41629	10,58371	23
38	9,40249	9,98568	9,41681	10,58319	22
39	9,40297	9,98565	9,41733	10,58267	21
40	9,40345	9,98561	9,41784	10,58216	20
41	9,40394	9,98558	9,41836	10,58164	19
42	9,40442	9,98555	9,41887	10,58113	18
43	9,40490	9,98551	9,41939	10,58061	17
44	9,40538	9,98548	9,41990	10,58010	16
45	9,40586	9,98545	9,42041	10,57958	15
46	9,40634	9,98541	9,42093	10,57907	14
47	9,40682	9,98538	9,42144	10,57856	13
48	9,40730	9,98535	9,42195	10,57805	12
49	9,40778	9,98531	9,42246	10,57754	11
50	9,40825	9,98528	9,42297	10,57703	10
51	9,40873	9,98525	9,42348	10,57652	9
52	9,40921	9,98521	9,42399	10,57601	8
53	9,40968	9,98518	9,42450	10,57550	7
54	9,41016	9,98515	9,42501	10,57499	6
55	9,41063	9,98511	9,42552	10,57448	5
56	9,41111	9,98508	9,42603	10,57397	4
57	9,41158	9,98504	9,42653	10,57347	3
58	9,41205	9,98501	9,42704	10,57296	2
59	9,41252	9,98498	9,42755	10,57245	1
60	9,41300	9,98494	9,42805	10,57195	0
	Co-fine	Sine	Co-tan.	Tangent	M

75 Degrees

15 Degrees

M	Sine	Co-fine	Tangent	Co-tang.	M
0	9,41300	9,98494	9,42805	10,57195	60
1	9,41347	9,98491	9,42856	10,57144	59
2	9,41394	9,98488	9,42906	10,57094	58
3	9,41441	9,98484	9,42957	10,57043	57
4	9,41488	9,98481	9,43007	10,56993	56
5	9,41535	9,98477	9,43057	10,56943	55
6	9,41581	9,98474	9,43107	10,56892	54
7	9,41628	9,98471	9,43158	10,56842	53
8	9,41675	9,98467	9,43208	10,56792	52
9	9,41722	9,98464	9,43258	10,56742	51
10	9,41768	9,98460	9,43308	10,56692	50
11	9,41815	9,98457	9,43358	10,56642	49
12	9,41861	9,98453	9,43408	10,56592	48
13	9,41908	9,98450	9,43458	10,56542	47
14	9,41954	9,98447	9,43508	10,56492	46
15	9,42001	9,98443	9,43558	10,56442	45
16	9,42047	9,98440	9,43607	10,56393	44
17	9,42093	9,98436	9,43657	10,56343	43
18	9,42139	9,98433	9,43707	10,56293	42
19	9,42186	9,98429	9,43756	10,56244	41
20	9,42232	9,98426	9,43806	10,56194	40
21	9,42278	9,98422	9,43855	10,56145	39
22	9,42324	9,98419	9,43905	10,56095	38
23	9,42370	9,98415	9,43954	10,56046	37
24	9,42416	9,98412	9,44004	10,55996	36
25	9,42461	9,98408	9,44053	10,55947	35
26	9,42507	9,98405	9,44102	10,55898	34
27	9,42553	9,98401	9,44151	10,55848	33
28	9,42599	9,98398	9,44201	10,55799	32
29	9,42644	9,98394	9,44250	10,55750	31
30	9,42690	9,98391	9,44299	10,55701	30
31	9,42735	9,98387	9,44348	10,55652	29
32	9,42781	9,98384	9,44397	10,55603	28
33	9,42826	9,98380	9,44446	10,55554	27
34	9,42872	9,98377	9,44495	10,55505	26
35	9,42917	9,98373	9,44543	10,55456	25
36	9,42962	9,98370	9,44592	10,55408	24
37	9,43007	9,98366	9,44641	10,55359	23
38	9,43053	9,98363	9,44690	10,55310	22
39	9,43098	9,98359	9,44738	10,55262	21
40	9,43143	9,98356	9,44787	10,55213	20
41	9,43188	9,98352	9,44836	10,55164	19
42	9,43233	9,98349	9,44884	10,55116	18
43	9,43278	9,98345	9,44933	10,55067	17
44	9,43323	9,98342	9,44981	10,55019	16
45	9,43367	9,98338	9,45029	10,54971	15
46	9,43412	9,98334	9,45078	10,54922	14
47	9,43457	9,98331	9,45126	10,54874	13
48	9,43502	9,98327	9,45174	10,54826	12
49	9,43546	9,98324	9,45222	10,54777	11
50	9,43591	9,98320	9,45271	10,54729	10
51	9,43635	9,98317	9,45319	10,54681	9
52	9,43680	9,98313	9,45367	10,54633	8
53	9,43724	9,98309	9,45415	10,54585	7
54	9,43769	9,98306	9,45463	10,54537	6
55	9,43813	9,98302	9,45511	10,54489	5
56	9,43857	9,98299	9,45559	10,54441	4
57	9,43901	9,98295	9,45606	10,54394	3
58	9,43946	9,98291	9,45654	10,54346	2
59	9,43990	9,98288	9,45702	10,54298	1
60	9,44034	9,98284	9,45750	10,54250	0
	Co-fine	Sine	Co-tan.	Tangent	M

74 Degrees

16 Degrees

M	Sine	Co-sine	Tangent	Co-tang	
0	9,44034	9,98284	9,45750	10,54250	60
1	9,44078	9,98280	9,45797	10,54203	59
2	9,44122	9,98277	9,45845	10,54155	58
3	9,44166	9,98273	9,45892	10,54107	57
4	9,44210	9,98270	9,45940	10,54060	56
5	9,44253	9,98266	9,45987	10,54012	55
6	9,44297	9,98262	9,46035	10,53965	54
7	9,44341	9,98259	9,46082	10,53918	53
8	9,44385	9,98255	9,46130	10,53870	52
9	9,44428	9,98251	9,46177	10,53823	51
10	9,44472	9,98248	9,46224	10,53776	50
11	9,44515	9,98244	9,46271	10,53728	49
12	9,44559	9,98240	9,46319	10,53681	48
13	9,44602	9,98237	9,46366	10,53634	47
14	9,44646	9,98233	9,46413	10,53587	46
15	9,44689	9,98229	9,46460	10,53540	45
16	9,44733	9,98226	9,46507	10,53493	44
17	9,44776	9,98222	9,46554	10,53446	43
18	9,44819	9,98218	9,46601	10,53399	42
19	9,44862	9,98215	9,46648	10,53352	41
20	9,44905	9,98211	9,46694	10,53305	40
21	9,44948	9,98207	9,46741	10,53259	39
22	9,44991	9,98203	9,46788	10,53212	38
23	9,45034	9,98200	9,46835	10,53165	37
24	9,45077	9,98196	9,46881	10,53119	36
25	9,45120	9,98192	9,46928	10,53072	35
26	9,45163	9,98189	9,46975	10,53025	34
27	9,45206	9,98185	9,47021	10,52979	33
28	9,45249	9,98181	9,47068	10,52932	32
29	9,45291	9,98177	9,47114	10,52886	31
30	9,45334	9,98174	9,47160	10,52839	30
31	9,45377	9,98170	9,47207	10,52793	29
32	9,45419	9,98166	9,47253	10,52747	28
33	9,45462	9,98162	9,47299	10,52700	27
34	9,45504	9,98159	9,47340	10,52654	26
35	9,45547	9,98155	9,47392	10,52608	25
36	9,45589	9,98151	9,47438	10,52562	24
37	9,45632	9,98148	9,47484	10,52516	23
38	9,45674	9,98144	9,47530	10,52470	22
39	9,45716	9,98140	9,47576	10,52424	21
40	9,45758	9,98136	9,47622	10,52378	20
41	9,45801	9,98132	9,47668	10,52332	19
42	9,45843	9,98128	9,47714	10,52286	18
43	9,45885	9,98125	9,47760	10,52240	17
44	9,45927	9,98121	9,47806	10,52194	16
45	9,45969	9,98117	9,47852	10,52148	15
46	9,46011	9,98113	9,47897	10,52102	14
47	9,46053	9,98109	9,47943	10,52057	13
48	9,46095	9,98106	9,47989	10,52011	12
49	9,46136	9,98102	9,48034	10,51965	11
50	9,46178	9,98098	9,48080	10,51920	10
51	9,46220	9,98094	9,48126	10,51874	9
52	9,46266	9,98090	9,48171	10,51829	8
53	9,46303	9,98087	9,48217	10,51783	7
54	9,46345	9,98083	9,48262	10,51738	6
55	9,46386	9,98079	9,48307	10,51692	5
56	9,46428	9,98075	9,48353	10,51647	4
57	9,46469	9,98071	9,48398	10,51602	3
58	9,46511	9,98067	9,48443	10,51556	2
59	9,46552	9,98063	9,48489	10,51511	1
60	9,46593	9,98060	9,48534	10,51466	0
	Co-sine	Sine	Co-tan.	Tangent	M

73 Degrees

17 Degrees

M	Sine	Co-sine	Tangent	Co-tang.	
0	9,46593	9,98060	9,48534	10,51466	60
1	9,46635	9,98056	9,48579	10,51421	59
2	9,46676	9,98052	9,48624	10,51376	58
3	9,46717	9,98048	9,48669	10,51331	57
4	9,46758	9,98044	9,48714	10,51285	56
5	9,46800	9,98040	9,48759	10,51241	55
6	9,46841	9,98036	9,48804	10,51196	54
7	9,46882	9,98032	9,48849	10,51151	53
8	9,46923	9,98029	9,48894	10,51106	52
9	9,46964	9,98025	9,48939	10,51061	51
10	9,47004	9,98021	9,48984	10,51016	50
11	9,47045	9,98017	9,49029	10,50971	49
12	9,47086	9,98013	9,49073	10,50927	48
13	9,47127	9,98009	9,49118	10,50882	47
14	9,47168	9,98005	9,49163	10,50837	46
15	9,47209	9,98001	9,49207	10,50793	45
16	9,47249	9,97997	9,49252	10,50748	44
17	9,47290	9,97993	9,49296	10,50703	43
18	9,47330	9,97989	9,49341	10,50659	42
19	9,47371	9,97985	9,49385	10,50614	41
20	9,47411	9,97982	9,49430	10,50570	40
21	9,47452	9,97978	9,49474	10,50526	39
22	9,47492	9,97974	9,49519	10,50481	38
23	9,47533	9,97970	9,49563	10,50437	37
24	9,47573	9,97966	9,49607	10,50393	36
25	9,47613	9,97962	9,49651	10,50348	35
26	9,47654	9,97958	9,49696	10,50304	34
27	9,47694	9,97954	9,49740	10,50260	33
28	9,47734	9,97950	9,49784	10,50216	32
29	9,47774	9,97946	9,49828	10,50172	31
30	9,47814	9,97942	9,49872	10,50128	30
31	9,47854	9,97938	9,49916	10,50084	29
32	9,47894	9,97934	9,49960	10,50040	28
33	9,47934	9,97930	9,50004	10,49996	27
34	9,47974	9,97926	9,50048	10,49952	26
35	9,48014	9,97922	9,50092	10,49908	25
36	9,48054	9,97918	9,50136	10,49864	24
37	9,48094	9,97914	9,50180	10,49820	23
38	9,48133	9,97910	9,50223	10,49776	22
39	9,48173	9,97906	9,50267	10,49733	21
40	9,48213	9,97902	9,50311	10,49689	20
41	9,48252	9,97898	9,50355	10,49645	19
42	9,48292	9,97894	9,50398	10,49602	18
43	9,48332	9,97890	9,50442	10,49558	17
44	9,48371	9,97886	9,50485	10,49515	16
45	9,48411	9,97882	9,50529	10,49471	15
46	9,48450	9,97878	9,50572	10,49428	14
47	9,48489	9,97874	9,50616	10,49384	13
48	9,48529	9,97870	9,50659	10,49341	12
49	9,48568	9,97865	9,50703	10,49297	11
50	9,48607	9,97861	9,50746	10,49254	10
51	9,48647	9,97857	9,50789	10,49211	9
52	9,48686	9,97853	9,50833	10,49167	8
53	9,48725	9,97849	9,50876	10,49124	7
54	9,48764	9,97845	9,50919	10,49081	6
55	9,48803	9,97841	9,50962	10,49038	5
56	9,48842	9,97837	9,51005	10,48995	4
57	9,48881	9,97833	9,51048	10,48951	3
58	9,48920	9,97829	9,51092	10,48908	2
59	9,48959	9,97825	9,51135	10,48865	1
60	9,48998	9,97821	9,51178	10,48822	0
	Co-sine	Sine	Co-tan.	Tangent	

72 Degrees

18 Degrees

19 Degrees

M	Sine	Co-sine	Tangent	Co-tang.	M	Sine	Co-sine	Tangent	Co-tang.
0	9,48998	9,97821	9,51178	10,48822	60	9,51264	9,97567	9,53697	10,46303
1	9,49037	9,97817	9,51221	10,48779	59	9,51301	9,97563	9,53738	10,46262
2	9,49076	9,97812	9,51263	10,48736	58	9,51337	9,97558	9,53779	10,46221
3	9,49115	9,97808	9,51306	10,48694	57	9,51374	9,97554	9,53820	10,46180
4	9,49153	9,97805	9,51349	10,48651	56	9,51411	9,97550	9,53861	10,46139
5	9,49192	9,97800	9,51392	10,48608	55	9,51447	9,97545	9,53902	10,46098
6	9,49231	9,97796	9,51435	10,48565	54	9,51484	9,97541	9,53943	10,46057
7	9,49269	9,97792	9,51478	10,48522	53	9,51520	9,97536	9,53984	10,46016
8	9,49308	9,97788	9,51520	10,48480	52	9,51557	9,97532	9,54024	10,45975
9	9,49347	9,97783	9,51563	10,48437	51	9,51593	9,97528	9,54065	10,45935
10	9,49385	9,97779	9,51606	10,48394	50	9,51629	9,97523	9,54106	10,45894
11	9,49424	9,97775	9,51648	10,48352	49	9,51666	9,97519	9,54147	10,45853
12	9,49462	9,97771	9,51691	10,48309	48	9,51702	9,97514	9,54187	10,45812
13	9,49500	9,97767	9,51733	10,48266	47	9,51738	9,97510	9,54228	10,45772
14	9,49539	9,97763	9,51776	10,48224	46	9,51774	9,97506	9,54269	10,45731
15	9,49577	9,97759	9,51818	10,48181	45	9,51811	9,97501	9,54309	10,45691
16	9,49615	9,97754	9,51861	10,48139	44	9,51847	9,97497	9,54350	10,45650
17	9,49654	9,97750	9,51903	10,48097	43	9,51883	9,97492	9,54390	10,45609
18	9,49692	9,97746	9,51946	10,48054	42	9,51919	9,97488	9,54431	10,45569
19	9,49730	9,97742	9,51988	10,48012	41	9,51955	9,97484	9,54471	10,45528
20	9,49768	9,97738	9,52030	10,47969	40	9,51991	9,97479	9,54512	10,45488
21	9,49806	9,97733	9,52073	10,47927	39	9,52027	9,97475	9,54552	10,45448
22	9,49844	9,97729	9,52115	10,47885	38	9,52063	9,97470	9,54593	10,45407
23	9,49882	9,97725	9,52157	10,47843	37	9,52099	9,97466	9,54633	10,45367
24	9,49920	9,97721	9,52199	10,47800	36	9,52135	9,97461	9,54673	10,45326
25	9,49958	9,97717	9,52242	10,47758	35	9,52171	9,97457	9,54714	10,45286
26	9,49996	9,97712	9,52284	10,47716	34	9,52207	9,97452	9,54754	10,45246
27	9,50034	9,97708	9,52326	10,47674	33	9,52242	9,97448	9,54794	10,45206
28	9,50073	9,97704	9,52368	10,47632	32	9,52278	9,97444	9,54834	10,45165
29	9,50110	9,97700	9,52410	10,47590	31	9,52314	9,97439	9,54875	10,45125
30	9,50148	9,97696	9,52452	10,47548	30	9,52349	9,97435	9,54915	10,45085
31	9,50185	9,97691	9,52494	10,47506	29	9,52385	9,97430	9,54955	10,45045
32	9,50223	9,97687	9,52536	10,47464	28	9,52421	9,97426	9,54995	10,45005
33	9,50261	9,97683	9,52578	10,47422	27	9,52456	9,97421	9,55035	10,44965
34	9,50298	9,97679	9,52620	10,47380	26	9,52492	9,97417	9,55075	10,44925
35	9,50336	9,97674	9,52661	10,47338	25	9,52527	9,97412	9,55115	10,44885
36	9,50373	9,97670	9,52703	10,47297	24	9,52563	9,97408	9,55155	10,44845
37	9,50411	9,97666	9,52745	10,47255	23	9,52598	9,97403	9,55195	10,44805
38	9,50448	9,97662	9,52787	10,47213	22	9,52634	9,97399	9,55235	10,44765
39	9,50486	9,97657	9,52828	10,47171	21	9,52669	9,97394	9,55275	10,44725
40	9,50523	9,97653	9,52870	10,47130	20	9,52705	9,97390	9,55315	10,44685
41	9,50561	9,97649	9,52912	10,47088	19	9,52740	9,97385	9,55355	10,44645
42	9,50598	9,97645	9,52953	10,47046	18	9,52775	9,97381	9,55395	10,44605
43	9,50635	9,97640	9,52995	10,47005	17	9,52810	9,97376	9,55434	10,44566
44	9,50673	9,97636	9,53037	10,46963	16	9,52846	9,97372	9,55474	10,44526
45	9,50710	9,97632	9,53078	10,46922	15	9,52881	9,97367	9,55514	10,44486
46	9,50747	9,97627	9,53120	10,46880	14	9,52916	9,97362	9,55554	10,44446
47	9,50784	9,97623	9,53161	10,46839	13	9,52951	9,97358	9,55593	10,44407
48	9,50821	9,97619	9,53202	10,46797	12	9,52986	9,97353	9,55633	10,44367
49	9,50858	9,97615	9,53244	10,46756	11	9,53021	9,97349	9,55673	10,44327
50	9,50896	9,97610	9,53285	10,46715	10	9,53056	9,97344	9,55712	10,44288
51	9,50933	9,97606	9,53327	10,46673	9	9,53091	9,97340	9,55752	10,44248
52	9,50970	9,97602	9,53368	10,46632	8	9,53126	9,97335	9,55791	10,44209
53	9,51006	9,97597	9,53409	10,46591	7	9,53161	9,97331	9,55831	10,44169
54	9,51043	9,97593	9,53450	10,46550	6	9,53196	9,97326	9,55870	10,44130
55	9,51080	9,97589	9,53492	10,46508	5	9,53231	9,97321	9,55910	10,44090
56	9,51117	9,97584	9,53533	10,46467	4	9,53266	9,97317	9,55949	10,44051
57	9,51154	9,97580	9,53574	10,46426	3	9,53301	9,97312	9,55988	10,44011
58	9,51191	9,97576	9,53615	10,46385	2	9,53336	9,97308	9,56028	10,43972
59	9,51227	9,97571	9,53656	10,46344	1	9,53370	9,97303	9,56067	10,43933
60	9,51264	9,97567	9,53698	10,46303	0	9,53405	9,97399	9,56107	10,43893
	Co-sine	Sine	Co-tan.	Tangent		Co-sine	Sine	Co-tan.	Tangent

71 Degrees

70 Degrees

20 Degrees

M	Sine	Co-sine	Tangent	Co-tang.	M
0	9,53405	9,97299	9,56107	10,43893	60
1	9,53440	9,97294	9,56146	10,43854	59
2	9,53474	9,97289	9,56185	10,43815	58
3	9,53509	9,97285	9,56224	10,43776	57
4	9,53544	9,97280	9,56264	10,43736	56
5	9,53578	9,97275	9,56303	10,43697	55
6	9,53613	9,97271	9,56342	10,43658	54
7	9,53647	9,97266	9,56381	10,43619	53
8	9,53682	9,97262	9,56420	10,43580	52
9	9,53716	9,97257	9,56459	10,43541	51
10	9,53751	9,97252	9,56498	10,43502	50
11	9,53785	9,97248	9,56537	10,43463	49
12	9,53819	9,97243	9,56576	10,43424	48
13	9,53854	9,97238	9,56615	10,43385	47
14	9,53888	9,97234	9,56654	10,43346	46
15	9,53922	9,97229	9,56693	10,43307	45
16	9,53956	9,97224	9,56732	10,43268	44
17	9,53991	9,97220	9,56771	10,43229	43
18	9,54025	9,97215	9,56810	10,43190	42
19	9,54059	9,97210	9,56849	10,43151	41
20	9,54093	9,97206	9,56887	10,43113	40
21	9,54127	9,97201	9,56926	10,43074	39
22	9,54161	9,97196	9,56965	10,43035	38
23	9,54195	9,97192	9,57003	10,42996	37
24	9,54229	9,97187	9,57042	10,42958	36
25	9,54263	9,97182	9,57081	10,42919	35
26	9,54297	9,97178	9,57119	10,42880	34
27	9,54331	9,97173	9,57158	10,42842	33
28	9,54365	9,97168	9,57197	10,42803	32
29	9,54399	9,97163	9,57235	10,42764	31
30	9,54432	9,97159	9,57274	10,42726	30
31	9,54466	9,97154	9,57312	10,42688	29
32	9,54500	9,97149	9,57351	10,42649	28
33	9,54534	9,97145	9,57389	10,42611	27
34	9,54567	9,97140	9,57428	10,42572	26
35	9,54601	9,97135	9,57466	10,42534	25
36	9,54635	9,97130	9,57504	10,42496	24
37	9,54668	9,97126	9,57543	10,42457	23
38	9,54702	9,97121	9,57581	10,42419	22
39	9,54735	9,97116	9,57619	10,42381	21
40	9,54769	9,97111	9,57658	10,42342	20
41	9,54802	9,97106	9,57696	10,42304	19
42	9,54836	9,97102	9,57734	10,42266	18
43	9,54869	9,97097	9,57772	10,42228	17
44	9,54903	9,97092	9,57810	10,42190	16
45	9,54936	9,97087	9,57849	10,42151	15
46	9,54969	9,97083	9,57887	10,42113	14
47	9,55003	9,97078	9,57925	10,42075	13
48	9,55036	9,97073	9,57963	10,42037	12
49	9,55069	9,97068	9,58001	10,41999	11
50	9,55102	9,97063	9,58039	10,41961	10
51	9,55136	9,97059	9,58077	10,41923	9
52	9,55169	9,97054	9,58115	10,41885	8
53	9,55202	9,97049	9,58153	10,41847	7
54	9,55235	9,97044	9,58191	10,41809	6
55	9,55268	9,97039	9,58229	10,41771	5
56	9,55301	9,97034	9,58266	10,41733	4
57	9,55334	9,97030	9,58304	10,41696	3
58	9,55367	9,97025	9,58342	10,41658	2
59	9,55400	9,97020	9,58380	10,41620	1
60	9,55433	9,97015	9,58418	10,41582	0
	Co-sine	Sine	Co-tan.	Tangent	M

69 Degrees

21 Degrees

M	Sine	Co-sine	Tangent	Co-tang.	M
0	9,55433	9,97015	9,58418	10,41582	60
1	9,55466	9,97010	9,58455	10,41544	59
2	9,55499	9,97005	9,58493	10,41507	58
3	9,55531	9,97001	9,58531	10,41469	57
4	9,55564	9,96996	9,58569	10,41431	56
5	9,55597	9,96991	9,58606	10,41394	55
6	9,55630	9,96986	9,58644	10,41356	54
7	9,55663	9,96981	9,58681	10,41318	53
8	9,55695	9,96976	9,58719	10,41281	52
9	9,55728	9,96971	9,58757	10,41243	51
10	9,55761	9,96966	9,58794	10,41206	50
11	9,55793	9,96962	9,58832	10,41168	49
12	9,55826	9,96957	9,58869	10,41131	48
13	9,55858	9,96952	9,58907	10,41093	47
14	9,55891	9,96947	9,58944	10,41056	46
15	9,55923	9,96942	9,58981	10,41019	45
16	9,55956	9,96937	9,59019	10,40981	44
17	9,55988	9,96932	9,59056	10,40944	43
18	9,56021	9,96927	9,59093	10,40906	42
19	9,56053	9,96922	9,59131	10,40869	41
20	9,56085	9,96917	9,59168	10,40832	40
21	9,56118	9,96912	9,59205	10,40795	39
22	9,56150	9,96907	9,59243	10,40757	38
23	9,56182	9,96902	9,59280	10,40720	37
24	9,56215	9,96898	9,59317	10,40683	36
25	9,56247	9,96893	9,59354	10,40646	35
26	9,56279	9,96888	9,59391	10,40609	34
27	9,56311	9,96883	9,59428	10,40571	33
28	9,56343	9,96878	9,59466	10,40534	32
29	9,56375	9,96873	9,59503	10,40497	31
30	9,56407	9,96868	9,59540	10,40460	30
31	9,56440	9,96863	9,59577	10,40423	29
32	9,56472	9,96858	9,59614	10,40386	28
33	9,56504	9,96853	9,59651	10,40349	27
34	9,56536	9,96848	9,59688	10,40312	26
35	9,56568	9,96843	9,59725	10,40275	25
36	9,56599	9,96838	9,59762	10,40238	24
37	9,56631	9,96833	9,59798	10,40201	23
38	9,56663	9,96828	9,59835	10,40165	22
39	9,56695	9,96823	9,59872	10,40128	21
40	9,56727	9,96818	9,59909	10,40091	20
41	9,56759	9,96813	9,59946	10,40054	19
42	9,56790	9,96808	9,59983	10,40017	18
43	9,56822	9,96803	9,60019	10,39981	17
44	9,56854	9,96798	9,60056	10,39944	16
45	9,56885	9,96793	9,60093	10,39907	15
46	9,56917	9,96788	9,60130	10,39870	14
47	9,56949	9,96783	9,60166	10,39834	13
48	9,56980	9,96777	9,60203	10,39797	12
49	9,57012	9,96772	9,60239	10,39760	11
50	9,57043	9,96767	9,60276	10,39724	10
51	9,57075	9,96762	9,60313	10,39687	9
52	9,57107	9,96757	9,60349	10,39651	8
53	9,57138	9,96752	9,60386	10,39614	7
54	9,57169	9,96747	9,60422	10,39578	6
55	9,57201	9,96742	9,60459	10,39541	5
56	9,57232	9,96737	9,60495	10,39505	4
57	9,57264	9,96732	9,60532	10,39468	3
58	9,57295	9,96727	9,60568	10,39432	2
59	9,57326	9,96722	9,60605	10,39395	1
60	9,57357	9,96717	9,60641	10,39359	0
	Co-sine	Sine	Co-tan.	Tangent	M

68 Degrees

22 Degrees

23 Degrees

M	Sine	Co-sine	Tangent	Co-tang.	M	Sine	Co-sine	Tangent	Co-tang.
0	9,57357	9,96717	9,60641	10,39359	60	9,59188	9,96403	9,62785	10,37215
1	9,57389	9,96711	9,60677	10,39323	59	9,59217	9,96397	9,62820	10,37180
2	9,57420	9,96706	9,60714	10,39286	58	9,59247	9,96392	9,62855	10,37145
3	9,57451	9,96701	9,60750	10,39250	57	9,59277	9,96386	9,62890	10,37109
4	9,57482	9,96696	9,60786	10,39214	56	9,59307	9,96381	9,62925	10,37074
5	9,57514	9,96691	9,60822	10,39177	55	9,59336	9,96376	9,62961	10,37039
6	9,57545	9,96686	9,60859	10,39141	54	9,59366	9,96370	9,62996	10,37004
7	9,57576	9,96681	9,60895	10,39105	53	9,59395	9,96365	9,63031	10,36969
8	9,57607	9,96676	9,60931	10,39069	52	9,59425	9,96360	9,63066	10,36934
9	9,57638	9,96670	9,60967	10,39033	51	9,59455	9,96354	9,63100	10,36899
10	9,57669	9,96665	9,61004	10,38996	50	9,59484	9,96349	9,63135	10,36864
11	9,57700	9,96660	9,61040	10,38960	49	9,59514	9,96343	9,63170	10,36830
12	9,57731	9,96655	9,61076	10,38924	48	9,59543	9,96338	9,63205	10,36795
13	9,57762	9,96650	9,61112	10,38888	47	9,59573	9,96332	9,63240	10,36760
14	9,57793	9,96645	9,61148	10,38852	46	9,59602	9,96327	9,63275	10,36725
15	9,57824	9,96639	9,61184	10,38816	45	9,59631	9,96322	9,63310	10,36690
16	9,57854	9,96634	9,61220	10,38780	44	9,59661	9,96316	9,63344	10,36655
17	9,57885	9,96629	9,61256	10,38744	43	9,59690	9,96311	9,63379	10,36620
18	9,57916	9,96624	9,61292	10,38708	42	9,59720	9,96305	9,63414	10,36586
19	9,57947	9,96619	9,61328	10,38672	41	9,59749	9,96300	9,63449	10,36551
20	9,57978	9,96614	9,61364	10,38636	40	9,59778	9,96294	9,63484	10,36516
21	9,58008	9,96608	9,61400	10,38600	39	9,59807	9,96289	9,63518	10,36481
22	9,58039	9,96603	9,61436	10,38564	38	9,59837	9,96284	9,63553	10,36447
23	9,58070	9,96598	9,61472	10,38528	37	9,59866	9,96278	9,63588	10,36412
24	9,58100	9,96593	9,61508	10,38492	36	9,59895	9,96273	9,63623	10,36377
25	9,58131	9,96588	9,61543	10,38456	35	9,59924	9,96267	9,63657	10,36343
26	9,58162	9,96582	9,61579	10,38421	34	9,59954	9,96262	9,63692	10,36308
27	9,58192	9,96577	9,61615	10,38385	33	9,59983	9,96256	9,63726	10,36273
28	9,58223	9,96572	9,61651	10,38349	32	9,60012	9,96251	9,63761	10,36239
29	9,58253	9,96567	9,61687	10,38313	31	9,60040	9,96245	9,63796	10,36204
30	9,58284	9,96561	9,61722	10,38278	30	9,60070	9,96240	9,63830	10,36170
31	9,58314	9,96556	9,61758	10,38242	29	9,60099	9,96234	9,63865	10,36135
32	9,58345	9,96551	9,61794	10,38206	28	9,60128	9,96229	9,63899	10,36101
33	9,58375	9,96546	9,61829	10,38170	27	9,60157	9,96223	9,63934	10,36066
34	9,58406	9,96541	9,61865	10,38135	26	9,60186	9,96218	9,63968	10,36032
35	9,58436	9,96535	9,61901	10,38099	25	9,60215	9,96212	9,64003	10,35997
36	9,58466	9,96530	9,61936	10,38063	24	9,60244	9,96207	9,64037	10,35963
37	9,58497	9,96525	9,61972	10,38028	23	9,60273	9,96201	9,64072	10,35928
38	9,58527	9,96519	9,62008	10,37992	22	9,60302	9,96196	9,64106	10,35894
39	9,58557	9,96514	9,62043	10,37957	21	9,60330	9,96190	9,64140	10,35860
40	9,58588	9,96509	9,62079	10,37921	20	9,60359	9,96185	9,64175	10,35825
41	9,58618	9,96504	9,62114	10,37886	19	9,60388	9,96179	9,64209	10,35791
42	9,58648	9,96498	9,62150	10,37850	18	9,60417	9,96173	9,64243	10,35757
43	9,58678	9,96493	9,62185	10,37815	17	9,60446	9,96168	9,64278	10,35722
44	9,58708	9,96488	9,62221	10,37779	16	9,60474	9,96162	9,64312	10,35688
45	9,58739	9,96483	9,62256	10,37744	15	9,60503	9,96157	9,64346	10,35654
46	9,58769	9,96477	9,62291	10,37708	14	9,60532	9,96151	9,64381	10,35619
47	9,58799	9,96472	9,62327	10,37673	13	9,60561	9,96146	9,64415	10,35585
48	9,58829	9,96467	9,62362	10,37638	12	9,60589	9,96140	9,64449	10,35551
49	9,58859	9,96461	9,62398	10,37602	11	9,60618	9,96135	9,64483	10,35517
50	9,58889	9,96456	9,62433	10,37567	10	9,60646	9,96129	9,64517	10,35483
51	9,58919	9,96451	9,62468	10,37532	9	9,60675	9,96123	9,64552	10,35448
52	9,58949	9,96445	9,62504	10,37496	8	9,60704	9,96118	9,64586	10,35414
53	9,58979	9,96440	9,62539	10,37461	7	9,60732	9,96112	9,64620	10,35380
54	9,59009	9,96435	9,62574	10,37426	6	9,60761	9,96107	9,64654	10,35346
55	9,59039	9,96429	9,62609	10,37391	5	9,60789	9,96101	9,64688	10,35312
56	9,59069	9,96424	9,62644	10,37355	4	9,60818	9,96095	9,64722	10,35278
57	9,59098	9,96419	9,62680	10,37320	3	9,60846	9,96090	9,64756	10,35244
58	9,59128	9,96413	9,62715	10,37285	2	9,60874	9,96084	9,64790	10,35210
59	9,59158	9,96408	9,62750	10,37250	1	9,60903	9,96079	9,64824	10,35176
60	9,59188	9,96403	9,62785	10,37215	0	9,60931	9,96073	9,64858	10,35142
Co-sine	Sine	Co-tan.	Tangent	M	Co-sine	Sine	Co-tan.	Tangent	M

67 Degrees

66 Degrees

24 Degrees

M	Sine	Co-fine	Tangent	Co-tang.	M
0	9,60931	9,96073	9,64858	10,35142	60
1	9,60960	9,96067	9,64892	10,35108	59
2	9,60988	9,96062	9,64926	10,35074	58
3	9,61016	9,96056	9,64960	10,35040	57
4	9,61045	9,96050	9,64994	10,35006	56
5	9,61073	9,96045	9,65028	10,34972	55
6	9,61101	9,96039	9,65062	10,34938	54
7	9,61129	9,96033	9,65096	10,34904	53
8	9,61158	9,96028	9,65130	10,34870	52
9	9,61186	9,96022	9,65164	10,34836	51
10	9,61214	9,96016	9,65197	10,34803	50
11	9,61242	9,96011	9,65231	10,34769	49
12	9,61270	9,96005	9,65265	10,34735	48
13	9,61298	9,95999	9,65299	10,34701	47
14	9,61326	9,95994	9,65333	10,34667	46
15	9,61354	9,95988	9,65366	10,34634	45
16	9,61382	9,95982	9,65400	10,34600	44
17	9,61410	9,95977	9,65434	10,34566	43
18	9,61438	9,95971	9,65467	10,34533	42
19	9,61466	9,95965	9,65501	10,34499	41
20	9,61494	9,95960	9,65535	10,34465	40
21	9,61522	9,95954	9,65568	10,34432	39
22	9,61550	9,95948	9,65602	10,34398	38
23	9,61578	9,95942	9,65636	10,34364	37
24	9,61606	9,95937	9,65669	10,34331	36
25	9,61634	9,95931	9,65703	10,34297	35
26	9,61662	9,95925	9,65736	10,34264	34
27	9,61689	9,95919	9,65770	10,34230	33
28	9,61717	9,95914	9,65803	10,34197	32
29	9,61745	9,95908	9,65837	10,34163	31
30	9,61773	9,95902	9,65870	10,34130	30
31	9,61800	9,95896	9,65904	10,34096	29
32	9,61828	9,95891	9,65937	10,34063	28
33	9,61856	9,95885	9,65971	10,34029	27
34	9,61883	9,95879	9,66004	10,33996	26
35	9,61911	9,95873	9,66038	10,33962	25
36	9,61939	9,95868	9,66071	10,33929	24
37	9,61966	9,95862	9,66104	10,33896	23
38	9,61994	9,95856	9,66138	10,33862	22
39	9,62021	9,95850	9,66171	10,33829	21
40	9,62049	9,95844	9,66204	10,33796	20
41	9,62076	9,95838	9,66238	10,33762	19
42	9,62104	9,95833	9,66271	10,33729	18
43	9,62131	9,95827	9,66304	10,33696	17
44	9,62159	9,95821	9,66337	10,33662	16
45	9,62186	9,95815	9,66371	10,33629	15
46	9,62213	9,95810	9,66404	10,33596	14
47	9,62241	9,95804	9,66437	10,33563	13
48	9,62268	9,95798	9,66470	10,33530	12
49	9,62296	9,95792	9,66503	10,33496	11
50	9,62323	9,95786	9,66537	10,33463	10
51	9,62350	9,95780	9,66570	10,33430	9
52	9,62377	9,95775	9,66603	10,33397	8
53	9,62405	9,95769	9,66636	10,33364	7
54	9,62432	9,95763	9,66669	10,33331	6
55	9,62459	9,95757	9,66702	10,33298	5
56	9,62486	9,95751	9,66735	10,33265	4
57	9,62513	9,95745	9,66768	10,33232	3
58	9,62541	9,95739	9,66801	10,33199	2
59	9,62568	9,95733	9,66834	10,33166	1
60	9,62595	9,95728	9,66867	10,33133	0
	Co fine	Sine	Co-tan.	Tangent	M

65 Degrees

25 Degrees

M	Sine	Co-fine	Tangent	Co-tang.	M
0	9,62595	9,95728	9,66867	10,33133	60
1	9,62622	9,95722	9,66900	10,33100	59
2	9,62649	9,95716	9,66933	10,33067	58
3	9,62676	9,95710	9,66966	10,33034	57
4	9,62703	9,95704	9,66999	10,33001	56
5	9,62730	9,95698	9,67032	10,32968	55
6	9,62757	9,95692	9,67065	10,32935	54
7	9,62784	9,95686	9,67098	10,32902	53
8	9,62811	9,95680	9,67131	10,32869	52
9	9,62838	9,95674	9,67163	10,32836	51
10	9,62865	9,95668	9,67196	10,32804	50
11	9,62892	9,95662	9,67229	10,32771	49
12	9,62918	9,95657	9,67262	10,32738	48
13	9,62945	9,95651	9,67295	10,32705	47
14	9,62972	9,95645	9,67327	10,32672	46
15	9,62999	9,95639	9,67360	10,32640	45
16	9,63025	9,95633	9,67393	10,32607	44
17	9,63052	9,95627	9,67426	10,32574	43
18	9,63079	9,95621	9,67458	10,32542	42
19	9,63106	9,95615	9,67491	10,32509	41
20	9,63133	9,95609	9,67524	10,32476	40
21	9,63160	9,95603	9,67556	10,32444	39
22	9,63186	9,95597	9,67589	10,32411	38
23	9,63212	9,95591	9,67622	10,32378	37
24	9,63239	9,95585	9,67654	10,32346	36
25	9,63266	9,95579	9,67687	10,32313	35
26	9,63292	9,95573	9,67719	10,32281	34
27	9,63319	9,95567	9,67752	10,32248	33
28	9,63345	9,95561	9,67785	10,32215	32
29	9,63372	9,95555	9,67817	10,32183	31
30	9,63398	9,95549	9,67850	10,32150	30
31	9,63425	9,95543	9,67882	10,32118	29
32	9,63451	9,95537	9,67915	10,32085	28
33	9,63478	9,95531	9,67947	10,32053	27
34	9,63504	9,95525	9,67979	10,32020	26
35	9,63531	9,95519	9,68012	10,31988	25
36	9,63557	9,95513	9,68044	10,31956	24
37	9,63583	9,95506	9,68077	10,31923	23
38	9,63610	9,95500	9,68109	10,31891	22
39	9,63636	9,95494	9,68142	10,31858	21
40	9,63662	9,95488	9,68174	10,31826	20
41	9,63689	9,95482	9,68207	10,31794	19
42	9,63715	9,95476	9,68239	10,31761	18
43	9,63741	9,95470	9,68271	10,31729	17
44	9,63767	9,95464	9,68303	10,31697	16
45	9,63793	9,95458	9,68336	10,31664	15
46	9,63820	9,95452	9,68368	10,31632	14
47	9,63846	9,95446	9,68400	10,31600	13
48	9,63872	9,95440	9,68432	10,31568	12
49	9,63898	9,95433	9,68465	10,31535	11
50	9,63924	9,95427	9,68497	10,31503	10
51	9,63950	9,95421	9,68529	10,31471	9
52	9,63976	9,95415	9,68561	10,31439	8
53	9,64002	9,95409	9,68593	10,31407	7
54	9,64028	9,95403	9,68625	10,31374	6
55	9,64054	9,95397	9,68658	10,31342	5
56	9,64080	9,95391	9,68690	10,31310	4
57	9,64106	9,95384	9,68722	10,31278	3
58	9,64132	9,95378	9,68754	10,31246	2
59	9,64158	9,95372	9,68786	10,31214	1
60	9,64184	9,95366	9,68818	10,31182	0
	Co-fine	Sine	Co-tan.	Tangent	M

64 Degrees

26 Degrees

27 Degrees

M	Sine	Co-sine	Tangent	Co-tang.		M	Sine	Co-sine	Tangent	Co-tang.	
0	9,64184	9,95366	9,68818	10,31182	60	0	9,65705	9,94988	9,70717	10,29283	60
1	9,64210	9,95360	9,68850	10,31150	59	1	9,65729	9,94982	9,70748	10,29252	59
2	9,64236	9,95354	9,68882	10,31118	58	2	9,65754	9,94975	9,70779	10,29221	58
3	9,64262	9,95347	9,68914	10,31086	57	3	9,65779	9,94969	9,70810	10,29190	57
4	9,64288	9,95341	9,68946	10,31054	56	4	9,65804	9,94962	9,70841	10,29159	56
5	9,64313	9,95335	9,68978	10,31022	55	5	9,65828	9,94956	9,70873	10,29127	55
6	9,64339	9,95329	9,69010	10,30990	54	6	9,65853	9,94949	9,70904	10,29096	54
7	9,64365	9,95323	9,69042	10,30957	53	7	9,65878	9,94943	9,70935	10,29065	53
8	9,64391	9,95317	9,69074	10,30926	52	8	9,65902	9,94936	9,70966	10,29034	52
9	9,64416	9,95310	9,69106	10,30894	51	9	9,65927	9,94930	9,70997	10,29003	51
10	9,64442	9,95304	9,69138	10,30862	50	10	9,65952	9,94923	9,71028	10,28972	50
11	9,64468	9,95298	9,69170	10,30830	49	11	9,65974	9,94917	9,71059	10,28941	49
12	9,64494	9,95292	9,69202	10,30798	48	12	9,66001	9,94910	9,71090	10,28910	48
13	9,64519	9,95285	9,69234	10,30766	47	13	9,66025	9,94904	9,71121	10,28878	47
14	9,64545	9,95279	9,69266	10,30734	46	14	9,66050	9,94897	9,71152	10,28847	46
15	9,64571	9,95273	9,69297	10,30702	45	15	9,66075	9,94891	9,71184	10,28816	45
16	9,64596	9,95267	9,69329	10,30671	44	16	9,66099	9,94884	9,71215	10,28785	44
17	9,64622	9,95261	9,69361	10,30639	43	17	9,66124	9,94878	9,71246	10,28754	43
18	9,64647	9,95254	9,69393	10,30607	42	18	9,66148	9,94871	9,71277	10,28723	42
19	9,64673	9,95248	9,69425	10,30575	41	19	9,66173	9,94865	9,71308	10,28692	41
20	9,64698	9,95242	9,69457	10,30543	40	20	9,66197	9,94858	9,71339	10,28661	40
21	9,64724	9,95236	9,69488	10,30512	39	21	9,66221	9,94852	9,71370	10,28630	39
22	9,64749	9,95229	9,69520	10,30480	38	22	9,66246	9,94845	9,71400	10,28599	38
23	9,64775	9,95223	9,69552	10,30448	37	23	9,66270	9,94839	9,71431	10,28568	37
24	9,64800	9,95217	9,69583	10,30416	36	24	9,66295	9,94832	9,71462	10,28538	36
25	9,64826	9,95210	9,69615	10,30385	35	25	9,66319	9,94826	9,71493	10,28507	35
26	9,64851	9,95204	9,69647	10,30353	34	26	9,66343	9,94819	9,71524	10,28476	34
27	9,64877	9,95198	9,69679	10,30321	33	27	9,66368	9,94813	9,71555	10,28445	33
28	9,64902	9,95192	9,69710	10,30290	32	28	9,66392	9,94806	9,71586	10,28414	32
29	9,64927	9,95185	9,69742	10,30258	31	29	9,66416	9,94799	9,71617	10,28383	31
30	9,64953	9,95179	9,69774	10,30226	30	30	9,66441	9,94793	9,71648	10,28352	30
31	9,64978	9,95173	9,69805	10,30195	29	31	9,66465	9,94786	9,71678	10,28321	29
32	9,65003	9,95166	9,69837	10,30163	28	32	9,66489	9,94780	9,71709	10,28291	28
33	9,65029	9,95160	9,69868	10,30131	27	33	9,66513	9,94773	9,71740	10,28260	27
34	9,65054	9,95154	9,69900	10,30100	26	34	9,66537	9,94766	9,71771	10,28229	26
35	9,65079	9,95148	9,69932	10,30068	25	35	9,66562	9,94760	9,71802	10,28198	25
36	9,65104	9,95141	9,69963	10,30037	24	36	9,66586	9,94753	9,71832	10,28167	24
37	9,65130	9,95135	9,69995	10,30005	23	37	9,66610	9,94747	9,71863	10,28137	23
38	9,65155	9,95129	9,70026	10,29974	22	38	9,66634	9,94740	9,71894	10,28106	22
39	9,65180	9,95122	9,70058	10,29942	21	39	9,66658	9,94733	9,71925	10,28075	21
40	9,65205	9,95116	9,70089	10,29911	20	40	9,66682	9,94727	9,71955	10,28044	20
41	9,65230	9,95110	9,70120	10,29879	19	41	9,66706	9,94720	9,71986	10,28014	19
42	9,65255	9,95103	9,70152	10,29848	18	42	9,66730	9,94714	9,72017	10,27983	18
43	9,65281	9,95097	9,70184	10,29816	17	43	9,66755	9,94707	9,72048	10,27952	17
44	9,65306	9,95090	9,70215	10,29785	16	44	9,66779	9,94700	9,72078	10,27922	16
45	9,65331	9,95084	9,70247	10,29753	15	45	9,66803	9,94694	9,72109	10,27891	15
46	9,65356	9,95078	9,70278	10,29722	14	46	9,66827	9,94687	9,72140	10,27860	14
47	9,65381	9,95071	9,70309	10,29690	13	47	9,66851	9,94680	9,72170	10,27830	13
48	9,65406	9,95065	9,70341	10,29659	12	48	9,66875	9,94674	9,72201	10,27799	12
49	9,65431	9,95059	9,70372	10,29628	11	49	9,66899	9,94667	9,72231	10,27768	11
50	9,65456	9,95052	9,70404	10,29596	10	50	9,66922	9,94660	9,72262	10,27738	10
51	9,65481	9,95046	9,70435	10,29565	9	51	9,66946	9,94654	9,72293	10,27707	9
52	9,65506	9,95039	9,70466	10,29534	8	52	9,66970	9,94647	9,72323	10,27677	8
53	9,65531	9,95033	9,70498	10,29502	7	53	9,66994	9,94640	9,72354	10,27646	7
54	9,65556	9,95027	9,70529	10,29471	6	54	9,67018	9,94634	9,72384	10,27616	6
55	9,65580	9,95020	9,70560	10,29440	5	55	9,67042	9,94627	9,72415	10,27585	5
56	9,65605	9,95014	9,70592	10,29408	4	56	9,67066	9,94620	9,72445	10,27555	4
57	9,65630	9,95007	9,70623	10,29377	3	57	9,67090	9,94614	9,72476	10,27524	3
58	9,65655	9,95001	9,70654	10,29346	2	58	9,67113	9,94607	9,72506	10,27493	2
59	9,65680	9,95994	9,70685	10,29315	1	59	9,67137	9,94600	9,72537	10,27463	1
60	9,65705	9,95988	9,70717	10,29283	0	60	9,67161	9,94593	9,72567	10,27433	0
	Co-sine	Sine	Co-tan.	Tangent	M		Co-sine	Sine	Co-tan.	Tangent	M

63 Degrees

62 Degrees

G g g g

28 Degrees

29 Degrees

M	Sine	Co-sine	Tangent	Co-tang.		M	Sine	Co-sine	Tangent	Co-tang.	
0	9,67161	9,94593	9,72567	10,27433	60	0	9,68557	9,94182	9,74375	10,25625	60
1	9,67185	9,94587	9,72598	10,27402	59	1	9,68580	9,94175	9,74405	10,25595	59
2	9,67208	9,94580	9,72628	10,27372	58	2	9,68603	9,94168	9,74435	10,25565	58
3	9,67232	9,94573	9,72659	10,27341	57	3	9,68625	9,94161	9,74464	10,25535	57
4	9,67256	9,94567	9,72689	10,27311	56	4	9,68648	9,94154	9,74494	10,25506	56
5	9,67279	9,94560	9,72720	10,27280	55	5	9,68671	9,94147	9,74524	10,25476	55
6	9,67303	9,94553	9,72750	10,27250	54	6	9,68694	9,94140	9,74554	10,25446	54
7	9,67327	9,94546	9,72780	10,27219	53	7	9,68716	9,94133	9,74583	10,25416	53
8	9,67350	9,94540	9,72811	10,27189	52	8	9,68739	9,94126	9,74613	10,25387	52
9	9,67374	9,94533	9,72841	10,27159	51	9	9,68762	9,94119	9,74643	10,25357	51
10	9,67398	9,94526	9,72872	10,27128	50	10	9,68784	9,94112	9,74673	10,25327	50
11	9,67421	9,94519	9,72902	10,27098	49	11	9,68807	9,94105	9,74702	10,25298	49
12	9,67445	9,94512	9,72932	10,27068	48	12	9,68829	9,94097	9,74732	10,25268	48
13	9,67468	9,94506	9,72963	10,27037	47	13	9,68852	9,94090	9,74762	10,25238	47
14	9,67492	9,94499	9,72993	10,27007	46	14	9,68875	9,94083	9,74791	10,25209	46
15	9,67515	9,94492	9,73023	10,26977	45	15	9,68897	9,94076	9,74821	10,25179	45
16	9,67539	9,94485	9,73053	10,26946	44	16	9,68920	9,94069	9,74850	10,25149	44
17	9,67562	9,94479	9,73084	10,26916	43	17	9,68942	9,94062	9,74880	10,25120	43
18	9,67586	9,94472	9,73114	10,26886	42	18	9,68965	9,94055	9,74910	10,25090	42
19	9,67609	9,94465	9,73144	10,26856	41	19	9,68987	9,94048	9,74939	10,25061	41
20	9,67633	9,94458	9,73175	10,26825	40	20	9,69010	9,94041	9,74969	10,25031	40
21	9,67656	9,94451	9,73205	10,26795	39	21	9,69032	9,94034	9,74996	10,25001	39
22	9,67680	9,94445	9,73235	10,26765	38	22	9,69055	9,94027	9,75028	10,24972	38
23	9,67703	9,94438	9,73265	10,26735	37	23	9,69077	9,94020	9,75058	10,24942	37
24	9,67726	9,94431	9,73295	10,26704	36	24	9,69100	9,94012	9,75087	10,24913	36
25	9,67750	9,94424	9,73326	10,26674	35	25	9,69122	9,94005	9,75117	10,24883	35
26	9,67773	9,94417	9,73356	10,26644	34	26	9,69144	9,93998	9,75146	10,24854	34
27	9,67796	9,94410	9,73386	10,26614	33	27	9,69167	9,93991	9,75176	10,24824	33
28	9,67820	9,94404	9,73416	10,26584	32	28	9,69189	9,93984	9,75205	10,24795	32
29	9,67843	9,94397	9,73446	10,26554	31	29	9,69211	9,93977	9,75235	10,24765	31
30	9,67866	9,94390	9,73476	10,26524	30	30	9,69234	9,93970	9,75264	10,24736	30
31	9,67889	9,94383	9,73507	10,26493	29	31	9,69256	9,93962	9,75294	10,24706	29
32	9,67913	9,94376	9,73537	10,26463	28	32	9,69278	9,93955	9,75323	10,24677	28
33	9,67936	9,94369	9,73567	10,26433	27	33	9,69301	9,93948	9,75353	10,24647	27
34	9,67959	9,94362	9,73597	10,26403	26	34	9,69323	9,93941	9,75382	10,24618	26
35	9,67982	9,94355	9,73627	10,26373	25	35	9,69345	9,93934	9,75411	10,24588	25
36	9,68006	9,94349	9,73657	10,26343	24	36	9,69368	9,93927	9,75441	10,24559	24
37	9,68023	9,94342	9,73687	10,26313	23	37	9,69390	9,93919	9,75470	10,24530	23
38	9,68052	9,94335	9,73717	10,26283	22	38	9,69412	9,93912	9,75500	10,24500	22
39	9,68075	9,94328	9,73747	10,26253	21	39	9,69434	9,93905	9,75529	10,24471	21
40	9,68098	9,94321	9,73777	10,26223	20	40	9,69456	9,93898	9,75558	10,24441	20
41	9,68121	9,94314	9,73807	10,26193	19	41	9,69479	9,93891	9,75588	10,24412	19
42	9,68144	9,94307	9,73837	10,26163	18	42	9,69501	9,93884	9,75617	10,24383	18
43	9,68167	9,94300	9,73867	10,26133	17	43	9,69523	9,93876	9,75646	10,24353	17
44	9,68190	9,94293	9,73897	10,26103	16	44	9,69545	9,93869	9,75676	10,24324	16
45	9,68213	9,94286	9,73927	10,26073	15	45	9,69567	9,93862	9,75705	10,24295	15
46	9,68236	9,94279	9,73957	10,26043	14	46	9,69589	9,93855	9,75734	10,24265	14
47	9,68259	9,94272	9,73987	10,26013	13	47	9,69611	9,93847	9,75764	10,24236	13
48	9,68282	9,94266	9,74017	10,25983	12	48	9,69633	9,93840	9,75793	10,24207	12
49	9,68305	9,94259	9,74047	10,25953	11	49	9,69655	9,93833	9,75822	10,24178	11
50	9,68328	9,94252	9,74077	10,25923	10	50	9,69677	9,93826	9,75852	10,24148	10
51	9,68351	9,94245	9,74107	10,25893	9	51	9,69699	9,93818	9,75881	10,24119	9
52	9,68374	9,94238	9,74136	10,25863	8	52	9,69721	9,93811	9,75910	10,24090	8
53	9,68397	9,94231	9,74166	10,25834	7	53	9,69743	9,93804	9,75939	10,24060	7
54	9,68420	9,94224	9,74196	10,25803	6	54	9,69765	9,93797	9,75969	10,24031	6
55	9,68443	9,94217	9,74226	10,25774	5	55	9,69787	9,93789	9,75998	10,24002	5
56	9,68466	9,94210	9,74256	10,25744	4	56	9,69809	9,93782	9,76027	10,23973	4
57	9,68489	9,94203	9,74286	10,25714	3	57	9,69831	9,93775	9,76056	10,23944	3
58	9,68511	9,94196	9,74316	10,25684	2	58	9,69853	9,93768	9,76086	10,23914	2
59	9,68534	9,94189	9,74345	10,25655	1	59	9,69875	9,93760	9,76115	10,23885	1
60	9,68557	9,94182	9,74375	10,25625	0	60	9,69897	9,93753	9,76144	10,23856	0
	Co-sine	Sine	Co-tan.	Tangent	M		Co-sine	Sine	Co-tan.	Tangent	M

31 Degrees

60 Degrees

30 Degrees

31 Degrees

M	Sine	Co-fine	Tangent	Co-tang.	M	Sine	Co-fine	Tangent	Co-tang.	M	Sine	Co-fine	Tangent	Co-tang.
0	9,69897	9,93753	9,76144	10,23856	60	0	9,71184	9,93307	9,77877	10,22123	60	0	9,71184	9,93307
1	9,69919	9,93746	9,76173	10,23827	59	1	9,71205	9,93299	9,77906	10,22094	59	1	9,71205	9,93299
2	9,69941	9,93738	9,76202	10,23798	58	2	9,71226	9,93291	9,77935	10,22065	58	2	9,71226	9,93291
3	9,69963	9,93731	9,76231	10,23769	57	3	9,71247	9,93284	9,77963	10,22037	57	3	9,71247	9,93284
4	9,69984	9,93724	9,76261	10,23739	56	4	9,71268	9,93276	9,77992	10,22008	56	4	9,71268	9,93276
5	9,70000	9,93716	9,76290	10,23710	55	5	9,71289	9,93268	9,78020	10,21980	55	5	9,71289	9,93268
6	9,70028	9,93709	9,76319	10,23681	54	6	9,71310	9,93261	9,78049	10,21951	54	6	9,71310	9,93261
7	9,70050	9,93702	9,76348	10,23652	53	7	9,71331	9,93253	9,78077	10,21922	53	7	9,71331	9,93253
8	9,70072	9,93695	9,76377	10,23623	52	8	9,71352	9,93246	9,78106	10,21894	52	8	9,71352	9,93246
9	9,70093	9,93687	9,76406	10,23594	51	9	9,71373	9,93238	9,78135	10,21865	51	9	9,71373	9,93238
10	9,70115	9,93680	9,76435	10,23565	50	10	9,71393	9,93230	9,78163	10,21837	50	10	9,71393	9,93230
11	9,70137	9,93672	9,76464	10,23536	49	11	9,71414	9,93223	9,78192	10,21808	49	11	9,71414	9,93223
12	9,70158	9,93665	9,76493	10,23507	48	12	9,71435	9,93215	9,78220	10,21780	48	12	9,71435	9,93215
13	9,70180	9,93658	9,76522	10,23478	47	13	9,71456	9,93207	9,78249	10,21751	47	13	9,71456	9,93207
14	9,70202	9,93650	9,76551	10,23449	46	14	9,71477	9,93200	9,78277	10,21723	46	14	9,71477	9,93200
15	9,70224	9,93643	9,76580	10,23419	45	15	9,71498	9,93192	9,78306	10,21694	45	15	9,71498	9,93192
16	9,70245	9,93636	9,76609	10,23390	44	16	9,71519	9,93184	9,78334	10,21666	44	16	9,71519	9,93184
17	9,70267	9,93628	9,76638	10,23361	43	17	9,71539	9,93177	9,78363	10,21637	43	17	9,71539	9,93177
18	9,70288	9,93621	9,76667	10,23332	42	18	9,71560	9,93169	9,78391	10,21609	42	18	9,71560	9,93169
19	9,70310	9,93614	9,76696	10,23303	41	19	9,71581	9,93161	9,78419	10,21580	41	19	9,71581	9,93161
20	9,70332	9,93606	9,76725	10,23274	40	20	9,71602	9,93154	9,78448	10,21552	40	20	9,71602	9,93154
21	9,70353	9,93599	9,76754	10,23245	39	21	9,71622	9,93146	9,78479	10,21524	39	21	9,71622	9,93146
22	9,70375	9,93591	9,76783	10,23217	38	22	9,71643	9,93138	9,78505	10,21495	38	22	9,71643	9,93138
23	9,70396	9,93584	9,76812	10,23188	37	23	9,71663	9,93131	9,78533	10,21467	37	23	9,71663	9,93131
24	9,70418	9,93577	9,76841	10,23159	36	24	9,71685	9,93123	9,78562	10,21438	36	24	9,71685	9,93123
25	9,70439	9,93569	9,76870	10,23130	35	25	9,71705	9,93115	9,78590	10,21410	35	25	9,71705	9,93115
26	9,70461	9,93562	9,76899	10,23101	34	26	9,71726	9,93107	9,78618	10,21382	34	26	9,71726	9,93107
27	9,70482	9,93554	9,76928	10,23072	33	27	9,71747	9,93100	9,78647	10,21353	33	27	9,71747	9,93100
28	9,70504	9,93547	9,76957	10,23043	32	28	9,71767	9,93092	9,78675	10,21325	32	28	9,71767	9,93092
29	9,70525	9,93539	9,76986	10,23014	31	29	9,71788	9,93084	9,78704	10,21296	31	29	9,71788	9,93084
30	9,70547	9,93532	9,77015	10,22985	30	30	9,71808	9,93077	9,78732	10,21268	30	30	9,71808	9,93077
31	9,70568	9,93525	9,77044	10,22956	29	31	9,71829	9,93069	9,78760	10,21240	29	31	9,71829	9,93069
32	9,70590	9,93517	9,77073	10,22927	28	32	9,71850	9,93061	9,78789	10,21211	28	32	9,71850	9,93061
33	9,70611	9,93510	9,77101	10,22898	27	33	9,71870	9,93053	9,78817	10,21183	27	33	9,71870	9,93053
34	9,70633	9,93502	9,77130	10,22870	26	34	9,71891	9,93046	9,78845	10,21155	26	34	9,71891	9,93046
35	9,70654	9,93495	9,77159	10,22841	25	35	9,71911	9,93038	9,78874	10,21126	25	35	9,71911	9,93038
36	9,70675	9,93487	9,77188	10,22812	24	36	9,71932	9,93030	9,78902	10,21098	24	36	9,71932	9,93030
37	9,70697	9,93480	9,77217	10,22783	23	37	9,71952	9,93022	9,78930	10,21070	23	37	9,71952	9,93022
38	9,70718	9,93472	9,77246	10,22754	22	38	9,71973	9,93014	9,78958	10,21041	22	38	9,71973	9,93014
39	9,70739	9,93465	9,77274	10,22725	21	39	9,71993	9,93007	9,78987	10,21013	21	39	9,71993	9,93007
40	9,70761	9,93457	9,77303	10,22697	20	40	9,72014	9,92999	9,79015	10,20985	20	40	9,72014	9,92999
41	9,70782	9,93450	9,77332	10,22668	19	41	9,72034	9,92991	9,79043	10,20957	19	41	9,72034	9,92991
42	9,70803	9,93442	9,77361	10,22639	18	42	9,72055	9,92983	9,79072	10,20928	18	42	9,72055	9,92983
43	9,70824	9,93435	9,77390	10,22610	17	43	9,72075	9,92975	9,79100	10,20900	17	43	9,72075	9,92975
44	9,70846	9,93427	9,77418	10,22581	16	44	9,72096	9,92968	9,79128	10,20872	16	44	9,72096	9,92968
45	9,70867	9,93420	9,77447	10,22553	15	45	9,72116	9,92960	9,79156	10,20844	15	45	9,72116	9,92960
46	9,70888	9,93412	9,77476	10,22524	14	46	9,72137	9,92952	9,79185	10,20815	14	46	9,72137	9,92952
47	9,70909	9,93405	9,77505	10,22495	13	47	9,72157	9,92944	9,79213	10,20787	13	47	9,72157	9,92944
48	9,70931	9,93397	9,77533	10,22467	12	48	9,72177	9,92936	9,79241	10,20759	12	48	9,72177	9,92936
49	9,70952	9,93390	9,77562	10,22438	11	49	9,72198	9,92929	9,79269	10,20731	11	49	9,72198	9,92929
50	9,70973	9,93382	9,77591	10,22409	10	50	9,72218	9,92921	9,79297	10,20703	10	50	9,72218	9,92921
51	9,70994	9,93375	9,77619	10,22380	9	51	9,72238	9,92913	9,79326	10,20674	9	51	9,72238	9,92913
52	9,71015	9,93367	9,77648	10,22352	8	52	9,72259	9,92905	9,79354	10,20646	8	52	9,72259	9,92905
53	9,71036	9,93360	9,77677	10,22323	7	53	9,72279	9,92897	9,79382	10,20618	7	53	9,72279	9,92897
54	9,71057	9,93352	9,77705	10,22294	6	54	9,72299	9,92889	9,79410	10,20590	6	54	9,72299	9,92889
55	9,71079	9,93344	9,77734	10,22266	5	55	9,72320	9,92881	9,79438	10,20562	5	55	9,72320	9,92881
56	9,71100	9,93337	9,77763	10,22237	4	56	9,72340	9,92874	9,79466	10,20534	4	56	9,72340	9,92874
57	9,71121	9,93329	9,77791	10,22208	3	57	9,72360	9,92866	9,79494	10,20505	3	57	9,72360	9,92866
58	9,71142	9,93322	9,77820	10,22180	2	58	9,72380	9,92858	9,79523	10,20477	2	58	9,72380	9,92858
59	9,71163	9,93314	9,77849	10,22151	1	59	9,72401	9,92850	9,79551	10,20449	1	59	9,72401	9,92850
60	9,71184	9,93307	9,77877	10,22123	0	60	9,72421	9,92842	9,79579	10,20421	0	60	9,72421	9,92842
	Co-fine	Sine	Co-tan.	Tangent	M		Co-fine	Sine	Co-tan.	Tangent	M		Co-fine	Sine

59 Degrees

58 Degrees

G g g g 2

32 Degrees

M	Sine	Co-fine	Tangent	Co-tang.	M
0	9,72421	9,92842	9,795 9	10,20421	60
1	9,72441	9,92834	9,79607	10,20393	59
2	9,72461	9,92825	9,79635	10,20365	58
3	9,72482	9,92818	9,79663	10,20337	57
4	9,72502	9,92810	9,79691	10,20309	56
5	9,72522	9,92802	9,79719	10,20281	55
6	9,72542	9,92795	9,79747	10,20252	54
7	9,72562	9,92787	9,79775	10,20224	53
8	9,72582	9,92779	9,79804	10,20196	52
9	9,72602	9,92770	9,79832	10,20168	51
10	9,72622	9,92763	9,79860	10,20140	50
11	9,72643	9,92755	9,79888	10,20112	49
12	9,72663	9,92747	9,79916	10,20084	48
13	9,72683	9,92739	9,79944	10,20056	47
14	9,72703	9,92731	9,79972	10,20028	46
15	9,72723	9,92723	9,80000	10,20000	45
16	9,72743	9,92715	9,80028	10,19972	44
17	9,72763	9,92707	9,80056	10,19944	43
18	9,72783	9,92699	9,80084	10,19916	42
19	9,72803	9,92691	9,80112	10,19888	41
20	9,72823	9,92683	9,80140	10,19860	40
21	9,72843	9,92675	9,80167	10,19832	39
22	9,72863	9,92667	9,80195	10,19804	38
23	9,72882	9,92659	9,80223	10,19777	37
24	9,72902	9,92651	9,80251	10,19749	36
25	9,72922	9,92643	9,80279	10,19721	35
26	9,72942	9,92635	9,80307	10,19693	34
27	9,72962	9,92627	9,80335	10,19665	33
28	9,72982	9,92619	9,80363	10,19637	32
29	9,73002	9,92611	9,80391	10,19609	31
30	9,73022	9,92603	9,80419	10,19581	30
31	9,73041	9,92595	9,80447	10,19553	29
32	9,73061	9,92587	9,80474	10,19525	28
33	9,73081	9,92579	9,80502	10,19498	27
34	9,73101	9,92571	9,80530	10,19470	26
35	9,73121	9,92563	9,80558	10,19442	25
36	9,73140	9,92554	9,80586	10,19414	24
37	9,73160	9,92546	9,80614	10,19386	23
38	9,73180	9,92538	9,80641	10,19358	22
39	9,73200	9,92530	9,80669	10,19331	21
40	9,73219	9,92522	9,80697	10,19303	20
41	9,73239	9,92514	9,80725	10,19275	19
42	9,73259	9,92505	9,80753	10,19247	18
43	9,73278	9,92498	9,80780	10,19219	17
44	9,73298	9,92490	9,80808	10,19192	16
45	9,73318	9,92482	9,80836	10,19164	15
46	9,73337	9,92473	9,80864	10,19136	14
47	9,73357	9,92465	9,80892	10,19108	13
48	9,73376	9,92457	9,80919	10,19081	12
49	9,73396	9,92449	9,80947	10,19053	11
50	9,73416	9,92441	9,80975	10,19025	10
51	9,73435	9,92433	9,81002	10,18997	9
52	9,73455	9,92425	9,81030	10,18970	8
53	9,73474	9,92416	9,81058	10,18942	7
54	9,73494	9,92408	9,81086	10,18914	6
55	9,73513	9,92401	9,81113	10,18887	5
56	9,73533	9,92392	9,81141	10,18859	4
57	9,73552	9,92384	9,81169	10,18831	3
58	9,73572	9,92375	9,81196	10,18804	2
59	9,73591	9,92367	9,81224	10,18775	1
60	9,73611	9,92359	9,81252	10,18748	0
	Co fine	Sine	Co-tan.	Tangent	M

57 Degrees

33 Degrees

M	Sine	Co-fine	Tangent	Co-tang.	M
0	9,73611	9,92359	9,81252	10,18748	60
1	9,73630	9,92351	9,81279	10,18721	59
2	9,73650	9,92343	9,81307	10,18693	58
3	9,73669	9,92334	9,81335	10,18665	57
4	9,73689	9,92326	9,81362	10,18638	56
5	9,73708	9,92318	9,81390	10,18610	55
6	9,73727	9,92310	9,81417	10,18582	54
7	9,73747	9,92302	9,81445	10,18555	53
8	9,73766	9,92293	9,81473	10,18527	52
9	9,73785	9,92285	9,81500	10,18500	51
10	9,73805	9,92277	9,81528	10,18472	50
11	9,73824	9,92269	9,81555	10,18444	49
12	9,73843	9,92260	9,81583	10,18417	48
13	9,73863	9,92252	9,81611	10,18389	47
14	9,73882	9,92244	9,81638	10,18362	46
15	9,73901	9,92235	9,81666	10,18334	45
16	9,73920	9,92227	9,81693	10,18307	44
17	9,73940	9,92219	9,81721	10,18279	43
18	9,73959	9,92211	9,81748	10,18252	42
19	9,73978	9,92202	9,81776	10,18224	41
20	9,73997	9,92194	9,81803	10,18196	40
21	9,74017	9,92186	9,81831	10,18169	39
22	9,74036	9,92177	9,81858	10,18141	38
23	9,74057	9,92169	9,81886	10,18114	37
24	9,74074	9,92161	9,81913	10,18086	36
25	9,74093	9,92152	9,81941	10,18059	35
26	9,74112	9,92144	9,81968	10,18032	34
27	9,74132	9,92136	9,81996	10,18004	33
28	9,74151	9,92127	9,82023	10,17977	32
29	9,74170	9,92119	9,82051	10,17949	31
30	9,74189	9,92111	9,82078	10,17922	30
31	9,74208	9,92102	9,82106	10,17894	29
32	9,74227	9,92094	9,82133	10,17867	28
33	9,74246	9,92085	9,82161	10,17839	27
34	9,74265	9,92077	9,82188	10,17812	26
35	9,74284	9,92069	9,82215	10,17784	25
36	9,74303	9,92060	9,82243	10,17757	24
37	9,74322	9,92052	9,82270	10,17730	23
38	9,74341	9,92044	9,82298	10,17702	22
39	9,74360	9,92035	9,82325	10,17675	21
40	9,74379	9,92027	9,82352	10,17648	20
41	9,74398	9,92018	9,82380	10,17620	19
42	9,74417	9,92010	9,82407	10,17593	18
43	9,74436	9,92001	9,82434	10,17565	17
44	9,74455	9,91993	9,82462	10,17538	16
45	9,74474	9,91985	9,82489	10,17511	15
46	9,74493	9,91976	9,82517	10,17483	14
47	9,74512	9,91968	9,82544	10,17456	13
48	9,74531	9,91959	9,82571	10,17429	12
49	9,74549	9,91951	9,82599	10,17401	11
50	9,74568	9,91942	9,82626	10,17374	10
51	9,74587	9,91934	9,82653	10,17347	9
52	9,74606	9,91925	9,82680	10,17319	8
53	9,74625	9,91917	9,82708	10,17292	7
54	9,74644	9,91908	9,82735	10,17265	6
55	9,74662	9,91900	9,82762	10,17238	5
56	9,74681	9,91891	9,82790	10,17210	4
57	9,74700	9,91883	9,82817	10,17183	3
58	9,74719	9,91874	9,82844	10,17156	2
59	9,74737	9,91866	9,82871	10,17128	1
60	9,74756	9,91857	9,82899	10,17101	0
	Co-fine	Sine	Co-tan.	Tangent	M

56 Degrees

34 Degrees

35 Degrees

M	Sine	Co-sine	Tangent	Co-tang.	M	Sine	Co-sine	Tangent	Co-tang.		
0	9,74756	9,91857	9,82899	10,17101	60	9,75859	9,91336	9,84523	10,15477		
1	9,74775	9,91849	9,82926	10,17074	59	9,75877	9,91328	9,84550	10,15450		
2	9,74794	9,91840	9,82953	10,17047	58	9,75895	9,91319	9,84576	10,15424		
3	9,74812	9,91832	9,82980	10,17019	57	9,75913	9,91310	9,84603	10,15397		
4	9,74831	9,91823	9,83008	10,16992	56	9,75931	9,91301	9,84630	10,15370		
5	9,74850	9,91815	9,83035	10,16965	55	9,75949	9,91292	9,84657	10,15343		
6	9,74868	9,91806	9,83062	10,16938	54	9,75967	9,91283	9,84684	10,15316		
7	9,74887	9,91798	9,83089	10,16911	53	9,75985	9,91274	9,84711	10,15289		
8	9,74906	9,91789	9,83116	10,16883	52	9,76003	9,91265	9,84738	10,15262		
9	9,74924	9,91780	9,83144	10,16856	51	9,76021	9,91257	9,84764	10,15236		
10	9,74943	9,91772	9,83171	10,16829	50	9,76039	9,91248	9,84791	10,15209		
11	9,74961	9,91763	9,83198	10,16802	49	9,76057	9,91239	9,84818	10,15182		
12	9,74980	9,91755	9,83225	10,16775	48	9,76075	9,91230	9,84845	10,15155		
13	9,74999	9,91746	9,83252	10,16747	47	9,76093	9,91221	9,84872	10,15128		
14	9,75017	9,91738	9,83279	10,16720	46	9,76111	9,91212	9,84898	10,15101		
15	9,75036	9,91729	9,83307	10,16693	45	9,76128	9,91203	9,84925	10,15075		
16	9,75054	9,91720	9,83334	10,16666	44	9,76146	9,91194	9,84952	10,15048		
17	9,75073	9,91712	9,83361	10,16639	43	9,76164	9,91185	9,84979	10,15021		
18	9,75091	9,91703	9,83388	10,16612	42	9,76182	9,91176	9,85006	10,14994		
19	9,75110	9,91694	9,83415	10,16585	41	9,76200	9,91167	9,85032	10,14967		
20	9,75128	9,91686	9,83442	10,16557	40	9,76218	9,91158	9,85059	10,14941		
21	9,75147	9,91677	9,83470	10,16530	39	9,76236	9,91149	9,85086	10,14914		
22	9,75165	9,91668	9,83497	10,16503	38	9,76253	9,91140	9,85113	10,14887		
23	9,75184	9,91660	9,83524	10,16476	37	9,76271	9,91131	9,85140	10,14860		
24	9,75202	9,91651	9,83551	10,16449	36	9,76289	9,91123	9,85166	10,14834		
25	9,75221	9,91643	9,83578	10,16422	35	9,76307	9,91114	9,85193	10,14807		
26	9,75239	9,91634	9,83605	10,16395	34	9,76324	9,91105	9,85220	10,14780		
27	9,75258	9,91625	9,83632	10,16368	33	9,76342	9,91096	9,85247	10,14753		
28	9,75276	9,91617	9,83659	10,16341	32	9,76360	9,91087	9,85273	10,14727		
29	9,75294	9,91608	9,83686	10,16314	31	9,76378	9,91078	9,85300	10,14700		
30	9,75313	9,91599	9,83713	10,16287	30	9,76395	9,91069	9,85327	10,14673		
31	9,75331	9,91591	9,83740	10,16259	29	9,76413	9,91060	9,85353	10,14646		
32	9,75349	9,91582	9,83767	10,16232	28	9,76431	9,91051	9,85380	10,14619		
33	9,75368	9,91573	9,83795	10,16205	27	9,76448	9,91041	9,85407	10,14593		
34	9,75386	9,91565	9,83822	10,16178	26	9,76466	9,91032	9,85434	10,14566		
35	9,75405	9,91556	9,83849	10,16151	25	9,76484	9,91023	9,85460	10,14540		
36	9,75423	9,91547	9,83876	10,16124	24	9,76501	9,91014	9,85487	10,14513		
37	9,75441	9,91538	9,83903	10,16097	23	9,76519	9,91005	9,85514	10,14486		
38	9,75459	9,91530	9,83930	10,16070	22	9,76537	9,90996	9,85540	10,14460		
39	9,75478	9,91521	9,83957	10,16043	21	9,76554	9,90987	9,85567	10,14433		
40	9,75496	9,91512	9,83984	10,16016	20	9,76572	9,90978	9,85594	10,14406		
41	9,75514	9,91503	9,84011	10,15989	19	9,76590	9,90969	9,85620	10,14380		
42	9,75533	9,91495	9,84038	10,15962	18	9,76607	9,90960	9,85647	10,14353		
43	9,75551	9,91486	9,84065	10,15935	17	9,76625	9,90951	9,85674	10,14326		
44	9,75569	9,91477	9,84092	10,15908	16	9,76642	9,90942	9,85700	10,14300		
45	9,75587	9,91468	9,84119	10,15881	15	9,76660	9,90933	9,85727	10,14273		
46	9,75605	9,91460	9,84146	10,15854	14	9,76677	9,90924	9,85754	10,14246		
47	9,75624	9,91451	9,84173	10,15827	13	9,76695	9,90915	9,85780	10,14220		
48	9,75642	9,91442	9,84200	10,15800	12	9,76712	9,90905	9,85807	10,14193		
49	9,75660	9,91433	9,84227	10,15773	11	9,76729	9,90896	9,85834	10,14166		
50	9,75678	9,91425	9,84253	10,15746	10	9,76747	9,90887	9,85860	10,14140		
51	9,75696	9,91416	9,84280	10,15719	9	9,76765	9,90878	9,85887	10,14113		
52	9,75714	9,91407	9,84307	10,15693	8	9,76782	9,90869	9,85913	10,14087		
53	9,75732	9,91398	9,84334	10,15666	7	9,76800	9,90860	9,85940	10,14060		
54	9,75751	9,91389	9,84361	10,15639	6	9,76817	9,90851	9,85967	10,14033		
55	9,75769	9,91381	9,84388	10,15612	5	9,76835	9,90842	9,85993	10,14006		
56	9,75787	9,91372	9,84415	10,15585	4	9,76852	9,90832	9,86020	10,13980		
57	9,75805	9,91363	9,84442	10,15558	3	9,76870	9,90823	9,86046	10,13954		
58	9,75823	9,91354	9,84469	10,15531	2	9,76887	9,90814	9,86073	10,13927		
59	9,75841	9,91345	9,84496	10,15504	1	9,76904	9,90805	9,86099	10,13900		
60	9,75859	9,91336	9,84523	10,15477	0	9,76922	9,90796	9,86126	10,13874		
	Co-sine	Sine	Co-tan.	Tangent	M		Co-sine	Sine	Co-tan.	Tangent	M

35 Degrees

34 Degrees

36 Degrees

M	Sine	Co-sine	Tangent	Co-tang.	M
0	9,76922	9,90796	9,86126	10,13874	60
1	9,76939	9,90787	9,86153	10,13847	59
2	9,76957	9,90777	9,86179	10,13821	58
3	9,76974	9,90768	9,862 6	10,13794	57
4	9,76991	9,90759	9,86232	10,13768	56
5	9,77009	9,90750	9,86259	10,13741	55
6	9,77026	9,90741	9,86285	10,13715	54
7	9,77043	9,90731	9,86312	10,13688	53
8	9,77061	9,90722	9,86338	10,13661	52
9	9,77078	9,90713	9,86365	10,13635	51
10	9,77095	9,90704	9,86391	10,13608	50
11	9,77112	9,90694	9,86418	10,13582	49
12	9,77130	9,90685	9,86444	10,13555	48
13	9,77147	9,90676	9,86471	10,13529	47
14	9,77164	9,90667	9,86497	10,13502	46
15	9,77181	9,90657	9,86524	10,13476	45
16	9,77199	9,90648	9,86550	10,13449	44
17	9,77216	9,90639	9,86577	10,13423	43
18	9,77233	9,90630	9,86604	10,13396	42
19	9,77250	9,90620	9,86630	10,13370	41
20	9,77267	9,90611	9,86656	10,13344	40
21	9,77285	9,90602	9,86683	10,13317	39
22	9,77302	9,90592	9,86709	10,13291	38
23	9,77319	9,90583	9,86736	10,13264	37
24	9,77336	9,90574	9,86762	10,13238	36
25	9,77353	9,90564	9,86789	10,13211	35
26	9,77370	9,90555	9,86815	10,13185	34
27	9,77387	9,90546	9,86842	10,13158	33
28	9,77405	9,90537	9,86868	10,13132	32
29	9,77422	9,90527	9,86894	10,13105	31
30	9,77439	9,90518	9,86921	10,13079	30
31	9,77456	9,90508	9,86947	10,13053	29
32	9,77473	9,90499	9,86974	10,13026	28
33	9,77490	9,90490	9,87000	10,13000	27
34	9,77507	9,90480	9,87026	10,12973	26
35	9,77524	9,90471	9,87053	10,12947	25
36	9,77541	9,90462	9,87079	10,12921	24
37	9,77558	9,90453	9,87106	10,12894	23
38	9,77575	9,90443	9,87132	10,12868	22
39	9,77592	9,90433	9,87158	10,12841	21
40	9,77609	9,90424	9,87185	10,12815	20
41	9,77626	9,90415	9,87211	10,12789	19
42	9,77643	9,90405	9,87238	10,12762	18
43	9,77660	9,90396	9,87264	10,12736	17
44	9,77677	9,90386	9,87290	10,12710	16
45	9,77694	9,90377	9,87317	10,12683	15
46	9,77711	9,90368	9,87343	10,12657	14
47	9,77727	9,90358	9,87369	10,12631	13
48	9,77744	9,90349	9,87396	10,12604	12
49	9,77761	9,90339	9,87422	10,12578	11
50	9,77778	9,90330	9,87448	10,12551	10
51	9,77795	9,90320	9,87475	10,12525	9
52	9,77812	9,90311	9,87501	10,12499	8
53	9,77829	9,90301	9,87527	10,12473	7
54	9,77845	9,90293	9,87554	10,12446	6
55	9,77862	9,90282	9,87580	10,12420	5
56	9,77879	9,90273	9,87606	10,12394	4
57	9,77896	9,90263	9,87633	10,12367	3
58	9,77913	9,90254	9,87659	10,12341	2
59	9,77929	9,90244	9,87685	10,12315	1
60	9,77946	9,90235	9,87711	10,12289	0
	Co-sine	Sine	Co-tan.	Tangent	M

53 Degrees

37 Degrees

M	Sine	Co-sine	Tangent	Co-tang.	M
0	9,77946	9,90235	9,87711	10,12289	60
1	9,77963	9,90225	9,87738	10,12262	59
2	9,77980	9,90216	9,87764	10,12236	58
3	9,77996	9,90206	9,87790	10,12210	57
4	9,78013	9,90197	9,87816	10,12183	56
5	9,78030	9,90187	9,87843	10,12157	55
6	9,78047	9,90178	9,87869	10,12131	54
7	9,78063	9,90168	9,87895	10,12105	53
8	9,78080	9,90158	9,87921	10,12078	52
9	9,78097	9,90149	9,87948	10,12052	51
10	9,78113	9,90139	9,87974	10,12026	50
11	9,78130	9,90130	9,88000	10,12000	49
12	9,78147	9,90120	9,88026	10,11973	48
13	9,78163	9,90111	9,88053	10,11947	47
14	9,78180	9,90101	9,88080	10,11921	46
15	9,78197	9,90091	9,88105	10,11895	45
16	9,78213	9,90082	9,88131	10,11869	44
17	9,78230	9,90072	9,88158	10,11842	43
18	9,78246	9,90063	9,88184	10,11816	42
19	9,78263	9,90053	9,88210	10,11790	41
20	9,78280	9,90043	9,88236	10,11764	40
21	9,78296	9,90034	9,88262	10,11737	39
22	9,78313	9,90024	9,88289	10,11711	38
23	9,78329	9,90014	9,88315	10,11685	37
24	9,78346	9,90005	9,88341	10,11659	36
25	9,78362	9,89995	9,88367	10,11633	35
26	9,78379	9,89985	9,88393	10,11607	34
27	9,78395	9,89976	9,88420	10,11580	33
28	9,78412	9,89966	9,88445	10,11554	32
29	9,78428	9,89956	9,88472	10,11528	31
30	9,78445	9,89947	9,88498	10,11502	30
31	9,78461	9,89937	9,88524	10,11476	29
32	9,78478	9,89927	9,88550	10,11450	28
33	9,78494	9,89918	9,88576	10,11423	27
34	9,78510	9,89908	9,88603	10,11397	26
35	9,78527	9,89898	9,88629	10,11371	25
36	9,78543	9,89888	9,88655	10,11345	24
37	9,78560	9,89879	9,88681	10,11319	23
38	9,78576	9,89869	9,88707	10,11293	22
39	9,78593	9,89858	9,88733	10,11267	21
40	9,78609	9,89849	9,88759	10,11241	20
41	9,78625	9,89840	9,88785	10,11214	19
42	9,78642	9,89830	9,88812	10,11188	18
43	9,78658	9,89820	9,88838	10,11162	17
44	9,78674	9,89810	9,88864	10,11136	16
45	9,78691	9,89801	9,88890	10,11110	15
46	9,78707	9,89791	9,88916	10,11084	14
47	9,78723	9,89781	9,88942	10,11058	13
48	9,78739	9,89771	9,88968	10,11032	12
49	9,78756	9,89761	9,88994	10,11006	11
50	9,78772	9,89752	9,89020	10,10980	10
51	9,78788	9,89742	9,89046	10,10953	9
52	9,78804	9,89732	9,89072	10,10927	8
53	9,78821	9,89722	9,89099	10,10901	7
54	9,78837	9,89712	9,89125	10,10875	6
55	9,78853	9,89702	9,89151	10,10849	5
56	9,78869	9,89693	9,89177	10,10823	4
57	9,78886	9,89683	9,89203	10,10797	3
58	9,78902	9,89673	9,89229	10,10771	2
59	9,78918	9,89663	9,89255	10,10745	1
60	9,78934	9,89653	9,89281	10,10719	0
	Co-sine	Sine	Co-tan.	Tangent	M

52 Degrees

38 Degrees

M	Sine	Co-fine	Tangent	Co-tang.	M
0	9,78934	9,89653	9,89281	10,10719	60
1	9,78950	9,89643	9,89307	10,10693	59
2	9,78966	9,89633	9,89333	10,10667	58
3	9,78983	9,89624	9,89359	10,10641	57
4	9,78999	9,89614	9,89385	10,10615	56
5	9,79015	9,89604	9,89411	10,10589	55
6	9,79031	9,89594	9,89437	10,10563	54
7	9,79047	9,89584	9,89463	10,10537	53
8	9,79063	9,89574	9,89489	10,10511	52
9	9,79079	9,89564	9,89515	10,10485	51
10	9,79095	9,89554	9,89541	10,10459	50
11	9,79111	9,89544	9,89567	10,10433	49
12	9,79127	9,89534	9,89593	10,10407	48
13	9,79144	9,89524	9,89619	10,10381	47
14	9,79160	9,89514	9,89645	10,10355	46
15	9,79176	9,89504	9,89671	10,10329	45
16	9,79192	9,89494	9,89697	10,10303	44
17	9,79208	9,89485	9,89723	10,10277	43
18	9,79224	9,89475	9,89749	10,10250	42
19	9,79240	9,89465	9,89775	10,10225	41
20	9,79256	9,89455	9,89801	10,10199	40
21	9,79272	9,89445	9,89827	10,10173	39
22	9,79288	9,89435	9,89853	10,10147	38
23	9,79303	9,89425	9,89879	10,10121	37
24	9,79319	9,89415	9,89905	10,10095	36
25	9,79335	9,89405	9,89931	10,10069	35
26	9,79351	9,89395	9,89957	10,10043	34
27	9,79367	9,89385	9,89983	10,10017	33
28	9,79383	9,89374	9,90009	10,09991	32
29	9,79399	9,89364	9,90035	10,09965	31
30	9,79415	9,89354	9,90060	10,09939	30
31	9,79431	9,89344	9,90086	10,09913	29
32	9,79447	9,89334	9,90112	10,09888	28
33	9,79463	9,89324	9,90138	10,09862	27
34	9,79478	9,89314	9,90164	10,09836	26
35	9,79494	9,89304	9,90190	10,09810	25
36	9,79510	9,89294	9,90216	10,09784	24
37	9,79526	9,89284	9,90242	10,09758	23
38	9,79542	9,89274	9,90268	10,09732	22
39	9,79557	9,89264	9,90294	10,09706	21
40	9,79573	9,89254	9,90320	10,09680	20
41	9,79589	9,89243	9,90345	10,09654	19
42	9,79605	9,89233	9,90371	10,09629	18
43	9,79621	9,89223	9,90397	10,09603	17
44	9,79636	9,89213	9,90423	10,09577	16
45	9,79652	9,89203	9,90449	10,09551	15
46	9,79668	9,89193	9,90475	10,09525	14
47	9,79684	9,89183	9,90501	10,09499	13
48	9,79699	9,89173	9,90527	10,09473	12
49	9,79715	9,89162	9,90553	10,09447	11
50	9,79731	9,89152	9,90578	10,09421	10
51	9,79746	9,89142	9,90604	10,09396	9
52	9,79762	9,89132	9,90630	10,09370	8
53	9,79778	9,89122	9,90656	10,09344	7
54	9,79793	9,89111	9,90682	10,09318	6
55	9,79809	9,89101	9,90708	10,09292	5
56	9,79825	9,89091	9,90734	10,09266	4
57	9,79840	9,89081	9,90759	10,09241	3
58	9,79856	9,89071	9,90785	10,09215	2
59	9,79872	9,89060	9,90811	10,09189	1
60	9,79887	9,89050	9,90837	10,09163	0
	Co-fine	Sine	Co-tan.	Tangent	M

51 Degrees

39 Degrees

M	Sine	Co-fine	Tangent	Co-tang.	M
0	9,79887	9,89050	9,90837	10,09163	60
1	9,79903	9,89040	9,90863	10,09137	59
2	9,79918	9,89030	9,90889	10,09111	58
3	9,79934	9,89019	9,90914	10,09086	57
4	9,79949	9,89009	9,90940	10,09060	56
5	9,79965	9,88999	9,90966	10,09034	55
6	9,79981	9,88989	9,90992	10,09008	54
7	9,79996	9,88978	9,91018	10,08982	53
8	9,80012	9,88968	9,91043	10,08956	52
9	9,80027	9,88958	9,91069	10,08931	51
10	9,80043	9,88948	9,91095	10,08905	50
11	9,80058	9,88937	9,91121	10,08879	49
12	9,80074	9,88927	9,91147	10,08853	48
13	9,80089	9,88916	9,91172	10,08827	47
14	9,80105	9,88906	9,91198	10,08802	46
15	9,80120	9,88896	9,91224	10,08776	45
16	9,80136	9,88885	9,91250	10,08750	44
17	9,80151	9,88875	9,91276	10,08724	43
18	9,80166	9,88865	9,91301	10,08699	42
19	9,80182	9,88854	9,91327	10,08673	41
20	9,80197	9,88844	9,91353	10,08647	40
21	9,80213	9,88834	9,91379	10,08621	39
22	9,80228	9,88823	9,91404	10,08596	38
23	9,80243	9,88813	9,91430	10,08570	37
24	9,80259	9,88803	9,91456	10,08544	36
25	9,80274	9,88792	9,91482	10,08518	35
26	9,80290	9,88782	9,91507	10,08492	34
27	9,80305	9,88771	9,91533	10,08467	33
28	9,80320	9,88761	9,91559	10,08441	32
29	9,80336	9,88751	9,91585	10,08415	31
30	9,80351	9,88740	9,91610	10,08389	30
31	9,80366	9,88730	9,91636	10,08364	29
32	9,80382	9,88720	9,91662	10,08338	28
33	9,80397	9,88709	9,91688	10,08312	27
34	9,80412	9,88699	9,91713	10,08287	26
35	9,80428	9,88688	9,91739	10,08261	25
36	9,80443	9,88678	9,91765	10,08235	24
37	9,80458	9,88668	9,91790	10,08209	23
38	9,80473	9,88657	9,91816	10,08184	22
39	9,80489	9,88647	9,91842	10,08158	21
40	9,80504	9,88636	9,91868	10,08132	20
41	9,80519	9,88626	9,91893	10,08107	19
42	9,80534	9,88615	9,91919	10,08081	18
43	9,80549	9,88605	9,91945	10,08055	17
44	9,80565	9,88594	9,91970	10,08029	16
45	9,80580	9,88584	9,91996	10,08004	15
46	9,80595	9,88573	9,92022	10,07978	14
47	9,80610	9,88563	9,92048	10,07952	13
48	9,80625	9,88552	9,92073	10,07927	12
49	9,80641	9,88542	9,92099	10,07901	11
50	9,80656	9,88531	9,92125	10,07875	10
51	9,80671	9,88520	9,92150	10,07850	9
52	9,80686	9,88510	9,92176	10,07824	8
53	9,80701	9,88499	9,92202	10,07798	7
54	9,80716	9,88489	9,92227	10,07773	6
55	9,80731	9,88478	9,92253	10,07747	5
56	9,80746	9,88468	9,92279	10,07721	4
57	9,80761	9,88457	9,92304	10,07696	3
58	9,80777	9,88447	9,92330	10,07670	2
59	9,80792	9,88436	9,92356	10,07644	1
60	9,80806	9,88425	9,92381	10,07619	0
	Co-fine	Sine	Co-tan.	Tangent	M

50 Degrees

40 Degrees

M	Sine	Co-sine	Tangent	Co-tang.
0	9,80807	9,88425	9,92381	10,07619
1	9,80822	9,88415	9,92407	10,07593
2	9,80837	9,88404	9,92433	10,07567
3	9,80852	9,88394	9,92458	10,07542
4	9,80867	9,88383	9,92484	10,07516
5	9,80882	9,88372	9,92510	10,07490
6	9,80897	9,88362	9,92535	10,07465
7	9,80912	9,88351	9,92561	10,07439
8	9,80927	9,88340	9,92586	10,07413
9	9,80942	9,88330	9,92612	10,07388
10	9,80957	9,88319	9,92638	10,07362
11	9,80972	9,88308	9,92663	10,07337
12	9,80987	9,88298	9,92689	10,07311
13	9,81002	9,88287	9,92715	10,07285
14	9,81017	9,88276	9,92740	10,07260
15	9,81032	9,88266	9,92766	10,07234
16	9,81046	9,88255	9,92791	10,07208
17	9,81061	9,88244	9,92817	10,07183
18	9,81076	9,88234	9,92843	10,07157
19	9,81091	9,88223	9,92868	10,07132
20	9,81106	9,88212	9,92894	10,07106
21	9,81121	9,88201	9,92920	10,07080
22	9,81136	9,88191	9,92945	10,07055
23	9,81151	9,88180	9,92971	10,07029
24	9,81165	9,88169	9,92996	10,07004
25	9,81180	9,88158	9,93022	10,06978
26	9,81195	9,88148	9,93047	10,06952
27	9,81210	9,88137	9,93073	10,06927
28	9,81225	9,88126	9,93099	10,06901
29	9,81240	9,88115	9,93124	10,06876
30	9,81254	9,88105	9,93150	10,06850
31	9,81269	9,88094	9,93175	10,06824
32	9,81284	9,88083	9,93201	10,06799
33	9,81299	9,88072	9,93227	10,06773
34	9,81313	9,88061	9,93252	10,06748
35	9,81328	9,88050	9,93278	10,06722
36	9,81343	9,88040	9,93303	10,06697
37	9,81358	9,88029	9,93329	10,06671
38	9,81372	9,88018	9,93354	10,06645
39	9,81387	9,88007	9,93380	10,06620
40	9,81402	9,87996	9,93406	10,06594
41	9,81417	9,87985	9,93431	10,06569
42	9,81431	9,87975	9,93457	10,06543
43	9,81446	9,87964	9,93482	10,06518
44	9,81461	9,87953	9,93508	10,06492
45	9,81475	9,87942	9,93533	10,06467
46	9,81490	9,87931	9,93559	10,06441
47	9,81505	9,87920	9,93584	10,06416
48	9,81519	9,87909	9,93610	10,06390
49	9,81534	9,87898	9,93635	10,06364
50	9,81548	9,87887	9,93661	10,06339
51	9,81563	9,87877	9,93687	10,06313
52	9,81578	9,87866	9,93712	10,06288
53	9,81592	9,87855	9,93738	10,06262
54	9,81607	9,87844	9,93763	10,06237
55	9,81621	9,87833	9,93789	10,06211
56	9,81636	9,87822	9,93814	10,06186
57	9,81651	9,87811	9,93840	10,06160
58	9,81665	9,87800	9,93865	10,06135
59	9,81680	9,87789	9,93891	10,06109
60	9,81694	9,87778	9,93916	10,06084
M	Co-sine	Sine	Co-tan.	Tangent

49 Degrees

41 Degrees

M	Sine	Co-sine	Tangent	Co-tang.
0	9,81694	9,87778	9,93916	10,06084
1	9,81709	9,87767	9,93942	10,06058
2	9,81723	9,87756	9,93967	10,06033
3	9,81738	9,87745	9,93993	10,06007
4	9,81752	9,87734	9,94018	10,05982
5	9,81767	9,87723	9,94044	10,05956
6	9,81781	9,87712	9,94069	10,05931
7	9,81796	9,87701	9,94095	10,05905
8	9,81810	9,87690	9,94120	10,05880
9	9,81825	9,87679	9,94146	10,05854
10	9,81839	9,87668	9,94171	10,05829
11	9,81854	9,87657	9,94197	10,05803
12	9,81868	9,87646	9,94222	10,05778
13	9,81882	9,87635	9,94248	10,05752
14	9,81897	9,87624	9,94273	10,05727
15	9,81911	9,87612	9,94299	10,05701
16	9,81926	9,87601	9,94324	10,05676
17	9,81940	9,87590	9,94350	10,05650
18	9,81954	9,87579	9,94375	10,05625
19	9,81969	9,87568	9,94401	10,05599
20	9,81983	9,87557	9,94426	10,05574
21	9,81998	9,87546	9,94452	10,05548
22	9,82012	9,87535	9,94477	10,05523
23	9,82026	9,87524	9,94503	10,05497
24	9,82041	9,87513	9,94528	10,05472
25	9,82055	9,87501	9,94553	10,05446
26	9,82069	9,87490	9,94579	10,05421
27	9,82083	9,87479	9,94604	10,05395
28	9,82098	9,87468	9,94630	10,05370
29	9,82112	9,87457	9,94655	10,05345
30	9,82126	9,87446	9,94681	10,05319
31	9,82140	9,87434	9,94706	10,05294
32	9,82155	9,87423	9,94732	10,05268
33	9,82169	9,87412	9,94757	10,05243
34	9,82183	9,87401	9,94783	10,05217
35	9,82198	9,87390	9,94808	10,05192
36	9,82212	9,87378	9,94833	10,05166
37	9,82226	9,87367	9,94859	10,05141
38	9,82240	9,87356	9,94884	10,05116
39	9,82255	9,87345	9,94910	10,05090
40	9,82269	9,87333	9,94925	10,05065
41	9,82283	9,87322	9,94961	10,05039
42	9,82297	9,87311	9,94986	10,05014
43	9,82311	9,87300	9,95012	10,04988
44	9,82325	9,87288	9,95037	10,04963
45	9,82340	9,87277	9,95062	10,04937
46	9,82353	9,87266	9,95088	10,04912
47	9,82368	9,87255	9,95113	10,04887
48	9,82382	9,87243	9,95139	10,04861
49	9,82396	9,87232	9,95164	10,04836
50	9,82410	9,87221	9,95190	10,04810
51	9,82424	9,87209	9,95215	10,04785
52	9,82439	9,87198	9,95240	10,04759
53	9,82453	9,87187	9,95266	10,04734
54	9,82467	9,87175	9,95291	10,04709
55	9,82481	9,87164	9,95317	10,04683
56	9,82495	9,87153	9,95342	10,04658
57	9,82509	9,87141	9,95367	10,04632
58	9,82523	9,87130	9,95393	10,04607
59	9,82537	9,87119	9,95418	10,04582
60	9,82551	9,87107	9,95444	10,04556
M	Co-sine	Sine	Co-tan.	Tangent

48 Degrees

42 Degrees

43 Degrees

M	Sine	Co-sine	Tangent	Co-tang.	M	Sine	Co-sine	Tangent	Co-tang.		
0	9,82551	9,87107	9,95444	10,04556	60	9,83378	9,86413	9,96966	10,03034		
1	9,8265	9,87096	9,95469	10,04531	59	9,83392	9,86401	9,96991	10,03009		
2	9,82579	9,87085	9,95494	10,04505	58	9,83405	9,86389	9,97016	10,02984		
3	9,82593	9,87073	9,95520	10,04480	57	9,83419	9,86377	9,97041	10,02958		
4	9,82607	9,87062	9,95545	10,04455	56	9,83432	9,86366	9,97067	10,02933		
5	9,82621	9,87050	9,95571	10,04429	55	9,83446	9,86354	9,97092	10,02908		
6	9,82635	9,87039	9,95596	10,04404	54	9,83459	9,86342	9,97117	10,02882		
7	9,82649	9,87028	9,95621	10,04378	53	9,83473	9,86330	9,97143	10,02857		
8	9,82663	9,87016	9,95647	10,04353	52	9,83486	9,86318	9,97168	10,02832		
9	9,82677	9,87005	9,95672	10,04328	51	9,83500	9,86306	9,97193	10,02806		
10	9,82691	9,86993	9,95698	10,04302	50	9,83513	9,86295	9,97219	10,02781		
11	9,82705	9,86982	9,95723	10,04277	49	9,83527	9,86283	9,97244	10,02756		
12	9,82719	9,86970	9,95748	10,04251	48	9,83540	9,86271	9,97269	10,02730		
13	9,82733	9,86959	9,95774	10,04226	47	9,83554	9,86259	9,97295	10,02705		
14	9,82747	9,86947	9,95799	10,04201	46	9,83567	9,86247	9,97320	10,02680		
15	9,82761	9,86936	9,95825	10,04175	45	9,83581	9,86235	9,97345	10,02655		
16	9,82774	9,86924	9,95850	10,04150	44	9,83594	9,86223	9,97371	10,02629		
17	9,82788	9,86913	9,95875	10,04125	43	9,83607	9,86211	9,97396	10,02604		
18	9,82802	9,86901	9,95901	10,04099	42	9,83621	9,86200	9,97421	10,02579		
19	9,82816	9,86890	9,95926	10,04074	41	9,83634	9,86188	9,97447	10,02553		
20	9,82830	9,86878	9,95951	10,04048	40	9,83648	9,86176	9,97472	10,02528		
21	9,82844	9,86867	9,95977	10,04023	39	9,83661	9,86164	9,97497	10,02503		
22	9,82858	9,86855	9,96002	10,03998	38	9,83674	9,86152	9,97523	10,02477		
23	9,82872	9,86844	9,96028	10,03972	37	9,83688	9,86140	9,97548	10,02452		
24	9,82885	9,86832	9,96053	10,03947	36	9,83701	9,86128	9,97573	10,02427		
25	9,82899	9,86821	9,96078	10,03922	35	9,83715	9,86116	9,97598	10,02401		
26	9,82913	9,86809	9,96104	10,03896	34	9,83738	9,86104	9,97624	10,02376		
27	9,82927	9,86798	9,96129	10,03871	33	9,83741	9,86092	9,97649	10,02351		
28	9,82941	9,86786	9,96154	10,03845	32	9,83755	9,86080	9,97674	10,02326		
29	9,82954	9,86775	9,96180	10,03820	31	9,83768	9,86068	9,97700	10,02300		
30	9,82968	9,86763	9,96205	10,03795	30	9,83781	9,86056	9,97725	10,02275		
31	9,82982	9,86751	9,96231	10,03769	29	9,83794	9,86044	9,97750	10,02250		
32	9,82996	9,86740	9,96256	10,03744	28	9,83808	9,86032	9,97776	10,02224		
33	9,83010	9,86728	9,96281	10,03719	27	9,83821	9,86020	9,97801	10,02199		
34	9,83023	9,86717	9,96307	10,03693	26	9,83834	9,86008	9,97826	10,02174		
35	9,83037	9,86705	9,96332	10,03668	25	9,83848	9,85996	9,97851	10,02148		
36	9,83051	9,86693	9,96357	10,03643	24	9,83861	9,85984	9,97877	10,02123		
37	9,83065	9,86682	9,96383	10,03617	23	9,83874	9,85972	9,97902	10,02098		
38	9,83078	9,86670	9,96408	10,03592	22	9,83887	9,85960	9,97927	10,02073		
39	9,83092	9,86659	9,96433	10,03566	21	9,83901	9,85948	9,97953	10,02047		
40	9,83106	9,86647	9,96459	10,03541	20	9,83914	9,85936	9,97978	10,02022		
41	9,83119	9,86635	9,96484	10,03516	19	9,83927	9,85924	9,98003	10,01997		
42	9,83133	9,86624	9,96509	10,03490	18	9,83940	9,85912	9,98029	10,01971		
43	9,83147	9,86612	9,96535	10,03465	17	9,83954	9,85900	9,98054	10,01946		
44	9,83161	9,86600	9,96560	10,03440	16	9,83967	9,85888	9,98079	10,01921		
45	9,83174	9,86589	9,96585	10,03414	15	9,83980	9,85876	9,98104	10,01896		
46	9,83188	9,86577	9,96611	10,03389	14	9,83993	9,85863	9,98130	10,01870		
47	9,83201	9,86565	9,96636	10,03364	13	9,84006	9,85851	9,98155	10,01845		
48	9,83215	9,86553	9,96662	10,03338	12	9,84020	9,85839	9,98180	10,01820		
49	9,83229	9,86542	9,96687	10,03313	11	9,84033	9,85827	9,98206	10,01794		
50	9,83242	9,86530	9,96712	10,03288	10	9,84046	9,85815	9,98231	10,01769		
51	9,83256	9,86518	9,96738	10,03262	9	9,84059	9,85803	9,98256	10,01744		
52	9,83270	9,86507	9,96763	10,03237	8	9,84072	9,85791	9,98281	10,01718		
53	9,83283	9,86495	9,96788	10,03212	7	9,84085	9,85779	9,98307	10,01693		
54	9,83297	9,86483	9,96814	10,03186	6	9,84098	9,85766	9,98332	10,01668		
55	9,83310	9,86472	9,96839	10,03161	5	9,84112	9,85754	9,98357	10,01643		
56	9,83324	9,86460	9,96864	10,03136	4	9,84125	9,85742	9,98383	10,01617		
57	9,83338	9,86448	9,96890	10,03110	3	9,84138	9,85730	9,98408	10,01592		
58	9,83351	9,86436	9,96915	10,03085	2	9,84151	9,85718	9,98433	10,01567		
59	9,83365	9,86424	9,96940	10,03060	1	9,84164	9,85706	9,98458	10,01542		
60	9,83378	9,86413	9,96966	10,03034	0	9,84177	9,85693	9,98484	10,01516		
	Co fine	Sine	Co-tan.	Tangent	M		Co-fine	Sine	Co-tan.	Tangent	M

47 Degrees

46 Degrees

H h h h

44 Degrees

M	Sine	Co-fine	Tangent	Co-tang.	
0	9,84177	9,85693	9,98484	10,01516	60
1	9,84190	9,85681	9,98509	10,01491	59
2	9,84203	9,85669	9,98534	10,01466	58
3	9,84216	9,85657	9,98560	10,01440	57
4	9,84229	9,85644	9,98585	10,01415	56
5	9,84242	9,85632	9,98610	10,01390	55
6	9,84255	9,85620	9,98635	10,01365	54
7	9,84268	9,85608	9,98661	10,01339	53
8	9,84281	9,85596	9,98686	10,01314	52
9	9,84295	9,85583	9,98711	10,01289	51
10	9,84308	9,85571	9,98736	10,01263	50
11	9,84320	9,85559	9,98762	10,01238	49
12	9,84334	9,85546	9,98787	10,01213	48
13	9,84346	9,85534	9,98812	10,01188	47
14	9,84359	9,85522	9,98838	10,01162	46
15	9,84372	9,85510	9,98863	10,01137	45
16	9,84385	9,85497	9,98888	10,01112	44
17	9,84398	9,85485	9,98913	10,01087	43
18	9,84411	9,85473	9,98939	10,01061	42
19	9,84424	9,85460	9,98964	10,01036	41
20	9,84437	9,85448	9,98989	10,01011	40
21	9,84450	9,85436	9,99014	10,00985	39
22	9,84463	9,85423	9,99040	10,00960	38
23	9,84476	9,85411	9,99065	10,00935	37
24	9,84489	9,85398	9,99090	10,00909	36
25	9,84502	9,85386	9,99116	10,00884	35
26	9,84515	9,85374	9,99141	10,00859	34
27	9,84528	9,85361	9,99166	10,00834	33
28	9,84540	9,85349	9,99191	10,00809	32
29	9,84553	9,85337	9,99217	10,00783	31
30	9,84566	9,85324	9,99242	10,00758	30
31	9,84579	9,85312	9,99267	10,00733	29
32	9,84592	9,85299	9,99292	10,00707	28
33	9,84605	9,85287	9,99318	10,00682	27
34	9,84617	9,85274	9,99343	10,00657	26
35	9,84630	9,85262	9,99368	10,00632	25
36	9,84643	9,85250	9,99393	10,00606	24
37	9,84655	9,85237	9,99419	10,00581	23
38	9,84669	9,85225	9,99444	10,00556	22
39	9,84682	9,85212	9,99469	10,00531	21
40	9,84694	9,85200	9,99495	10,00505	20
41	9,84707	9,85187	9,99520	10,00480	19
42	9,84720	9,85175	9,99545	10,00455	18
43	9,84733	9,85162	9,99570	10,00429	17
44	9,84745	9,85150	9,99596	10,00404	16
45	9,84758	9,85137	9,99621	10,00379	15
46	9,84771	9,85125	9,99646	10,00354	14
47	9,84784	9,85112	9,99671	10,00328	13
48	9,84796	9,85100	9,99697	10,00303	12
49	9,84809	9,85087	9,99722	10,00278	11
50	9,84822	9,85074	9,99747	10,00253	10
51	9,84834	9,85062	9,99773	10,00207	9
52	9,84847	9,85049	9,99798	10,00202	8
53	9,84860	9,85037	9,99823	10,00177	7
54	9,84873	9,85024	9,99848	10,00152	6
55	9,84885	9,85012	9,99874	10,00126	5
56	9,84898	9,84999	9,99899	10,00101	4
57	9,84911	9,84986	9,99924	10,00076	3
58	9,84923	9,84974	9,99949	10,00050	2
59	9,84936	9,84961	9,99975	10,00025	1
60	9,84948	9,84948	10,00000	10,00000	0
	Co-fine	Sine	Co-tan.	Tangent	M

45 Degrees



A

T R E A T I S E

O F

M A R I N E F O R T I F I C A T I O N .

I. **B**Y MARINE FORTIFICATION, is meant the art of raising works fit for the defence of a harbour against the attacks of any kind of shipping.

As the works proper for this purpose depend in a great measure on the principles employed in the fortification of towns; therefore it will be necessary to give some general notions of the usual methods of fortifying, previous to the measures that seem most effectual to the securing of harbours.

It is not intended in this tract to give a compleat system of Fortification; and consequently the reader will not be perplexed with the different systems, as they are called, of *such* and *such* Engineers; many of them being only such persons, as having diverted themselves with conundrums and whimsical scratchings on paper, publish'd their conceits for a like amusement to others; and gravely compared their *pretty drawings* with the approved works of experienced Engineers: It is high time that most of their names and notions were forgot, and that those who hereafter may write on this subject, should confine themselves to the description of such works only, as are most in use, and which by experience have been found to be most conducive to the expected defence of the place, and retardation of the approaches of the enemy. And this is what is briefly attempted in the following pages.

P A R T I.

Of Land Fortifications.

2. FORTIFICATION is the art of making certain works about a place, whereby the persons within may be able to defend themselves against the attacks of a much greater number from without, more effectually and longer than if the place was not furnished with such works.

3. Defensive works usually consist of moats and banks covering the place which they defend.

4. A place is said to be covered, either when the enemy is prevented from coming to that place by a body of troops or other hindrance: Or, when a sufficiently strong bank raised before that place prevents those without from seeing them within.

5. Defensive works are of various kinds, the most simple are usually called lines.

S E C T I O N I.

Of Lines or Intrenchments.

6. A LINE, or an INTRENCHMENT, consists of a bank of earth and a ditch, and are of two sorts.

First, Such as the trenches carried on in a siege, where the bank is between the ditch and the enemy or town; and here the earth is thrown up against the town, because the workmen are better covered in carrying on the trenches.

Secondly, Such as serve to inclose a town or camp, secure a pass, cover the entrance into a country or other place, &c.; in these, the ditch is between the bank and the enemy.

The first sort are usually called trenches or approaches; and the second are generally called lines; which are the sort that will be here more particularly treated on.

7. Lines are generally made to shut up an avenue or entrance to some place; the sides of that entrance being rivers, woods, mountains, morasses, or other obstructions not easy to be passed by an army: Though there have been cases where lines were constructed in plain countries; but then they were carried quite round the place they were to defend, as the lines surrounding a camp, called lines of circumvallation. Lines are likewise thrown up to stop the progress of an army, as those made by Marshal Villars and others. However in what follows, it will be supposed that the line is to cover a pass that can only be attacked in front.

8. The making of lines greatly depends on the nature of the place, and the time that can be taken to construct them; those done at leisure being very different from those done in haste.

The

The works usually thrown up in haste are called *temporary*; they serving chiefly on a present emergency: Those made at leisure are generally intended to last many years, and are kept in repair under a probability of their being frequently attacked. But whether the work be temporary or lasting, it is necessary that it be adapted to the materials at hand, or that can be procured with the least expence and trouble.

A road or pass may be advantageously held by a few men against a much greater number, many ways; as by turning a water-course, planting of stakes, felling of trees, digging of ditches, making a fence of stones, &c. But the way now most usually practised is, by making a line after some such like method as contained in the following precepts.

9. *To throw up a Line across a pass.*

1st. Having chose the place judged most convenient for the line, let a rope, or haybands, or twisted straw or grass, be run quite across the way along the intended place of the line, pegging it to the ground at every 4 or 5 yards; and at the distance of about 10 or 12 feet before the line, towards the enemy, let such another line, or a row of stakes, be carried in a parallel position to the first rope.

2d. Range the labourers or troops within those limits, either single, in pairs, or treble, &c. according to the number there are to employ, or the time in which the work is to be done, each being furnished with proper digging tools.

3d. Let the earth, as they dig it up in this breadth, be thrown on the other side of the first rope, until a bank of about 5 or 6 feet thick and 6 or 7 feet high be raised; observing that the sides of the ditch be sloped in about the same manner as the earth naturally takes in rolling down the bank; and continue the digging till the ditch is about 5 or 6 feet deep, the breadth of the bottom being about $\frac{1}{3}$ of the breadth staked out at top: And while the ditch is digging, if the earth as it is thrown up be trod down, the bank will be more compact; or a few hand-rammers (such as the paviors use) kept constantly going, will render the work more firm or durable.

4th. Let the inner side of the bank be pared with the spade into such a slope, as a man standing upright may easily touch with his arm extended strait before him; and at the foot of this bank, let a *foot-bank* or step be raised, of such a height as a man standing on it may easily fire his musket over the bank; that is, let the top of the foot-bank be about four feet and a half lower than the top of the bank or *breast-work*.

5th. If there is time and opportunity, let the workmen throw up the loose and scattered earth upon the bank, on both sides, and flap or beat it with the flats of their shovels to make it the more neat and compact: Also make a gentle slope to the top of the *foot-bank*, that the troops may ascend it the more easily; and let the *crown* or *top* of the *breast-work* be sloped

sloped so, that a musket laid flat on it may strike the ground with its shot about 5 or 6 feet beyond the ditch.

When the soil is clay, sand, turf or mould, the workmen will need only spades and shovels: When gravelly or stony, then one man should break the ground with a pick-axe or mattock, while another throws it upon the bank, where a third should spread it level and tread it down.

10. A work of this kind being compleated, the troops behind the *line* may wait the coming of the enemy with less concern than without it: For 1st, the bank or breast-work secures them from the enemy's fire; 2d. And when they stand on the foot-bank they are more than $\frac{2}{3}$ covered, while the enemy are wholly exposed; and consequently the troops within may make three of their shots tell for one of the enemy's. 3d. They can, by going off the foot-bank, be quite covered while they load again. And on the whole, they are in no great danger of being forced from the lines while they have courage enough to keep possession of them, except the enemy are greatly superior in numbers and cannon.

The magnitude of lines are generally proportioned to the little time that can be spared to throw them up, and to the strength of the enemy who are to attack them. The following forts are what have been practised on several emergencies, and the rate at which men ordinarily work on such occasions.

II. *A TABLE of the dimensions of lines.*

Breast-work			Ditch			Expence	
Thickness at top	Height within	Height without	Upper breadth	Lower breadth	Depth	Solid content	Days works
4 feet	$7\frac{1}{2}$ feet	6 feet	8 ft.	$2\frac{2}{3}$ ft.	5 ft.	$4\frac{2}{3}$ feet	1
5	$7\frac{1}{2}$	6	10	$3\frac{1}{4}$	$5\frac{1}{2}$	$6\frac{1}{2}$	$1\frac{1}{2}$
6	$7\frac{1}{2}$	6	12	4	6	8	2
7	$7\frac{1}{2}$	6	14	$4\frac{2}{3}$	$6\frac{1}{2}$	10	$2\frac{1}{2}$
8	$7\frac{1}{2}$	6	16	$5\frac{1}{3}$	7	12	3
9	$7\frac{1}{2}$	6	18	6	$7\frac{1}{2}$	15	$3\frac{1}{2}$

The days works here, is for one yard in length.

In the first, 100 men will compleat 100 yards in length of this kind of lines in one day; 200 men in half a day; 400 men in a quarter of a day; and so on.

12. When lines are thrown up at leisure, then the ditch is usually 18 feet broad at top, 7 or 8 feet deep, and the sides of the ditch are so sloped as to leave only 6 feet breadth at bottom; the breast-work, or parapet, is about 7 feet thick on the top or crown, and 7 or 8 feet high.

In Plate XII. fig. 1. shews the heights, depths and breadths of the several parts of a line well designed and finished.

Where

Where IL represents the ground line, or surface of the place.

AB, the breadth of the ditch at the top.

CD, the breadth of the ditch at bottom.

FAC, is the slope or scarpe of the parapet and ditch.

DBK, is the counterscarpe.

EF, the top or crown of the parapet or breast-work.

EG, the inner slope of the parapet.

HG, the top of the foot-bank.

HI, the slope of the foot-bank.

BKL, a small sloping bank, called the glacis.

13. This figure is called a section or profile, and may be thus drawn.

1st. In the ground line, lay off from any scale of equal parts, the distances $Ia = 6$ feet, $ab = 4$ feet, $bc = 1\frac{1}{2}$ ft. $cd = 7$ ft. $dA = 4\frac{1}{2}$ ft. $Af = 6$ feet, $fg = 6$ feet, $gB = 6$ ft. $BL = 5$ ft.

2d. Thro' a, b, c, d, e, f, g, B , draw lines perpendicular to IL.

3d. Make $aH = 2\frac{1}{2}$ feet $= bG$, $cE = 7$ ft. $dF = 6$ ft. $fC = 8$ ft. $= gD$.

4th. Draw IH, HG, GE, EF, FAC, CD, DB, which continue till it meets the line FL, and the profile is constructed.

14. When lines are made to cover a camp or a large tract of land where a considerable body of troops are posted, the work is not made in one strait, or uniformly bending line: But at certain distances the lines project in salient angles towards the enemy. These projections, or salient angles are called REDENTS or FLANKERS; because the fire from them take the enemy in flank or sideways as they march to attack the lines.

The distance between the salient angles of the *flankers* is usually between the limits of 200 and 260 yards, the ordinary flight of a musket ball, point blank, being generally within those limits; altho' muskets a very little elevated will do effectual service at the distance of 360 yards.

15. A flanker consists of two lines called faces, meeting in a point, and forming an angle called the *salient angle*, or *flanked angle*.

That part of the line between two adjacent flankers is called the *curtin*.

The *gorge* of a flanker is the distance between its faces taken at their intersection with the curtin.

16. In Plate XII. fig. 2 shews the form of the usual lines; where the figures CAB, cab , are redents or flankers.

AC, AB, ac, ab , the faces.

CB, cb , the gorges.

AD, ad , the capitals.

Bb the curtin.

The angle CAB, cab , the salient or flanked angle.

The distance of the salient angles is about 240 yards on a mean.

The length of the capital is usually between 40 and 50 yards.

The length of the gorges is also about 60 or 70 yards.

17. To make a Plan of lines with redents. Plate XII. Fig. 3.

1st. Let the line EEEE, &c. be so drawn, if possible, that wherever there is a bend or angle, it may either be at once, or twice, or thrice, &c. the length of about 240 yards one from another, whereby a redent will always be where there is an angle.

2d. In this line lay off distances of 240 yards each, as from E to E, E to E, &c. reckoning from the bends towards each end, whether it happens that the line will, or will not be exactly measured by a repetition of the 240 yards.

3d. At each point E draw the *capital* EF in a perpendicular position to the direction of the line in that point, and make the *capitals* about 40 or 50 yards long.

4th. On each side of E take the *half gorge* EG, EG, each of about 30 or 35 yards, and draw the *faces* FG, FG; and the out-line, or *master-line* of the curtains and redents are formed.

5th. Parallel to each *curtin* and *face* draw lines within, at the distances from the *master-line* of 7 feet, 8 feet, 12 feet, and 18 feet.

Then the breadth of 7 feet represents the plan of the *parapet*.

of 1 foot

its inner slope.

of 4 feet

the top of the *foot-bank*.

of 6 feet

the foot-bank slope.

6th. On the outside of the *master-line*, draw lines at the distances of $10\frac{1}{2}$, $16\frac{1}{2}$, and $22\frac{1}{2}$ feet, parallel to each curtain and face; and these will represent the plans of the *scarpe*, *ditch*, and *counterscarpe*; observing that the salient angles of the counterscarpe are rounded before the angles of the redents.

When plans are drawn from a large scale, all the lines beforementioned are drawn: But when the plan is drawn from a small scale, as of 20 yards or more to an inch, then the plan is usually represented by four parallel lines; one without the master-line, representing the counterscarpe or out-line of the *ditch*; and two within, representing the breadths of the *parapet* and *foot-bank*.

Sometimes a short line is hastily formed by a *CHEVEAUX DE FRISE*; which is a beam of 6 or 8 inches square and 10 or 12 feet long; the sides being bored thro' with holes about an inch and half diameter, and 6 inches distant from one another; thro' each hole goes a staff of about 5 or 6 feet long, armed at both ends with an iron spike, so that which ever way it is laid along, one row of pikes is presented against the enemy: A line is formed of these pieces by chaining together a sufficient number of them end to end.

SECTION II.

18.

Of Batteries.

When lines are to be vigorously defended against a potent enemy, it is proper to raise *batteries*, that is, works from whence cannon may play on the troops as they advance to the attack: These batteries are either *open* or *covered*.

19. An OPEN BATTERY is nothing more than a number of cannon, generally *field pieces* (that is, such as carry a ball not exceeding 9 pounds weight) ranged in a line or row abreast of one another, on some natural small elevation of the ground; or on an artificial bank of about a yard or two high: These cannon are ranged at about 15 or 16 feet distant from one another, having their shot and loading utensils lying by their side, and the powder lodged in a hole at some distance behind the battery. A less distance than about 15 feet from gun to gun, might occasion danger, considering the hot service for which such batteries are erected.

20. A COVER'D BATTERY is when the cannon and gunners are covered by a bank made of brushwood, faggots and earth; of about 18 or 20 feet thick, and 7 or 8 feet high. The cannon used in such batteries are generally from 9 to 18 pounders; and sometimes 24 pounders are used in them.

21. The faggots, or, as they are most usually called, the FASCINES, are made of the middling sized loppings of trees, under an inch diameter, and are of three different lengths, 6, 9 and 12 feet, according to the work they are intended for: They are usually bound or tied round with wyths; the shortest in two places, the next in three, and the longest have four bands. These fascines are commonly 8 or 9 inches thick throughout, the ends being cut so as not to be tapering.

The fascines are fixed to the ground, and to one another by stakes called PICKETS, like birch broom-sticks, but stronger; they are about an inch, or inch and half thick, and from 3 to 6 feet long: The 6 feet ones are in number about $\frac{2}{3}$ of the 3 feet pickets, but much stronger.

All *field batteries* consist of four chief parts, namely, the *ditch*, the *parapet*, the *platform*, and the *magazine*.

22.

Of the Ditch.

The ditch is usually dug in the front and sides, and sometimes quite round; and serves not only to furnish the earth necessary for the parapet, but it also prevents the enemy from entering the battery so readily as he might otherwise do. The ditch in front is commonly 18 or 20 feet broad, and the depth about 7 or 8 feet; the sides sloping towards the bottom, which is about 6 feet wide: But the ditches on the sides are about 10 feet wide, and 6 feet deep. These are the dimensions usually given; but the general practice seems to be, only to get earth sufficient for the work, and not mind the regularity of the ditch: For as these works in attacks are usually done by night, the keeping strictly to the assigned dimensions is hardly practicable.

23.

Of the Parapet.

The parapet is generally raised about 3 or 4 feet distant from the brink of the ditch, the space left between, called the BERM or FORELAND, serving to lodge the rubbish beat down by the enemy's shot, that it may not fall into the ditch.

The thickness of the parapet should be about 18 or 20 feet, in order to be cannon proof, and about 7 or 8 feet high when the enemy has no command above the battery: But if they have a command over this height, the parapet should be raised high enough to cover the men while they load the guns.

The length of the parapet depends on the number of guns to be employed in the battery.

24. Thus for one gun, allow 8 yards in length.

And 6 yards more for every other gun.

So 2 guns has 14 yards; 3 guns 20 yards; 4 guns 26 yards, &c.

25. There should be great care taken that the battery be not *enfiladed* or *raked* by the enemy's cannon: Neglects of this kind are often fatal to the officers and gunners, as was well known at the siege of Carthage, anno 1741. But this is easily prevented by raising at one, or both ends of the parapet, a bank across the battery called an *epaulment*, of the same thickness and height with the parapet, and about 15 or 18 feet long.

26. The parapet consists of two parts, namely, the *wall* and the *merlons*.

The WALL is that part of the parapet which is contained in one piece from end to end, and is about $2\frac{1}{2}$ or 3 feet high.

The MERLONS are detached pieces of the parapet, leaving openings called EMBRASURES, thro' which the cannon deliver their shot.

The embrasures should as much as is possible be cut perpendicular to the parapet; therefore the battery should be parallel, or nearly so, to the object to be battered: For the direct shots have most force; and oblique embrasures weaken the merlons, or parts of the parapet standing between the embrasures.

27. The embrasures are usually about 2 or $2\frac{1}{2}$ feet wide on the inside, and about 9 or 10 feet on the outside; whereby the cannon may be traversed from the right to the left, so as to command a pretty large extent in front. The distance from the middle of one embrasure to the middle of the next should be about 18 feet, in order to leave sufficient room for the working of the guns, and the stowage of the shot and other necessaries.

28.

Of the Platform.

The PLATFORM is a floor made to facilitate the rolling of the carriage wheels, and to prevent them from sinking into the ground by the weight of the cannon, especially in wet weather.

Platforms are generally laid sloping towards the parapet 9 or 10 inches; this carries off the rain, prevents the gun from recoiling so much when fired as it would do if laid level; and when loaded, it is more easily brought to the embrasure.

29. In

29. In temporary batteries, the platforms are made of planks laid across some ground timbers or sleepers: There is usually a platform made to each gun; it is about 18 feet long, 8 feet broad next the parapet, and about 14 feet broad at the tail, the intermediate spaces between the platforms serving for the shot and other necessities.

30. When a platform is to be laid on marshy ground; first lay a floor or two of fascines; cover these with *burdles* of 12 or 15 feet long, and 6 or 7 broad; on these lay a floor of 3 or 4 inches of earth, and therein lay the sleepers, and over them the planks.

31. When a battery is built of stone or brick, the platform is generally a flat-stone pavement ranging the whole length of the battery: This, on account of its resisting the injuries of the weather for a long time, is to be preferred to planks; but in case of a bombardment such a platform is to be avoided, because the shells will not only break the pavement, but also by driving about the broken stones, do the troops much mischief.

32.

Of the Magazine.

The magazine to a field battery is usually made about 50 or 60 yards behind the platform: It is a cavity dug in the ground about 4 feet deep; the earth whereof is thrown between the pit and the platform; the sides of the pit are sometimes planked round to keep it dry, and prevent the earth from crumbling in; and the powder barrels placed herein are covered with hurdles and earth, or tanned hides, to preserve the powder from wet and fire.

The communication to the magazine is by a sloping trench beginning to descend about 5 or 6 yards behind the platform; and the earth thrown on that side where it will most conveniently cover the persons who remove the barrels of powder from the great magazine to the battery, or small magazine.

33. When there are many cannon in battery, and the service is quick, it is customary to have to every two pieces a small magazine to hold 20 or 30 barrels of powder: This is placed about 15 or 20 yards behind the platform, and against the merlon between the cannon; and as these barrels are used, they are replaced by others from the great magazine.

At each magazine a centinel is placed to prevent accidents.

And to prevent persons coming into the battery and magazines who have no business there, a trench is sometimes dug behind the magazine, and carried into the trenches which communicate between the magazine and battery.

34.

To construct a Fascine battery.

For one gun provide fascines of 9 feet, 600; of 6 feet, 100; of 3 feet, 120.

And for every other piece 400; 100; 100;
each fascine being about 9 inches thick; and let there be three or four pickets for every fascine, for many will break in driving.

Trace the limits of the parapet in two parallel lines about 18 feet distant, allowing 8 yards in length for one gun, and 6 yards for every other gun; and along these lines cut a trench about half spade deep.

Lay a row of 9 feet fascines along one trench, observing that their ends be well jammed one into the other, and let each be pegg'd down with two pickets, drove into the ground till the head is sunk into the fascine.

Close to this row lay another, letting the two end ones be of 6 feet, and the rest 9 feet, whereby the joinings in this row will not fall against the joinings in the first row: Let these be also staked down as before. Close to the second row lay a third all of 9 feet. Close to the third lay a fourth, the two end ones being 6 feet, and the rest 9 feet. In this manner lay the rows of fascines within the limits staked out, which will be covered with 24 rows; then over this floor of fascines sprinkle earth to fill up the hollows, and let the whole be trod or rammed down.

On the first floor lay a second, observing, that as in the first floor every second row ended with 6 feet fascines, so in the second floor let every odd row end with 6 feet ones, whereby the joinings of no two fascines in this floor will fall over the joining of the fascines below them. Let the outside rows in the second floor be so placed over those in the lower, that there be a little sloping preserved, the work narrowing upwards. The pickets which stake down this floor are to be drove up to the head, and the channels or hollows in the floor are to be filled up with earth. In this manner lay four floors, which will raise the work to about $2\frac{1}{2}$ or 3 feet; and that part of the breast-work called the wall will be compleated.

35.

To stake out the Merlons.

Measure from each end of the wall 12 feet, there stick a stake; and plant other stakes at every intermediate 18 feet: This being done on the inside of the wall, let other stakes be planted on the outside, either directly opposite the former, or in the line towards the place where the gun is more particularly intended to deliver its shot. Plant other stakes on the inside, one a foot distant on each side the former, and this will leave spaces of two feet each for the inner opening of the embrasures: Then on the outside, plant other stakes at 5 or 6 feet distance from the former ones, one on each side, and spaces of 10 or 12 feet will be marked out for the outside openings of the embrasures.

In the direction of the pickets which limit the inner and outer openings of the embrasures, let single rows of fascines be staked down across the wall, and these will be the sides of the embrasures: Fill the intermediate spaces or merlons with rows of fascines laid lengthwise to the wall, and this will be the first floor of the merlon, which is to be picketted down, and the hollows filled with earth as before directed. Let other floors be raised in like manner, until the merlons are carried up to about 5 or 6 feet, or more if necessary; and on the top of each let a bed or floor of earth be laid of about 8 or 12 inches thick.

Across the top of the inner opening of the embrasure, either lay a bundle of 10 or 12 fascines bound together, or as many stuff into a kind of basket open at both ends, and the bundle well staked to the merlons.

Provide a *blind* made of planks 3 or 4 inches thick, to fit the inner opening of the embrasure, which is to be put up while the gunners are loading the piece, to preserve them from the fire of the musketry.

The

The cheeks or sides of the embrasures are to slope so, as to have the top wider than the bottom; which not only leaves a greater opening for the blast of the gun, but these slopes in works of this kind give a greater degree of strength to support themselves against the thrust of the upper parts.

36. *To construct the Platforms.*

Against each embrasure lay on the ground five pieces of timber called sleepers, of about 18 feet long and 6 or 8 inches square; let the ends next the embrasure be placed at such a distance from one another, that the distance from out to out be about 6 feet, the other ends spreading about 12 or 14 feet from out to out; drive two stakes of about 3 inches square into the ground on both sides of each end of every sleeper, whereby they will be kept steady in their places, and let the earth be well rammed and beat up close to them: But observe to let the tails of the sleepers be raised about 6 or 8 inches higher than the ends next the wall, to prevent the cannon from recoiling too much when fired.

Cover the sleepers with planks of about 2 inches thick laid crosswise; that next the embrasures being about 8 feet long, the rest gradually increasing, so that the plank at the tail be about 14 feet long, and let the planks be nailed to the sleepers.

On this floor fix a piece of timber of about 6 or 8 inches square next the embrasure, to serve as a spur for the carriage wheels to knock against (and is therefore usually called the *knocker*), when the gun is run up to the wall; observing that the spur be laid parallel to the object to be battered.

The whole platform should be of oak, if it can be had.

37. If there is any danger of the battery's being raked by the enemy's cannon, let an *epaulment* or *screen* be raised at one or both ends of the parapet, joining to it, and constructed by floors of fascines as before shewn: Then all the stragling twigs of the fascines being cut off, the battery is constructed.

A battery thus made will do in case of haste, and where earth enough is not easily had: But as it is subject to be fired when the wood is dry, it is not so much in use as another fort called a *Coffer battery*; especially where wood is scarce and earth plenty.

A *COFFER BATTERY* is that where the sides of the wall and merlons only are formed of fascines, and all the cavities or included spaces filled with earth.

38. *To make a Coffer battery.*

The place of the battery being determined, mark out with a line of some kind, the limits of the parapet of about 18 or 20 feet thick; and 3 or 4 feet before the parapet, mark out with lines or stakes, the limits of the ditch of about 10 or 12 feet broad, or more if earth is wanted; allowing 8 yards in length for one gun, and 6 yards more for every other gun.

On the outlines of the parapet, cut a trench of about 5 or 6 inches wide and deep, and therein lay a row of fascines, the ends being jammed one into the other; let these be staked down: Lay on them another row,

so that the joinings of these be not directly over the joinings of the lower one, and the knots of all the bands turned inwards, stake these down; and on them lay in like manner a third and a fourth row, &c. until the height be about 3 feet. The same kind of work being done at the ends, and for the epaulment if wanted, the coffer for the wall will be made.

Then let the men be disposed along the place intended for the ditch, and with the proper tools break the ground and throw it into the coffer; where another set of men are, as the earth is thrown in, to spread it and stamp it down with rammers; and this work is to be continued until the coffer is filled.

39. When the wall is finished, let the embrasures be staked out as before (35), and a coffer formed in like manner, as for the wall, for each merlon, which is also to be filled with earth and rammed down.

The other articles in the preceding battery are to be followed in this, the only difference being in the making of the parapet.

When a proper place for a temporary battery is destitute both of dry earth and wood, then materials must be carried to the place; these usually are *gabions* and *earth-sacks*.

The EARTH-SACKS are only bags to carry earth in.

A GABION is a large basket without a bottom: They are of various sizes, as from 3 feet high and $2\frac{1}{2}$ or 3 feet diameter, to 6 or 8 feet high and 6 or 7 feet diameter.

40. *To make a Gabion battery.*

Along the line pitch'd out for the battery, let the gabions be planted in the places where the merlons are to be: The gabions used are of 5, 6 and 7 feet diameter, and 8 feet high. Each merlon must have 7; that is, 3 within of 6 feet diameter, next two of 7 feet diameter, and on the outside two of 5 feet diameter; observing to leave proper openings for the embrasures, of about 2 feet on the inside, and 9 or 10 on the outside.

Or thus: Let the merlons be made of gabions 5 feet diameter; then put four within, three in the middle, and two on the outside; this construction being rather stronger than the former.

The ends and epaulments are also formed by three rows of gabions.

The floors or bottoms of the embrasures are to be filled with gabions of about three feet high, and of a proper diameter to fill up the spaces between the merlons.

The gabions being placed, they are to be filled with earth brought, from the nearest places that will afford it, in the earth-sacks; or else they may be filled with dung mingled with sand; and in cases of necessity, they may be filled with large faggots, or billet wood, observing that the voids between the gabions are also filled.

Batteries of this construction are usually made on marshy or rocky ground.

41. The *Sieur Remy*, in his *Memoirs of Artillery*, has given a table for the ready finding of all the requisites for the construction of temporary batteries, and for their daily service, the pieces being 24 pounders; and although these batteries are calculated only for sieges, and are of the coffer-kind, yet from this table may be derived such notions as will greatly help young artists on other occasions.

It is also proper to mention the number of fascines and pickets that is usually expected every day from the labour of each man employed in that service.

Fascines of 5 or 6 feet long, and 5 or 6 inches thick, bound with two wyth bands each, one man will make 16 or 18 in a day, with two pickets to each.

Fascines of 8 or 9 feet long by 8 or 9 inches thick, and two pickets to each, one man usually makes 10 or 12 in a day.

Fascines of 12 feet long by 9 inches thick, with three pickets to each, 8 or 10 such are usually expected for one man's day's work.

The days here understood are such wherein the men may work about 12 hours.

In the following table C stands for hundred.

42. *A TABLE for constructing of Batteries.*

Number of 24 pounders in battery.	Length of the parapet in yards.	Number of workmen to construct the battery.	Number of workmens tools used at the battery, viz. shovels, spades, pick axes, mattocks, &c.	Fascine-makers, each a bill and hatchet.	Fascines of 8 or 9 feet by 8 or 9 inches.	Fascines of 12 feet by 8 or 9 inches.	Fascines of 5 or 6 feet by 5 or 6 inches, made by the cavalry.	Pickets from 3 to 6 feet long, and about 1½ inch thick.	Mallets to drive the pickets.	Handbills, 2 for each embrasure, beside 1 spare hatchet.	Planks for the platforms, of 2 or 2½ inches thick.	Gunners to serve in the battery.	Soldiers to assist the gunners.	Powder for 100 rounds, at 12 lb. each, per day.	Shot of 24 pounds for 100 rounds.
2	14	50	70	15	120	40	2 C	520	10	4	32	4	12	24 C	2 C
3	20	60	85	20	165	60	3 C	740	14	6	48	6	18	36 C	3 C
4	26	70	100	25	210	80	4 C	960	18	8	64	8	24	48 C	4 C
5	32	80	115	30	255	100	5 C	1180	22	10	80	10	30	60 C	5 C
6	38	90	130	35	300	120	6 C	1400	26	12	96	12	36	72 C	6 C
7	44	100	145	40	345	140	7 C	1620	30	14	112	14	42	84 C	7 C
8	50	110	160	45	390	160	8 C	1840	34	16	128	16	48	96 C	8 C
9	56	120	175	50	435	180	9 C	2060	38	18	144	18	54	108 C	9 C
10	62	130	190	55	480	200	10 C	2280	42	20	160	20	60	120 C	10 C
11	68	140	205	60	525	220	11 C	2500	46	22	176	22	66	132 C	11 C
12	74	150	220	65	570	240	12 C	2720	50	24	192	24	72	144 C	12 C
13	80	160	235	70	615	260	13 C	2940	54	26	208	26	78	156 C	13 C
14	86	170	250	75	660	280	14 C	3160	58	28	224	28	84	168 C	14 C
15	92	180	265	80	705	300	15 C	3380	62	30	240	30	90	180 C	15 C
16	98	190	280	85	750	320	16 C	3600	66	32	256	32	96	192 C	16 C

43. When

43. When batteries are erected at leisure, and wanted to stand for some years, then it is best to make them of stone or brick, or good loamy earth, according as the materials are nearest at hand, or can be most easily procured.

When made of stone, the thickness of the parapet is usually about 4 or 5 feet: But this thickness is varied according to the quality of the stone, or to the size of the cannon that may be brought by an enemy to fire on it.

Parapets of brick, are of 5, 6, 8, or even 10 feet thickness, according to the goodness of the materials, or the service it is to endure; and the angles are generally of stone.

A parapet made of good earth, and 18 or 20 feet thick, with the surface covered with green turf, will resist the injuries of the weather for several years, and stand the shock of an enemy's cannon the best of any.

44. As these perpetual batteries are generally raised somewhat above the level of the adjacent land or water which they are to command; so it is usual to make the floor of the embrasures sloping, the drip being on the outside, about a foot or foot and half below the sill of the inner opening, whereby the cannon may be pointed downwards.

When the embrasures are cut thro' an earthen parapet, they are commonly lined with brick; or at least the angles are made of masonry, these parts being the most subject to be injured by the weather.

The best embrasures are those made with a neck, whereby a part of the opening becomes more contracted, and consequently is better adapted to preserve the gunners and cannon from the enemy's shot, than when the cheeks of the embrasures are made strait.

45. Altho' the axis or middle line of an embrasure should, if possible, be at right angles to the length of the parapet, yet it may sometimes be necessary to have them oblique: When this happens, the inside of the parapet is notched, or so cut away, to leave proper room for the front of the platform, that the wheels of the carriage may be drawn up against the inner opening for the axis of the gun to lie in the axis of the embrasure: But then the outside of the parapet is to be strengthened, to compensate for what is cut away within.

The foregoing particulars, relating to embrasures, will be sufficiently illustrated by the following precepts for their construction.

46. *To construct the Plans of embrasures.*

FIRST. *For the Plain embrasure.* Plate XII. Fig. 4.

- 1st. Draw the axis AB perpendicular to the parapet.
- 2d. For the front opening, make $AC = AD$, each from 4 to 5 feet.
- 3d. For the inner opening, make $BF = BE$, each from $1\frac{1}{2}$ to 2 feet.
- 4th. Draw FD, EC, for the cheeks of the embrasure required.

47. SE-

47. SECONDLY. *For the neck'd embrasures.* Plate XII. Fig. 5.

1st. Having drawn the cheeks for a plain embrasure as before; then from B, the middle of the inner opening, draw BC, BD.

2d. In the axis AB, take $BG = 2\frac{1}{2}$ or 3 feet; thro' G draw IH at right angles to BA, cutting BC, BD, in H, I, and join EH, FI.

Then DIF, CHE, are the angular cheeks, IH being the neck or narrowing of the embrasure.

48. THIRDLY. *For an oblique embrasure.* Plate XII. Fig. 6.

1st. Draw the axis *ab* in the direction where the gun is to command, and thro' any convenient point *i*, taken in *ab*, draw NN at right angles to *ab*.

2d. Make $IN = 4$ or 5 feet for the front opening, $IP = 1\frac{1}{4}$ or 2 feet for the inner opening, and $IQ = 4$ feet for the front of the platform.

3d. Thro' N, P, Q, and parallel to *ab*, draw NC, ND, cutting the front of the parapet in c, d; and Pp, Qq, cutting the inside of the parapet in q, q.

4th. Thro' q draw *qr* parallel to NN, cutting the former lines in F, B, E, r; join ED, FC, and draw *rs* parallel to a line thro' B and c.

Then the space *qrs* being cut away, the wheels of the carriage can knock against the line *qr*, and the axis of the gun will lie in the line *Ba* intended for the axis of the embrasure.

It is evident that the parapet is considerably weakened by the cuts for these oblique embrasures, and therefore should be strengthened on the outside.

49. *When the embrasures are to be expressed in the parapets of a plan drawn from a small scale, it is done thus.* Plate XII. Fig. 8.

On the inside of the parapet RS, lay off as many times 6 yards, in the points *c, c, c*, &c. as the length of the parapet will admit, unless restrained by particular circumstances; and from the points *c, c, c*, &c. draw lines to the front of the parapet, dividing it into parts of 3 yards each; then taking the parts opposite the points *c* for the fronts of the embrasures, the other parts will represent the fronts of the merlons.

Shade all the merlons, leaving white angular spaces to express the embrasures, and strengthen the inner line of the parapet and the front lines of the merlons.

50. *Of the Profiles of batteries.* Plate XII. Fig. 7.

Let the ground line of the battery be AB, BD that of the parapet, whose inner slope DH, is formed by making $Da = 1\frac{1}{2}$ feet, and the perpendicular $aH = 6$ or 7 feet; the crown of the parapet HI is formed by making *bI* a foot or two lower than *aH*; and the front of the battery IB is found by making $bB = \frac{2}{3} bI$, when of earth, or $= \frac{1}{3}$ of *bI* when of masonry. Make $DC = 2\frac{1}{2}$ or 3 feet, gives c the fill of the embrasure, whose floor CG is to drip a foot or two below the level line CF.

The platform DE is 18 or 20 foot, the tail E rising about 6 inches above the level line AB, the lower double line represents the sleeper laid lengthwise, and the upper double line shaded with the lines across, express the ends of the planks laid on the sleepers.

A gun on its carriage, with the wheels against the knocker at D, is annexed to the figure, to strengthen the learner's notions.

K k k k

SECTION

SECTION III.

Of additional works.

51.

Of Ramps.

When a battery is raised above the level of the ground it is built on, there must be made gentle slopes for the cannon to be drawn up and down by, and also for the easy communication of the troops posted there; these slopes, to distinguish them from the slope usually given to elevated works for their better security of standing, are called *Ramps*.

The base of the slope commonly given to earth-banks for their support, is on the inside about a foot base for a foot of height, and on the outside is about 8 inches on a foot, or $\frac{2}{3}$ of the height; that is, they rise 12 inches, on 8 inches of base: But for the slopes of ramps, the rise is only about 2 inches on 12, or the length of the ramp's base is six times the height, and this is general, for the draught of carriages: But foot-ways need not be of so gentle a slope, a rise of one foot in three may do well enough; or, instead of ramps, stairs may be, and commonly are, used for the passage of the foot.

The breadth of a carriage-ramp is usually about 9 or 10 feet, that breadth being sufficient for the carriage and foot-way beside: But those for foot-passage only, need not be above 3 or 4 feet wide.

Ramps may either rise on the side of an elevated work, or against a salient angle of that work; their representations in plans may be expressed as in the following constructions.

52. *To construct a Ramp on the side of a bank.* Plate XII. Fig. 9.

Let xx , zz , be lines expressing in a plan, the slope of a bank equal to its height, which suppose to be 12 feet.

1st. Thro' A , the point assigned where the ramp is to end, draw CB perpendicular to xx , and make AC of about 10 feet.

2d. In zz take $BD = 4$ times AB , or 8 yards; thro' D draw AE , and thro' C draw CF parallel to AE , cutting xx in G , and zz in F .

3d. Thro' F draw, at right angles to GF , the line fE cutting xx in f , and AE in E ; join BE , and the lines GF , EA , EB express the ramp.

It is sometimes proper to have ramps rise on either hand for the convenience of carriages coming from the right or left; but they must then be so far asunder, that should a carriage be drawing up each at the same time, the horses of one may pass clear of those of the other.

To construct a Ramp in a salient angle of a bank. Plate XII. Fig. 10.

53. FIRST METHOD. This is done in the same manner as in the side of a bank; observing that the end A is taken 6 or 7 yards from the salient angle, when a ramp is to be constructed on both sides; otherwise the ramp may end at the salient angle x .

54. SE-

54. SECOND METHOD. Fig. 11. 1st. From x draw a line xN bisecting the salient angle; make $xN = 6$ times the height, or equal to 12 yards, and on both sides draw a parallel line QR at the distance from xN of half the breadth of the way; suppose at 5 feet distance from xN .

2d. Bisect the angle xQR with the line QY , cutting the line zz in Y ; join YR , and the plan is formed.

55. THIRD METHOD. Fig. 12. From x the salient angle, with a radius of 6 times the height, suppose of 12 yards, describe the arc LL ; and with 5 times the height, or 10 yards, describe the arc KK , cutting ZI in O , and draw OP parallel to KL .

Make $xM = 2$ or 3 yards, and draw the curved lines MO , MP , which in plans are best done by hand; or else draw right lines from M to O , and from M to P , and the plan of the ramp is made.

56. Of Barbets.

It often happens that by raising the floor of part of a battery, the guns placed thereon have an advantageous command over some part of the neighbourhood; and when the guns thus raised fire over the crown of the parapet without any embrasures, it is called an OPEN BATTERY, or *Battery en barbe*, or plainly a BARBET.

These barbets may be made either in a curtain, or at the salient angle of a flanker: They always should be $2\frac{1}{2}$ or 3 feet lower than the crown of the parapet, be about 8 or 9 yards broad at top, with a proper slope to the base, of a length proper for the number of guns to be mounted on them, allowing about 5 or 6 yards for each, and ascended at each end by a proper ramp.

57. To give a farther illustration to this article, here is added an explanation of the plan, and of the profile thro' the line BC . Plate XII. Fig. 13, 14, 15.

Let $PQRVX$ be a common bank of a line, whose parapet is $RSTV$; the inner slope RS being about 6 or 7 feet higher than QR : Then the bank $mnoR$, raised so high that the cannon can fire over the crown of the parapet ST , is the *barbet*, whose height np is about 3 or 4 feet.

On the top of the barbet a platform is raised as in other batteries.

Let the figs. 14, 15, represent part of the plan of a line, and one of its flankers, or of a battery constructed in such a form; where Aa is the length of the barbet or raised battery, suited to the number of guns to be used, which are to be drawn up the ramps put at the ends; the breadths being about 8 or 9 feet, and the length ab about 7 or 8 yards.

58. Of Cavaliers.

A cavalier is a battery raised above the other works, and covered by a parapet with embrasures.

Cavaliers are of various shapes, such as square (Plate XII.) fig. 17, or round, fig. 16, or like a horse-shoe, or a plain line; but when they are constructed in flankers, as is usually the case, fig. 18, they are of a

K k k k 2

like

like figure to that flanker; leaving a space of about 8 or 10 yards, or more, between the parapet of the flanker and the outline of the cavalier.

They should be faced with earth, or plank or brick, but not with stone, because the splinters knock'd off by the enemy's cannon would greatly annoy the troops that may be posted in the line below the cavalier.

Any other particulars will be sufficiently known from the figures; observing, in the plans, to represent their outside limits by a double line, to express the sloping of the earth, and to put a ramp in the gorge.

59.

Of Traverses.

A traverse is a bank of earth thrown perpendicularly across a line, or other work, to prevent the enemy's cannon from raking that line.

These traverses may be from 12 to 18 feet thick, in order to be cannon proof, and their height about 6 or 7 feet, or more if the line is exposed to any eminence from whence the enemy has a command.

And to preserve a communication, a passage of about 5 or 6 feet wide must be left at one end of the traverse.

60. The different ways of constructing these works are readily seen by the figures in Plate XII. fig. 19, 20, 21, 22.

Thus at A, fig. 19, it is formed by two equal short banks, whose ends jut one beyond the other about 3 or 4 feet, leaving a passage of about 6 feet.

At B, C, and figs. 20, 21, the end of the traverse juts about 2 or 3 feet into the parapet, beside the passage: But as this greatly weakens the parapet, it is to be practised only in such places where there is sufficient thickness left; and where nothing can be spared from the parapet, the traverse may be made as at E and F, fig. 22.

If any part of a work, thus shut in by one or more traverses, is likely to be defended by the musketry, then it is proper to add to the traverse one or more foot-banks within the defence, for the troops to mount on when they want to fire over the traverse, leaving the upper foot-bank $4\frac{1}{2}$ feet below the crown of the traverse.

61.

Of Palisades.

Palisades are strait wooden stakes of about 9 or 10 feet long and about the thickness of a middling sized leg, but of a triangular or rectangular shape, and have one of their ends pointed. Their use is to form a line before those places that are most likely to be taken by surprize.

The best are those made of oak, but where this is wanted, or on emergent occasions, the best are those which can most conveniently be procured.

In planting of palisades, it is usual to sink about three feet of them into the ground, the thick end downwards, and to set them about four inches from one another; the whole is braced together by pieces nailed across them near the tops, and secured by thick posts at the distance of every four or five yards.

Palisades

Palisades are found to be a very good defence against an enemy not greatly superior, and destitute of cannon; for the defendants may freely use their small arms at the time they are half covered by the palisades; whereas the assailants being quite exposed, attack with the disadvantage of, at least, one to two.

62.

Of Barriers.

A barrier is a kind of fence or gate serving to shut up the roads or passages in works of any kind, to prevent their being carried by surprise, and are of various kinds, as

1st. A strong chain drawn across the passage, at about $3\frac{1}{2}$ or 4 feet above the ground, and fastened at each end to an upright post.

2d. A beam laid horizontally across two upright posts.

3d. A BAR-GATE hanging by hinges to an upright post.

4th. A CHEVEAUX DE FRISE, which is a horizontal beam usually cut into 6 sides, each whereof is armed with iron spikes set within five or six inches one of the other: This beam is supported at each end by an upright post.

5th. A TURNSTILE which consists of two or three pieces of timber framed crosswise, making four or six rays like the spokes of a wheel; this frame is put on a post in the middle of a passage to turn horizontally, so that two of its rays always stretch across the passage, and prevent more than one person at a time from passing the same way.

6th. A HERRISON, which is a cheveaux de frise turning horizontally on a post in the middle of a passage like a turnstile.

7th. KLINKETS, are strong doors or gates made like palisades, and are hung and fastened as other gates are.

63.

Of Redouts.

A REDOUT is usually a temporary work, thrown up to strengthen the lines which shut up, or secure a *pass*: They are usually figures of three, four, five or six sides, encompassed with a ditch and a bank of earth.

The bank encompassing such works generally consists of two parts, called *rampart* and *parapet*.

64. The RAMPART is a large bank surrounding a work or place, of a height sufficient to cover the habitable parts of the houses (if any) from the direct fire of the enemy; or to raise the defendants so, as to command the neighbourhood; and of a thickness sufficient to resist the efforts of an enemy's cannon for some time.

65. The PARAPET, or breast-work, is a bank raised on the rampart close to its outer edge, to cover and defend the troops posted behind it, on the rampart.

66. On those sides of the work likely to be attacked, the parapet is cut into embrasures and merlons, of the same size and shape as has been shewn at arts. 35, 46.

Against

Against every merlon, a *foot-bank* is raised on the inside for the musketry to stand on when they are to fire over the crown of the parapet.

To every embrasure a platform is to be fitted, either of planks, as shewn in art. 36, or of stone, as described in art. 43.

When these redouts are intended to defend lines or intrenchments, their faces or fronts should be so placed, that the shot discharged from them may fly directly to the front of those lines; for hereby the enemy may be so galled in flank as to find it necessary to attack the redouts, and thereby either weaken his attack on the lines, or cause him to lose time: But when redouts are built more particularly to defend the spot whereon they stand, their faces must then front the places from whence they may be battered, and also command the ways by which they may be approached.

67. The inner sides of square redouts are usually between the limits of 12 yards and 32 yards; and when they are to be defended by musketry, the number of men necessary to the defence may be thus determined.

RULE. *Half the side squared gives the number of troops.*

EXAM. In a square redout, one of whose sides within is 24 yards: Required the number of troops necessary for its defence?

Half of 24 is 12: Then the square of 12 is 144 for the number of men.

AND, Twice the square root of a given number of men, shews the length in yards of the side of a square redout proper to contain them.

Thus to contain 100 men, whose square root is 10, the side of the square redout must be 20 yards.

68. *To construct a Square redout.* Plate XII. Fig. 23.

1st. Mark out a square whose side is adapted to the number of troops that may be allotted for the defence; as AB for the inside of the rampart.

2d. About this square, at the distance of 10 or 12 feet, describe another square, whose side CD is the inner boundary of the parapet.

3d. Make a parapet of about 9 or 10 feet thick, whose outline is the line EF; leave a berm of about 3 or 4 feet broad, whose side is GH, and dig a ditch of about 16 feet wide, and about 6 or 7 feet deep, which let be rounded before the angles of the redout.

4th. Make the rampart from 4 feet to 9 or 10 feet high, according to circumstances, let the parapet be 6 or 7 feet higher, and let the foot-bank be $4\frac{1}{2}$ feet lower than the crown of the parapet.

5th. On that side most secure from the enemy make a bridge across the ditch, and a passage thro' the rampart: The breadth of this way should be about 4 or 5 feet, if the defence is to be musketry only; but 9 or 10 feet if cannon are to be used; and in either case the passage is to be shut up by a strong gate.

But if the redout is to be defended by cannon, both the rampart and parapet should be at least 5 or 6 feet thicker.

69. As

69. As it is customary for troops to present their breasts to the parapet, and then fire direct before them, there will be a very considerable space before each angle of the work, that will be in a great measure exempt from the shot: To remedy this defect in these works, it has been proposed to round the inside of the angles of the parapet, and post three or four men there to defend the angle, for more cannot well be applied: But this appearing also defective, Mr. Clairac has shewn an ingenious method to make the fire pretty near equal in all parts; which is done by cutting the inside of the parapet into notches, whose two sides, of a yard each, are at right angles to one another, and make half right angles with the sides of the work; the manner whereof, and its defence, will be plain from seeing the figure 23, where the lines with dots at the ends represent the fire three different ways from the same side.

There are in many Treatises of Fortification directions to make a kind of temporary works called SCONCES, or FIELD-FORTS; some whereof, which are pretty well contrived for defence, will be here explained; but one species of sconces, called STAR-FORTS, are such *wretched baubles*, that nothing more will be mentioned of them here, than to recommend their plans as pretty things to stick on childrens kites.

70. *To construct a Flanked redout.* Plate XII. Figs. 24, 25, 26.

Having described the inner figure, where AB is an inner side of the redout, suppose of about 18 yards.

Parallel to this figure, at the distance of about four or five yards from it (or more if necessary), describe an outward figure, whose sides are ED, EF, &c.

Divide each side, as ED, EF, &c. into three equal parts, one whereof is EI, FL, &c.; and in these sides prolonged, take DG, EH, &c. each equal to one of those three parts.

From the points G, H, &c. thus found, draw the lines of defence to the angles of the outward figure, as HD, &c.

From the points I, L, &c. draw the flanks IK, LM, &c. perpendicular to the sides ED, EF, &c. meeting the lines of defence in K, M, &c.

Or, The flanks may be thus drawn.

In the line DI continued towards E, take IP equal to twice IE. And from P, as a centre with the radius PI, cut the line of defence in K, and draw the flank IK.

Then to each face (HK), flank (KI) and curtain (IG), make a parapet of about 7 feet thick (or more if necessary), which is expressed in the plan by drawing lines at that distance on the inside of the several parts.

Leave a proper berm of about 4 or 5 feet broad, and make a ditch of about 5 or 6 yards wide, whose outer line or counterscarp is to be parallel to the faces and curtains only.

A redout thus constructed is more capable of defence than one without flanks: For the curtain IG is well defended by the flank IK; and the face HK has some defence from the part DG.

71.

Of Redans.

A REDAN is a fortified line, broke into several curtains and flanks; the defences of the flanks being all directed towards one way.

This is a very good work for the defence of any passage; such as a hollow way, a road, a river, or a harbour's entrance.

If the defence is chiefly intended for cannon, the distance of the flanks from one another should not exceed 300 yards, that being about the point-blank flight of a musket shot: But if the defence is chiefly intended for musketry, the distance of the flanks may be about 50 yards or less. In general, the distance of the flanks should be regulated according to the number of them intended to be placed within the length of the line. The length of each flank should be about 15 or 18 feet, in order to admit one gun; but if more cannon than one are to be put in a flank, then the flanks must be made proportionally longer.

And if the flanks are made about two or three feet lower than the curtains, there will then be no danger of the troops, posted in the saliant angle of one flank, being hit by those in the entring angle of the flank next behind.

72.

To construct a Redan.

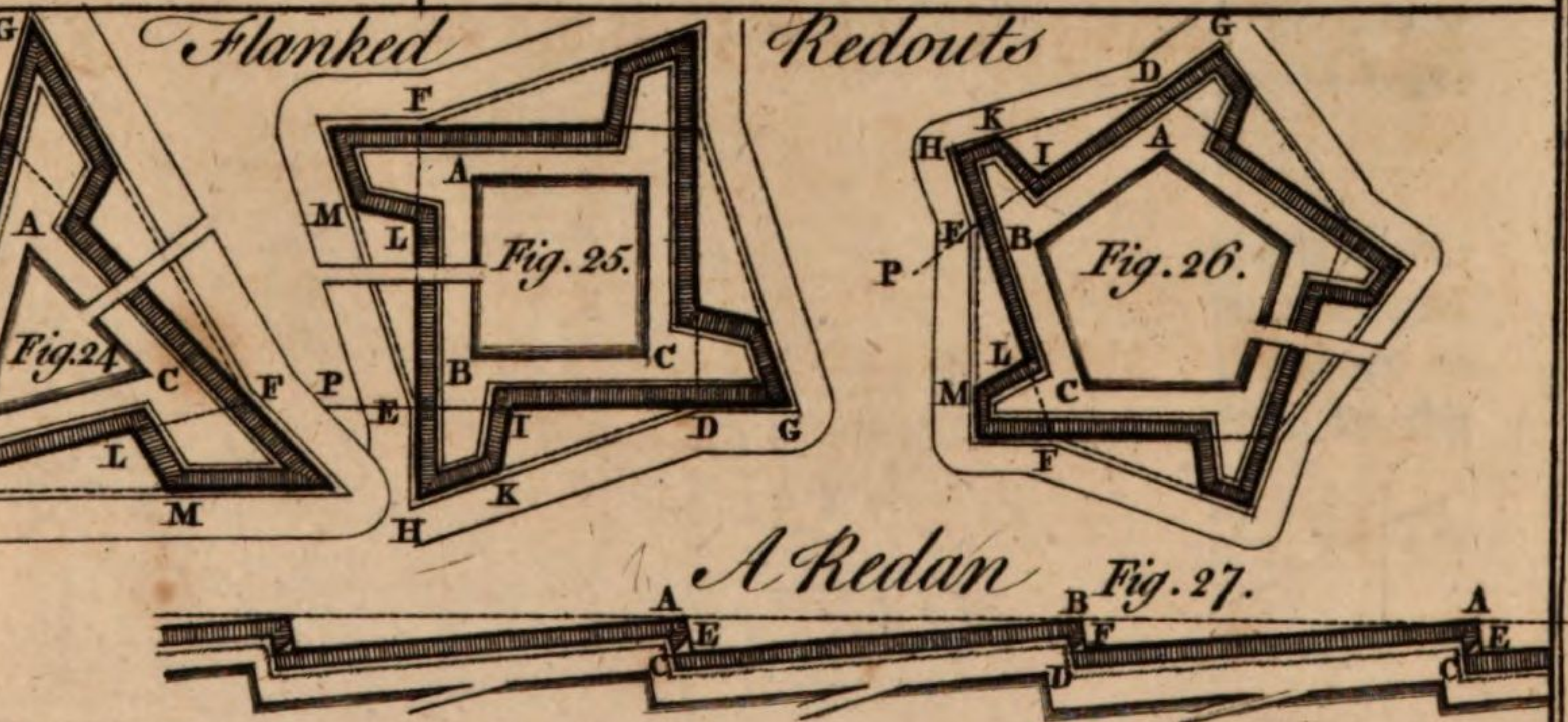
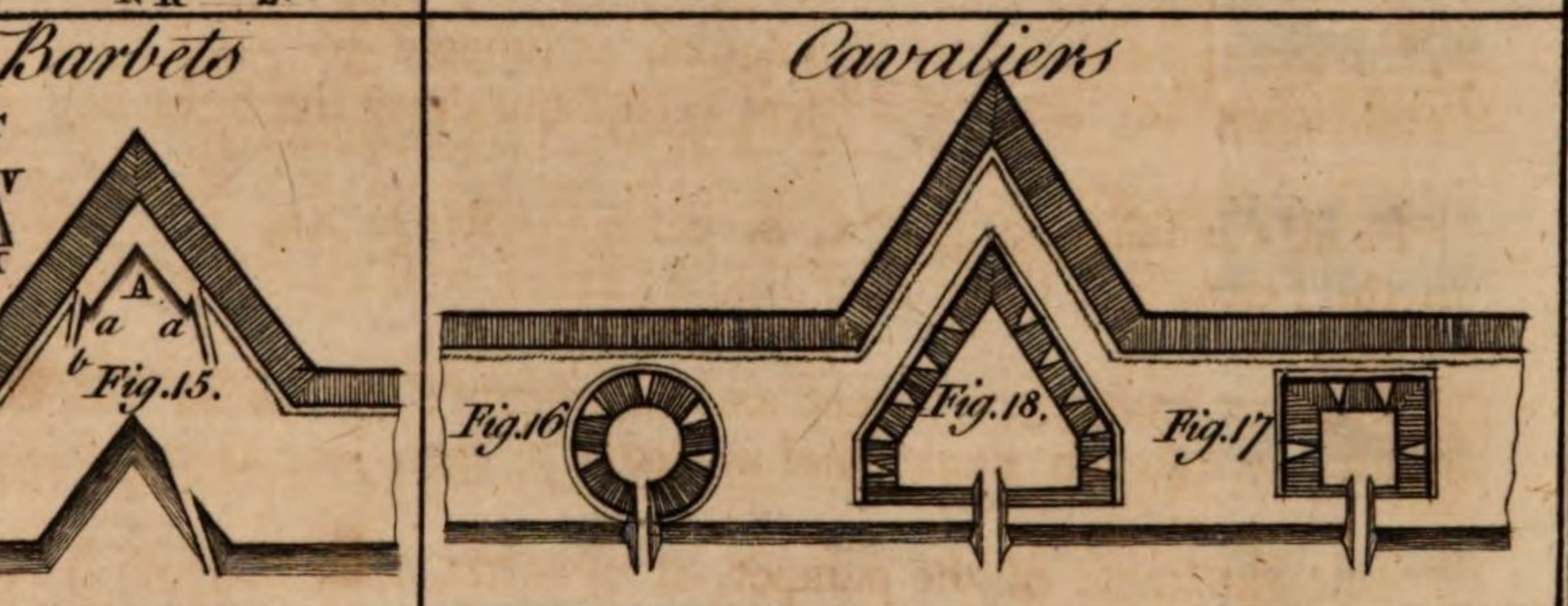
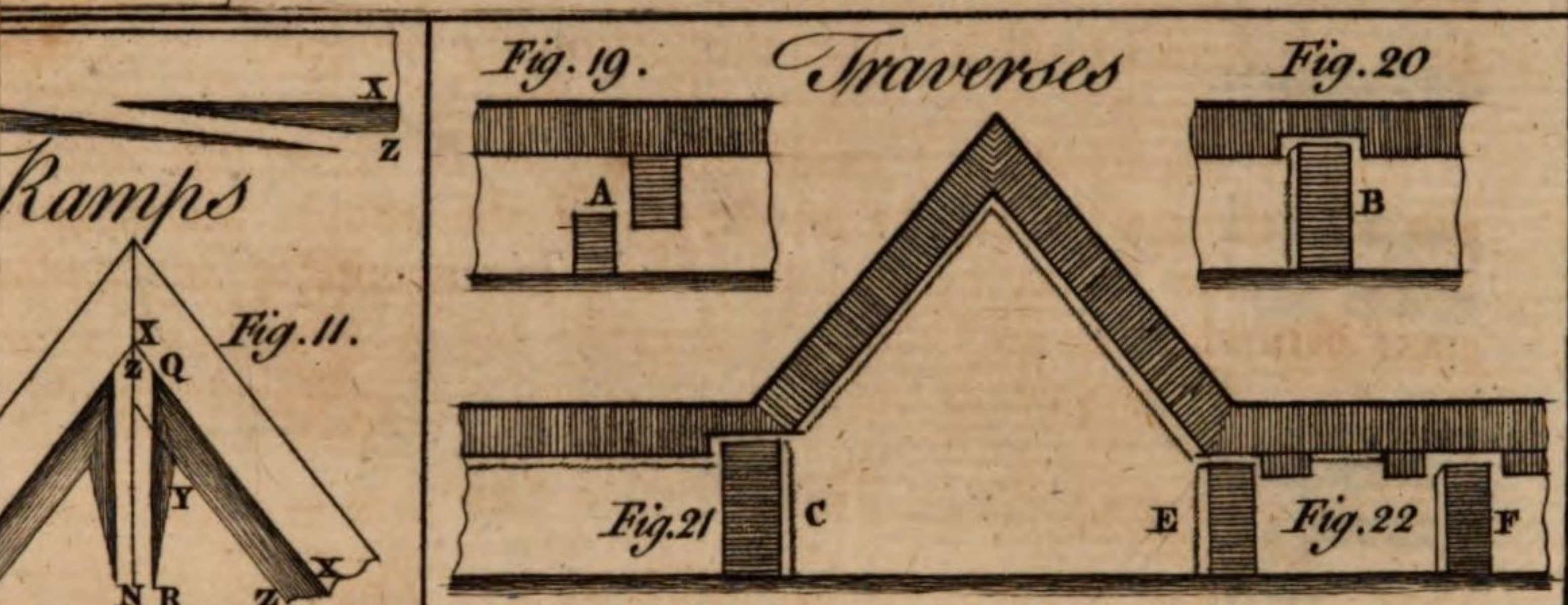
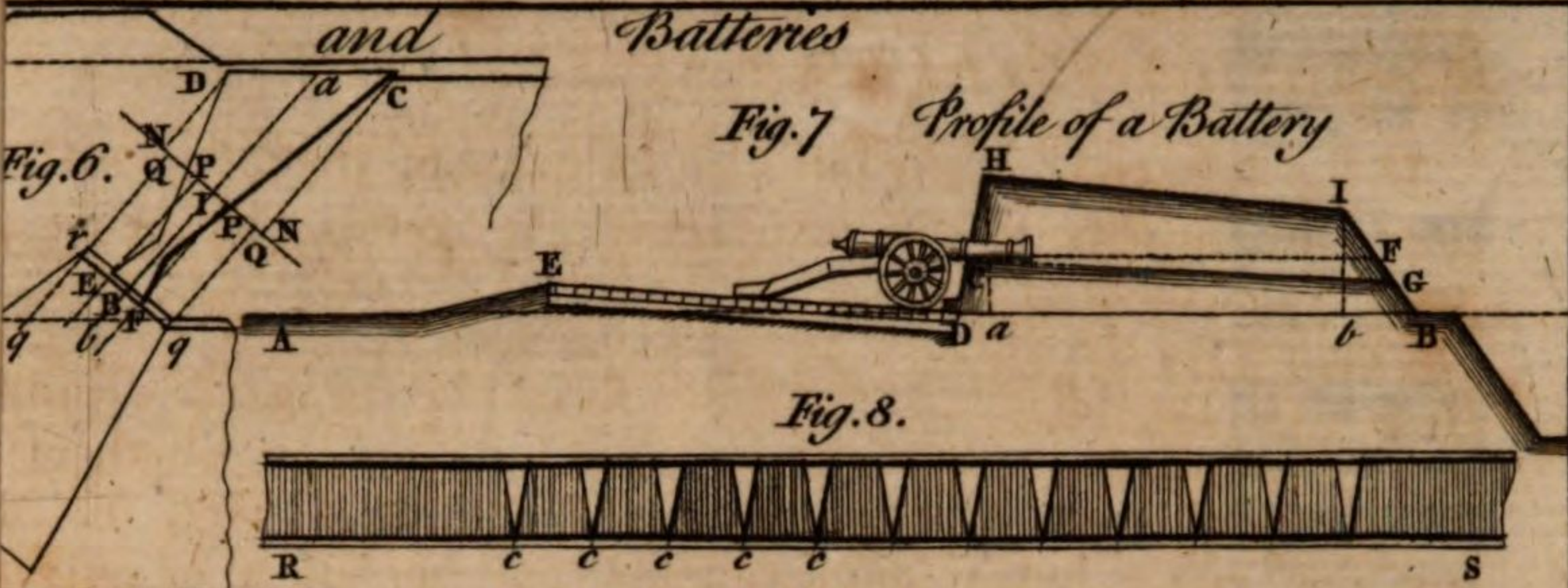
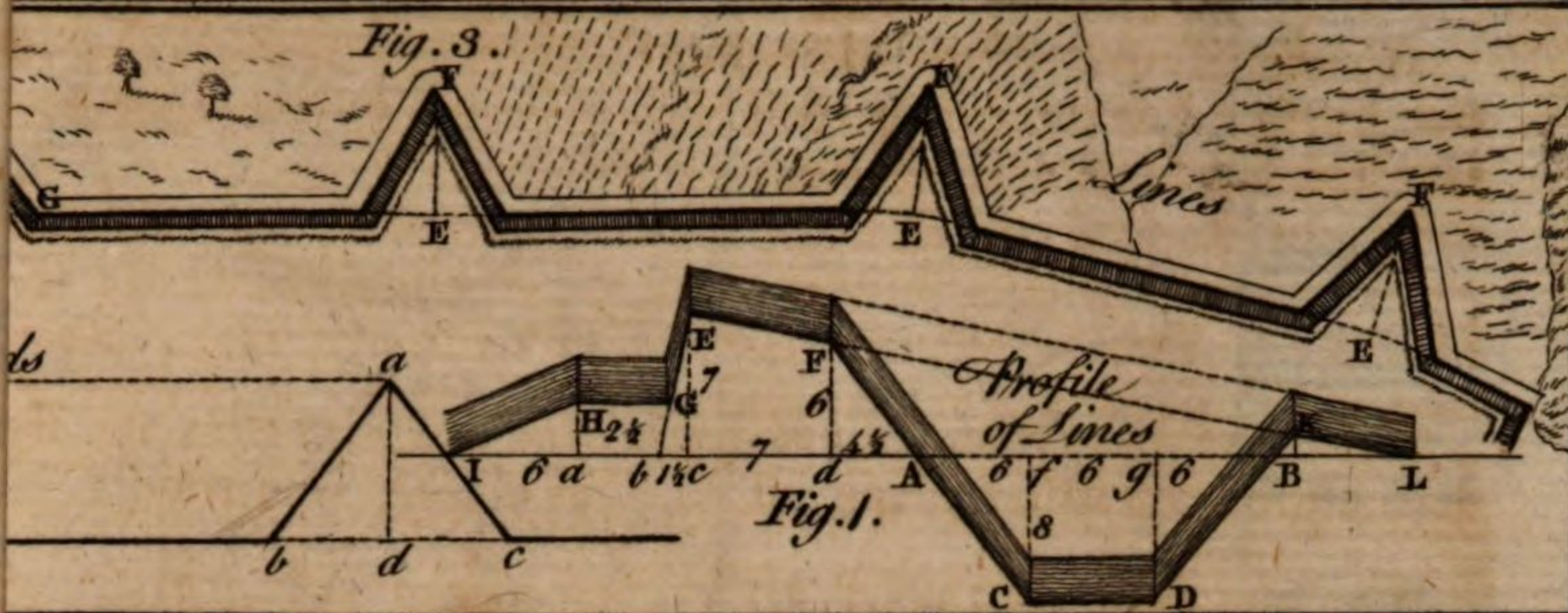
In the front line take the equal spaces AB, AB, &c. of the length proposed for the distance of the flanks, as suppose 50 yards; draw the perpendiculars AC, BD, of 18 feet each, and draw the faces BC, AD.

From the saliant angles A, B, draw the flanks AE, BF, at right angles to the faces, and the master line AEBFAE will be constructed.

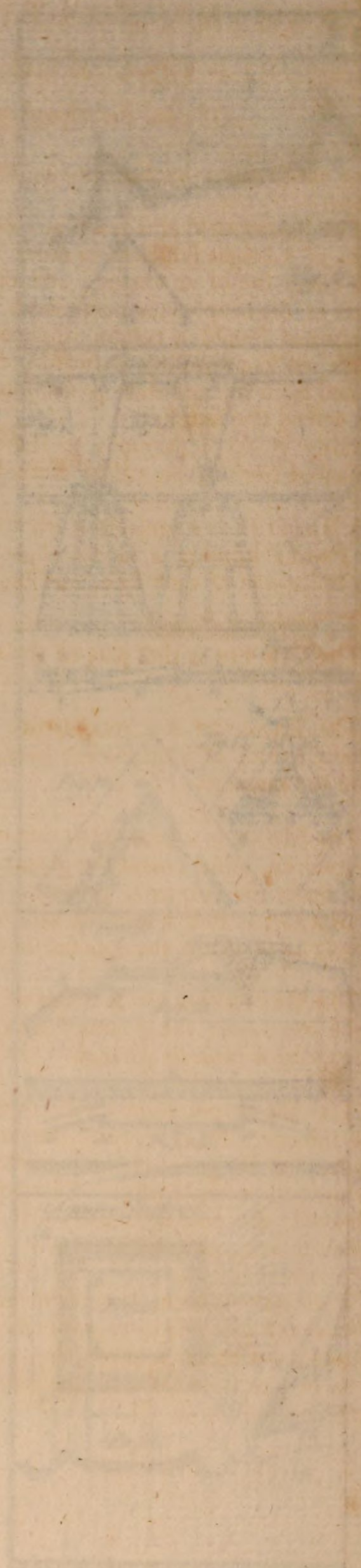
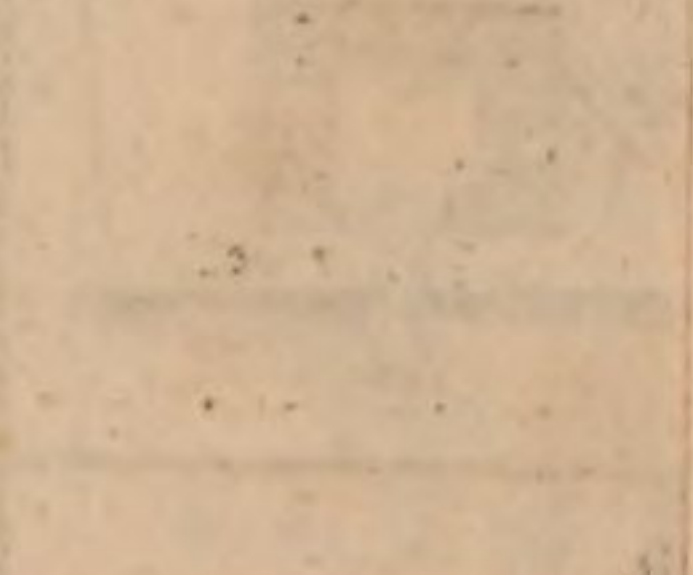
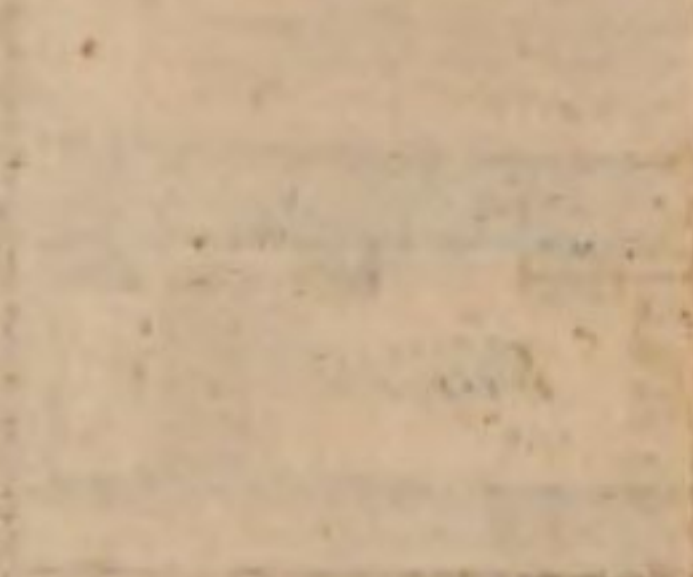
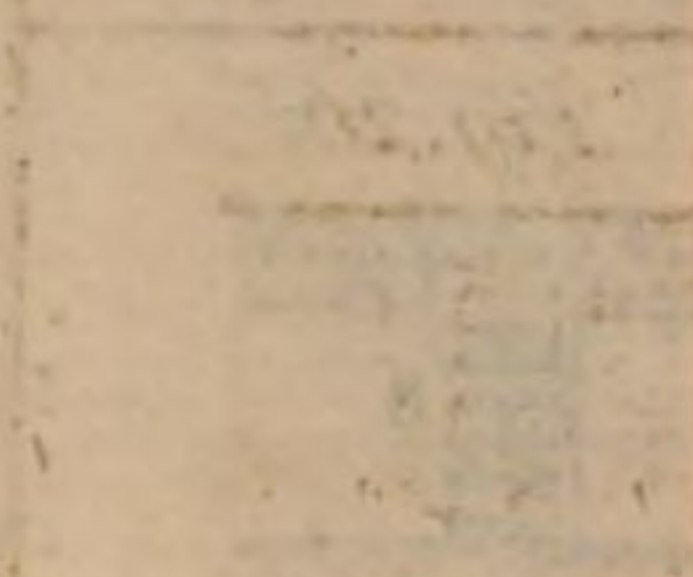
Make a rampart, parapet and foot-bank; ramps in the curtains, an embrasure in each flank, and as many in each face as convenient.

The thickness of the parapet, if of earth, may be from 9 to 12 feet; but if of masonry, about 4 or 5 feet will do.

SECTION



12



SECTION IV.

Of the Fortifying of Towns.

Towns are fortified by surrounding them with a rampart, moat and outworks.

73. The **RAMPART** of a town is a large bank of earth surrounding the place, of a height sufficient to cover the habitable parts of the houses from the direct fire of an enemy; and of a thickness sufficient to resist the efforts of his cannon for many days.

The rampart should at certain distances, within the reach of musket shot, have works projecting outwards, either contiguous to the rampart, or detached from it; and these projections are to be so contrived, as to flank or defend one another.

Projecting works contiguous to the rampart are called *bastions*, and those detached from it, are called *outworks*.

74. A **BASTION** is a projecting work consisting of four lines; namely, of two **FACES** forming a salient angle; and of two **FLANKS**, which join one to the end of each face, and have their other ends joined to the rampart.

75. The **CURTIN** is that part of the rampart which lies between two bastions.

76. The **PARAPET** is a breast-work ranging quite round the faces, flanks and curtins; of a height and strength proper to cover the troops posted on the rampart.

77. The **MOAT** is a large ditch dug on the outside of the rampart and some outworks, quite round the place: The earth that comes out of it serves to make the ramparts, parapets, and other elevated works.

The **SCARPE** is that border or wall of the moat next the place, and the **COUNTERSCARP** is the opposite border or wall.

78. The **COVER'D WAY** is a sort of passage or street running quite round the moat next the counterscarp; and is covered on the side next the country by a bank or parapet.

79. The **GLACIS** is a gentle slope of earth descending from the parapet of the cover'd way towards the country.

The *head of the glacis* is the highest part thereof, or the top of the parapet next the cover'd way. This part is sometimes called the counterscarp; but the soldiery usually call the cover'd way and glacis together the counterscarp.

80. The **OUTWORKS**, or those detached from the rampart, are of various forms and uses; some to cover the principal parts of the rampart, and to retard the enemy's approaches towards the town, and others to secure those places, which, if neglected, might be of advantage to the besiegers.

81. *The names of the principal parts of the Plan of a Fortrefs. Plate 13.*

The figure HFBMN represents the plan of a bastion.

<i>a g</i>	the thickness of the rampart.
<i>a d</i>	the breadth of the parapet.
<i>m m m</i>	the moat, ditch or foss.
<i>a a a a</i>	the cover'd way.
<i>b b b b</i>	the glacis.

82.

Of the principal Lines.

The line AB represents the exterior side of the polygon, and LA its radius.

IK the interior side and LI its radius.

IA the capital CD the normal.

GH the curtain AH the line of defence.

GI the demi-gorge, the breadth HN the gorge.

The line HF represents the flank } of the bastion.
And BF the face }

83.

Of the principal Angles.

The angle ALB represents the angle of the centre.

$\angle AAB$ the angle of the polygon, or $\angle$ of the gorge.

$\angle EGH$ the angle of the curtain, or $\angle$ of the flank.

$\angle GEA$ the angle of the shoulder.

$\angle FBM$ the angle of the bastion, or flanked angle.

$\angle BAH$ the diminished angle.

$\angle ADB$ } the exterior flanking angle; or
the angle of the tenail.

$\angle EHG$ the interior flanking angle.

a p a the angle of the counterscarp.

84. *The names of the principal parts of the Profile of a Fortrefs.*

Supposed to be taken thro' the line PR, Fig. 6.

The line A a represents the thickness of the rampart at its base.

AR the inner slope
BR the inner height } of the rampart.
BC the walk

CD the slope, and DE the top of the foot-bank.

EF the inner face or slope
FG the top or crown } of the parapet.

GH the outer slope or front

HI the face of the wall, called the scarp.

IK the breadth of the moat.

KL the outer wall of the moat, or counterscarp.

LM the breadth of the cover'd way.

MN the slope, and NO the top of the foot-bank.

OP the slope of the parapet, or inner face of the glacis.

PQ the gentle slope, and P the head of the glacis.

Military

Military works of defence should be conformable to the principles which have from time to time been collected and established as fundamental propositions, such are the following.

85. GENERAL PRINCIPLES, *usually called* MAXIMS.

I. The plan of every place that is to be fortified, ought first to be circumscribed with one of the most convenient polygonal figures that can be fitted to that place, and this polygon to be fortified.

II. The more regular the circumscribing polygon is, the better.

III. It is a disadvantage, to be necessitated to extend the works about a fortress.

IV. Every side of the polygon, as much as is possible, should be equally fortified; that the enemy may not have it in his choice to attack a part more weak than the rest.

V. All works ought, as much as is possible, to be easy to build, hard to destroy, and not difficult to be repaired.

VI. The works about a fortress should be so contrived, as to cover the defendants, discover the approaches of the enemy, and adapted to the common weapons of defence.

VII. Each work should be defended from as many as may be, and liable to be attacked from as few places as possible.

VIII. Those works made to defend one another should be sufficiently large to contain the troops and cannon necessary for that defence.

IX. Works which are to flank one another, ought not to be at a greater distance than the point-blank flight of a musket shot.

X. The works farthest distant from the body of the place should ever be lowest, that they may be commanded by those that are nearer.

XI. None of the works should, if it can be helped, be beyond the point-blank reach of the cannon within the town.

XII. Wet ditches are preferable for small places, dry ones for large places.

XIII. Bastions with large flanks and large gorges are most esteemed.

XIV. The nearer the flanked angle of a bastion is to a right one, the better.

XV. The faces of bastions should in length not differ much from 100 yards.

XVI. A curtain should not be much longer than 180 yards, nor much shorter than 100 yards.

XVII. The angle of the curtain should be between the limits of 90 and 100 degrees.

XVIII. The point-blank flight of a musket-ball is about 300 yards, or within the limits of 240 and 360 yards; and that of a battering cannon is about 1200 yards.

XIX. Battering cannon will drive a ball 14 or 15 feet into a bank of common earth at the distance of 300 or 400 yards.

From the foregoing maxims, constructions have been made for all the regular polygons from four to twelve sides inclusive; and the quantity of the several lines and angles have been obtained from trigonometrical operations; the results of which are contained in the following table

86. A TABLE shewing the measures of the principal lines and angles in all regular Fortresses from four to twelve sides inclusive. Calculated from the exterior side, normal and face being taken as known quantities in yards.

Names of the polygons.		Square	Penta- gon	Hexa- gon	Hepta- gon	Octagon	Nona- gon	Deca- gon	Undeca- gon	Dodeca- gon
The exterior side in yards.		360	360	360	360	360	360	360	360	360
Radius of exterior side in yards.		254.56	306.24	360.00	414.82	470.36	526.28	582.50	638.78	695.46
Interior side in yards.		231.00	247.90	261.26	272.44	280.06	285.86	288.52	292.60	296.20
Radius of interior side in yards.		163.34	210.90	261.26	313.96	365.96	417.86	466.90	519.36	572.14
Capital		91.22	95.34	98.68	100.96	104.40	108.42	115.60	119.42	123.32
Normal		45	54	60	64	68	72	78	82	86
Curtin		156.06	154.22	152.80	151.80	150.54	149.40	147.34	142.90	138.68
Flank		40.54	48.92	54.54	58.32	62.12	65.92	71.66	73.96	76.18
Face		100	100	100	100	100	100	100	102	104
Line of defence		265.96	268.42	270.28	271.62	272.86	274.32	276.46	276.30	276.34
Demigorge		37.48	46.84	54.26	60.32	64.76	68.24	70.58	74.84	78.76
Angle of the centre.		90° 00'	72° 00'	60° 00'	51° 26'	45° 00'	40° 00'	36° 00'	32° 44'	30° 00'
Angle of the polygon.		90 00	108 00	120 00	128 34	135 00	140 00	144 00	147 16	150 00
Angle of the curtain.		97 01	98 21	99 13	99 47	100 21	100 54	101 43	102 15	102 46
Angle of the shoulder.		111 03	115 03	117 39	119 21	121 03	122 42	125 09	126 45	128 18
Angle of bastion, or flanked angle.		61 56	74 36	83 08	89 26	93 36	96 24	97 08	98 16	98 56
Diminished angle.		14 02	16 42	18 26	19 34	20 42	21 48	23 26	24 30	25 32
Exterior flanking angle.		151 56	146 36	143 08	140 52	138 36	136 24	133 08	131 00	128 56
Breadth of the foss, in yards.		30 yds.	33 yds.	36 yds.	40 yds.	40 yds.	40 yds.	42 yds.	44 yds.	46 yds.

By the help of this table, every thing relative to the Master line of any of these Fortresses are readily constructed.

87.

P R O B L E M I.

To describe any of the polygons in the table.

FIRST. *To construct a polygon in a circle.* Plate XIII. Fig. 1.

- 1st. With the exterior radius of the given polygon describe a circle.
- 2d. Apply the exterior side equal to 360 yards within the circumference of that circle, from A to B, B to C, C to D, &c. as many times as the proposed polygon has sides.
- 3d. Draw the lines AB, BC, CD, &c. and the polygon will be constructed.

88. SECONDLY. *To construct a polygon upon the exterior side.* Plate XIII. Fig. 2.

- 1st. Draw the exterior side AB equal to 360 yards.
 - 2d. At the ends A, B, make angles equal to the angle of the polygon, and in that position draw the lines AE, BC, each of the length of AB.
 - 3d. At the points E, C, make angles as before, and in those positions draw the lines ED, CD, each of the same length as AB.
- And thus proceed until the polygon is constructed.

Note, The two last lines may be found by describing intersecting arcs D, with the radius AB, from the centres E and C.

89.

P R O B L E M II.

To fortify any regular polygon in the table.

Or, To draw the MASTER LINE of a regular fortress.

FIRST. *From the exterior side.* Plate XIII. Fig. 3.

- 1st. Make the exterior side AB equal to 360 yards, and find its middle c.
- 2d. Draw CD at right angles to AB, and make CD equal to the normal.
- 3d. Thro' D draw the lines of defence ADH, BDG.
- 4th. Make the faces AE, BF, each of their proper lengths.
- 5th. Transfer the distance FE from F to G, and from E to H.
- 6th. Draw the curtain GH, and the flanks EG, FH.

This work being done for every front or side, the master line of the fortress will be described.

This method is most fit, when the master line is to be laid down on a place where there were no works before.

90. SECONDLY. *To fortify from the curtain.* Fig. 3.

- 1st. Make the curtain GH of its proper length, for the given polygon.
- 2d. Make the angles of the curtain HGE, GHF, as per table.
- 3d. In the lines GE, HF, take the flanks GE, HF, of their proper length.
- 4th. Make the angles of the shoulders GEA, HFB, as per table.
- 5th. Make the faces FB, EA, of their proper lengths.

This

This method is proper to be used when a piece of an old fortification is to serve for a curtain, to save expence.

91. *THIRDLY. To fortify from the interior side. Fig. 3.*

1st. Make the interior side IK of the length given in the table for the proposed polygon.

2d. From each end set off its proper demigorge KH, and GI.

3d. On K and I as centres, with the extent of the capital, as per table, describe arcs at B and A.

4th. On H and G as centres, with the length of the line of defence, as per table, cut the former arcs in A and B.

5th. Draw the lines AH, BG; in which take the faces AE, BF, and draw the flanks EG, FH.

This method is useful when modern ramparts and bastions are to be annexed to simple walls furrounding a place.

In either of the above methods, the work here described for one front is to be applied to every front of the polygon; and it is recommended to the learner, that he applies the precepts given for one front, to a compleat polygon; and should a frequent repetition of these operations to a whole polygon become disagreeable, they may be done only on two fronts, which may be thus drawn.

Draw a capital line AL; make the angles LAB, LAA, each equal to half the angle of the polygon; and make the sides AB, AA, each equal to 360 yards.

92.

PROBLEM III.

To construct the plan of a bastion. Plate XIII. Fig. 4.

1st. Draw a capital line BK, and make the angles MBK, FBK, each equal to half the flanked angle.

2d. Take the lines BF, BM, each of the length of the face.

3d. At the points F, M, make the angles of the shoulder BFH, BMN.

4th. Make the lines FH, MN, each the length of the flank.

5th. At the points H, N, make the angles FHG, MNa, equal to the angle of the curtain, and draw the pieces of the curtain HG, Na.

This problem may be found useful on many occasions; particularly when the drawings of works in a bastion are to be represented with that bastion only, on a large scale; therefore it was here introduced, more especially for beginners, who are for the most part apt to be somewhat puzzled about constructing part of a work, independent of the whole.

93.

PROBLEM IV.

The master line of a fortress being given; to draw the plan of the rampart.

Plate XIII. Fig. 5.

1st. Draw the lines ag perpendicular to each face, flank and curtain.

2d. In each of the lines ag take $ag = 24$ yards, $ab = 20$ yards.

$ac = 8$ yards, $ad = 6$ yards.

3d. Thro

3d. Thro' the points d, c, b, g , draw lines parallel to the master line; that is, to each face, flank, and curtain, intersecting one another.

Then the breadth of the rampart is represented by $ag = 24$ yards.

of the parapet by $ad = 6$ yards.

of the foot-bank by $dc = 2$ yards.

of the walk of the rampart by $cb = 12$ yards.

And of the slope of the rampart by $bg = 4$ yards.

The breadths of these parts of the rampart are sometimes different, according to the quality of the earth, or other materials the work is to be built with.

Sometimes the bastions are made solid; that is, the rampart fills up the whole space contained within the faces, flanks and neck: The plans of such are represented by continuing the pair of lines bg , forming the inner slope of the rampart, against the adjacent curtains, till they meet the capital in the breast of the bastion, as at R : But then this pair of lines is to be omitted against the flanks and faces of such bastions.

In plans drawn from a large scale, it is usual to express the outside slope of the rampart by a pair of lines; that is, by drawing a parallel to the master line at about five or six feet distant from it on the outside.

The master line is to be drawn the thickest; the inside of the parapet something finer; and the other lines as fine as they can be drawn.

94. Before any of the lines are drawn in ink (for it should be remarked that every thing is first to be drawn in black lead pencil), let the ramps be put in the inner slope of the rampart (52); the embrasures in the parapets of the flanks (46); the barbets in the salient angles of the bastions, fig. 6. with their ramps (56); and the cavaliers, if any are to be, in the solid bastions, fig. 7. with their ramps and embrasures (58).

All these several parts being laid down with black lead pencil, let the lines be neatly drawn with Indian ink, taking care not to draw them beyond their proper terminations; for wherever superfluous ink lines are scraped off, those plans will never look well when they come to be washed with any colour.

When the plan is to be washed, that is coloured, either with tints of Indian ink or other colours, leave the lower line of the inner slope of the rampart in lead pencil, which will make it appear softer when the drawing is cleaned off.

REMARKS.

95. In the middle of bastions, fig. 8. it is usual to build magazines for the stores of powder, bombs and other fireworks, and also to contain the provision for the troops: For in these places they are not only more out of the way of doing damage to the town on an accident of blowing up, but they are also nearer at hand to be transported to the several works where they may be wanted.

Tacse

These magazines are usually built about 20 yards long by 12 broad in the clear, and about 8 feet high in the upright, besides the rise of the semicircular vault: The side walls are about 8 or 9 feet thick at the foundations, and about 8 at top; the end walls about 5 at bottom and 4 at top: And that they may be bomb proof, the work is about three feet thick in the *reins*, or just above the springing of the arch, and eight or nine feet thick in the crown or top; this is effected by building the roof sloping on the outside, like the roof of a house.

These buildings are sometimes strengthened on each side by five buttresses 6 feet by 4; and in each side wall, there should be left in the solidity of the wall five vent-holes to air the magazine: These vents wind round a kind of shaft of about a foot square. In each gable end, a small window is made to give light.

96. Magazines may be either in hollow or solid bastions; taking care in the latter to leave an area about their walls: And altho' there are those who would have the magazines at the foot of the rampart along the curtains, in order (as they say) that the bastion by being left clear, may be disputed with the enemy inch by inch at last; but as there are very few examples, since the modern practice of war, of such extraordinary resolution in the defendants after a breach is made, therefore the reasons for having the bastions thus unincumbered with buildings, are rather ideal than real, and consequently can have but little weight: However, when the ditch furnishes earth enough, or it can be otherwise easily had, it has been the practice of the most celebrated engineers to make full bastions, especially if they were small and had narrow necks.

97. In the salient angles of bastions, fig. 7. such as the flanked angle, and the angles of the shoulders, it is usual to make little centry-boxes either of wood or stone, for the centinels posted there to secure themselves from the inclemency of the weather and random shot: They are furnished in their sides with loopholes, through which may be seen what is doing in the ditch and cover'd-way, and also to fire through on occasion.

The plans of these centry-boxes may be either square, round, or pentagonal, and their floors are on a level with the walk of the rampart; their communication is by a passage of about 3 feet wide, brick'd on each side, cut through the parapet.

The centry-boxes built of stone, are usually ornamented with mouldings, and the arms of the prince or state.

98.

P R O B L E M V.

The master line of a fortress being given, to draw the plan of the foss and esplanade. Plate XIII. Fig. 4.

1st, On the flanked angle B of each bastion, with the radius BC equal to the breadth of the foss found in the table, describe an arc CC.

2d. Lay a ruler to touch each arc at C, and the angle E of the nearest shoulder of the next bastion, and in that direction draw the lines CP, meeting one another in P.

3d. From the flanked angle of each bastion, draw lines BQ at right angles to CP.

4th. In

4th. In each of the lines Bq , take $ca = 10$ yards, (Fig. 4.)
 $co = 12$ yards, and $cq = 60$ yds.

5th. Thro' the points a, o, q , draw lines parallel to cp , meeting before the middles of the curtains at c and t (Fig. 3.), and before the flanked angles of the bastions in v, x, y .

Then will BC represent the breadth of the fofs.

cq	of the esplanade.
co	of the cover'd-way.
ao	of the foot-bank.
oq	of the glacis.

The arcs cc represent the roundings of the fofs.

The lines cp the counterscarp.

xc the head

xy the ridges

st the vallies

And yt the foot

} of the glacis.

The head of the glacis is to be drawn strong, and all the other lines are to be fine.

99. When the plan is to be washed, if the foot of the glacis be drawn in lead pencil only, the termination of the glacis, or its concurrence with the adjoining lands, will appear more soft and neat.

In the colouring of the glacis, it is usual first to lay a very light tint of colour over every plane, and then to strengthen the colour on every second plane, beginning at any salient angle; the strongest tint to be next the head of the glacis, and to be washed gradually off; so that the limits of the lightest shade shall lie in a diagonal line from the foot of the glacis at a ridge, to the head of the valley, *as from y to s* .

Give a light touch of the pencil or brush, with a middling tint of Indian ink along the inside of the line, denoting the foot-bank.

Wash the borders of the ditch near the scarpe and counterscarp, softning the edge of the colour with a brush wet with fair water: In wet ditches the colour used is distilled verdigrease; but in dry ditches use bistre.

The parapet is generally coloured with a middling tint of Indian ink.

100. By this method of drawing the counterscarp, all the fofs is seen from the flanks, and consequently is defended by them, and must be more advantageous than if the counterscarp was directed to any other part of the flank, the intercepted part next the shoulder being thereby rendered useless for the defence of the ditch; nevertheless, if it was on any account necessary, the direction of the wall of the ditch may point within the shoulder by the thickness of the parapet of the face, and still the fofs would have the whole defence of the flank.

101. Fortifications in general would be very defective without fosses; for the wall or rampart need not be raised so high when there is a ditch, as they must be without one, to secure the place from surprizes by scaling, &c. Neither is a low wall so much exposed as a high one is in the modern methods of attack: Besides, a place is not so liable to the attempts of a surprize with a fofs as it would be without one; for the enemy, if discovered, may easily retreat while he is on a level with the country, which he would find some difficulty in doing from the bottom of a deep fofs

M m m m

bounded

bounded by walls or banks nearly upright: Add to these, that the earth taken out of the ditch serves to make the ramparts, parapets, glacis, &c.

102. It may appear to some persons, that the broader and deeper a foss is, the better, as it would throw more difficulties in the way of the besiegers when they wanted to pass it: But it must be observed that some advantages are best waved, when by taking them, greater disadvantages would ensue. For beside the great expence that the digging and removing of so much earth would occasion, the enemy might see the foot of the wall at a greater distance in a broad ditch; and in a deep one, the cannon of the flanks could not dip low enough to defend the bottom. And on the whole, if the foss by its magnitude furnished more earth than was wanted, some inconvenience might arise in the disposing of the overplus.

103. Those fosses are best which can be filled with water and drained at pleasure by the help of sluices or flood-gates; because all the advantages that may be reaped either from a wet or dry foss, are by these means obtained: But as these cannot be had every where, the ditch must be adapted to the situation of the place.

In dry fosses, a row of palisades is sometimes planted along the middle, either upright, or sloping towards the counterscarp 20° or 30° from the upright.

In some dry fosses, where water can be had, a trench called *CUVETTE* *nn* (Fig. 12.) is dug along the middle, of about 15 feet wide and 9 feet deep, for the water to run in.

The breadth of the foss, altho' unequal in itself, is reckoned from the flanked angles, before which it is always rounded; the depth here may be from 15 to 20 feet, and thence rise gradually to the middle of the curtain, where the depth is 4 or 5 feet less.

Of Gates.

104. A *GATE* in a fortified place, is a passage thro' the rampart, which can be shut up or opened by the means of *doors* and a *portcullis*.

105. A *PORTCULLIS*, is either a *HERSE* consisting of strong pieces of timber framed like a lattice, the lower ends of the upright pieces being shod with pointed iron, and the whole frame let down by chains or ropes: Or *ORGUES*, consisting of several long pieces of timber not framed together; each hanging by its particular rope or chain, whose ends are wound round a windlass which lets them all down together.

The use of a portcullis is to stop the gateway in case of a surprize; and in this, the orgues are by much the best contrivance; for a herse may be broke to pieces by a *petard*, or by cannon shot: But should a beam in the orgues be struck off in the middle, the upper part will descend to supply the place of the part taken away.

Gates are either public or private.

106. *PRIVATE* or *POSTERN GATES*, are those passages thro' the rampart by which the troops can go out of the town unseen by the enemy, when they pass to and from the relief of the duty in the outworks; or on any other occasion which is to be concealed from the besiegers.

107. *PUB-*

107. PUBLIC GATES, are those passages thro' the middles of such curtains to which the great roads or public ways lead.

They are put in the middles of curtains, as being the best defended place in the outline of the rampart.

The dimensions of a public gateway are usually about 13 or 14 feet high, and 9 or 10 feet wide, continued thro' the rampart, with proper recesses for the foot passengers to stand in while the wheel carriages pass on. It is usual to ornament the front of the gateway on the outside with architecture either of the Tuscan or Doric order, and to cover the passage with a strong vault: Over the vault, close to the town wall, is erected a building of about 18 or 20 feet square, in which is suspended the portcullis; and on the inside of the rampart there is generally raised another building, of about 100 feet front and 30 deep, and high enough to contain one or two floors of rooms for one of the town-officers, the ground floor serving for guard-rooms for the troops who are on duty at the gates.

108. *To construct the Plan of a gate with the adjoining buildings.* Pl. XIII.
Fig. 9.

Let the line ac represent the thickness of the base of the rampart, and the middle line of the gateway; in which take these measures in feet: $af = 4\frac{1}{2}$; $fr = 6$; $rb = 18$; $ci = 30$; and $cm = 15$: Thro' these point lines being drawn parallel to the curtain, take $fd = 4\frac{1}{2}$; $fb = 14$; $rg = 9$; $ck = 100$; and $kl = 6$ feet, and complete the figure.

Then bg represents the plan of the frontispiece; gb of the portcullis room; ik of the arcade or guard-house; and lk is the breadth of the stairs to ascend the rampart.

The plan to which these directions refer is on the right hand side of the plate.

109. *Of the disposition of the Streets and Buildings in a Fortress.*

When a new town is built and fortified, and all that relates to the rampart is fixed with regard to situation and size; then the plan of the streets are to be considered; and herein, some authors have thought it convenient to adapt the position of the streets to the climate the town is built in.

Thus in hot climates, they would have the streets run NE. and SW.; NW. and SE.; for then the streets will be shaded by the houses from the mid-day sun: In this manner *New Brisac*, part of *Malta*, and some other regular built towns are laid out. But in cold climates, they advise to place the streets so as to run north and south, east and west, in order to have as much sun heat as can be.

110. *To design a Plan of the streets and buildings.*

About the middle L of the town, describe a square, whose sides shall be parallel to the position of the streets (already determined); this square is called the great place of arms, whose size should be proportioned to

M m m m 2

that

that of the town, and to the number of troops designed for its defence. Thus in a square, make the side of the place of arms about 90 yards; in a pentagon, about 100; a hexagon, 120; a heptagon, 140; an octagon, about 160 yards, &c. In this place of arms the troops parade, or are assembled for their daily exercise.

In the continuation of each side of the square, draw streets of about 10 yards wide; likewise thro' the middle of each side draw a street of about 12 yards broad; cross these streets perpendicularly with others of about 8 yards wide; make the parallel distance of these cross streets about 40 yards, and continue all the lines to the rampart.

It is not necessary that the middle streets going from the great place of arms L should terminate directly against the gate E, fig. 9. But it is somewhat more convenient, when the circumstances of the place will admit it to be so.

Make streets of about 10 or 12 yards wide, parallel to the curtains, and bounded on one side by the rampart: About 15 yards distant from this street, within, draw another street of about 9 yards wide: In the space H between these two streets, the barracks for lodging the officers and soldiers are built; and in the street next the rampart, the piles of shot and bombs are sometimes placed.

The length of the plan of a barrack before a curtain is about 140 or 180 yards, and the plan of a pavilion I for the officers may be about 25 yards long by 20 yards broad.

Against those curtains where the public gates E are placed, the barracks are discontinued for about 40 yards on each side the gate, making thereby an area of about 80 yards long and about 36 broad, serving for a lesser place of arms where the guards of a gate may draw up, in case of a surprize: Moreover, this area is useful for the carriages that are going out to stop in, until those on the bridge coming in have entered the gate. In figures of 4, 5, or 6 sides, this area is less.

The extremities I of the barracks next the gates and bastions are usually assigned for the officers pavilions, being the most airy places.

III. Of the rectangular plats made by the intersections of the streets, the most conveniently situated ones are to be chose for the public buildings; such as the arsenal, church, town-house, market, governor, and for the other chief officers in the service of the state; and on the remaining plats, houses are built for the burghers.

Beside the guard-houses of the gates, there should be others made on the ramparts for the centinels appointed for the night watch; these guard-chambers are often placed at the centers of the necks of bastions, when in those places there are no cavaliers or magazines of powder: They are also well disposed, when set on the middles of those curtains where there are no public gates, especially when there is a water-gate thro' which runs a canal or river.

The arsenal should be erected in a place the least liable to accidents of fire; and if there is a conveniency of water carriage in the town, the arsenal should be placed as near the water as may be.

There are no general rules to be given for fixing the places on which the public buildings are to be erected, because the circumstances by which engineers regulate their plans are various: Therefore in the figure referred

ferred to, which is a regular pentagon fortified in the common way, the streets and buildings are laid out in imitation of their disposition in *New Brisac*; which being in general esteem, is usually proposed as a pattern for beginners.

There are many writers on this subject, who propose to make the great place of arms or parade, similar to the figure of the town when regular; and from the angles and sides of this parade, draw streets to the middle of the bastions and curtains: But as this disposition spoils most of the corner houses in the town, by making the walls at oblique angles to one another, it was thought sufficient to have barely mentioned such constructions.

Of Bridges.

112. The passage over the moats of fortified places is by bridges, most commonly made of timber, and supported by frames rising from the bottom of the ditch: Such bridges are either *private* or *public*.

PRIVATE BRIDGES serve for the easy communication between the outworks.

PUBLIC BRIDGES, are those by which carriages and passengers go over when they come into or depart from the town; and such bridges consist of two parts, namely, the *fixed* and *moveable*.

The FIXED or DORMANT parts of a bridge, are those parts which are strongly joined to the supporting piers; and are never moved but in the cases of repairs, or the danger of a siege or surprize.

The MOVEABLE or DRAW-BRIDGE, is a floor of timber framed together, one end of which serves as an axis, whereby the other end may be drawn up or let down by some easy contrivance: The most common way is by a kind of balance called *plyers*.

The PLYERS are two timber leavers about twice the length of the bridge they lift, and joined together by other timbers framed in the form of a St. Andrew's cross, which serves as a counterpoise. The plyers are supported by two upright jaumbs, on which they swing; and the draw-bridge is raised or let down by the means of chains joining the ends of the plyers and bridge.

The breadth of a public bridge is usually about 16 feet, including the rails and footway of about 3 feet on each side: But the breadth of the draw-bridge is only 9 or 10 feet, and its length about 15 feet.

The draw-bridge is generally placed close to the gate, whether it be the gate of the town or that of an outwork; and is then by the means of the plyers drawn up against the gate: But when a bridge is more than 40 yards long, it is usual to put, besides the draw-bridge at the gate, another about the middle of the bridge.

113. *To draw a bridge in the Plan of a Fortress.*

Before the gates draw four lines across the ditch, making three intervals or spaces, the middle one being 10 feet broad, and the space on each side three feet.

Divide

Divide the length of the bridge into parts of about 16 feet long each, to express the distance between pier and pier, or the bays of building, and against each division, on the outside of the lines, make a square of a foot, to express the ends of the piers supporting the bridge.

Allow the bay next the gate for a draw-bridge; and if the length of the bridge exceeds 40 yards, allow the middle bay for another draw-bridge: These are to be distinguished by drawing two diagonals across the middle interval of these bays; that of the other bays having lines drawn across them at a foot's distance from one another, and the space on each side left white for the footway; which are to be broke off at each draw-bridge. See figs. 13, 15.

114.

Of the Orillon and Retired flank.

When the besiegers of a fortress have made a breach in the face of a bastion sufficiently perfect, and have every thing ready for the storm, they then cross the ditch to the attack, if the besieged will suffer it to be brought on: In this attack, the fire from the flanks is the most effectual defence; and cannon placed there, loaded with grape or case shot, would then do great execution: But of this the besiegers are well aware, and therefore take care beforehand to dismount all the cannon flanking their intended passage of the foss; and as this cannot be well prevented by the besieged, especially when the flanks are plain ones, several of the most eminent engineers have proposed, and actually built part of the flanks retired behind the ordinary limits, leaving at the shoulder of the bastion a mass of earth, commonly called an Orillon, which is to serve as a skreen between the enemy's direct fire and a part of the flank. In this covered part are reserved one or two cannon, to be used only in the defence of the breach in the face of the next bastion.

This notion of covering part of the flank, is almost as old as the modern method of fortification.

115. *To construct the Orillon and Retired flank.* Plate XIII. Fig. 10.

1st. At right angles to the face As , draw from the angle of the shoulder the line sb equal to 8 or 9 yards, and make b the centre of the arc, sa forming the *orillon*.

2d. From a point z in the face of the next bastion, fig. 6, about 15 yards from the point A , draw thro' a a line zm ; in which, and also in the line of defence Ar produced, take the *brisures* or *breaks* am and rn , each 10 yards.

3d. From m and n , with the radius mn , describe arcs cutting in d , which make the centre of the arc mn , or *concave flank*.

4th. In dm produced lay the several breadths of the parts of the rampart; as the parapet = 6 yards, foot-bank = 2 yards, walk 12 yards, and the slope = 4 yards; then from the centre d , thro' those points describe arcs, meeting in the continuation of the corresponding lines from the face of the next bastion.

The

The parapet of the orillon is drawn parallel to $s a$ at 6 yards distance.

The axis or middle line of each embrasure should tend towards a point about the middle of the fofs opposite the flanked angle of the next bastion.

The wall in the upper brisure $a m$, as it cannot be battered, need not be above 2 or 3 feet thick to support the earth behind it.

Thro' this wall there should be a sally port coming out at the bottom of the fofs if a dry one, or a little above the water line in a wet fofs: The descent to this port is by a passage under the rampart of the flank; but in strait flanks, the sally port is in the curtain next the angle of the flank, the passage to it being under the rampart of the curtain.

116.

Of Double flanks.

As the progress of the works of the besiegers generally depend on their having a greater army than is in the town besieged, and of forming a greater fire on the town than they can return; therefore some engineers have endeavoured to hinder the progress of the besiegers batteries, by furnishing the town with an equivalent, or rather a superior fire; and this has been proposed to be effected, by making second and third flanks one behind another, gradually more elevated than those before: For as it is found convenient to silence the fire of the flanks, before the besiegers can avail themselves of the breach they make in the face of the bastion; therefore batteries to ruin the flanks are made, and play at the same times as those for the breaches in the faces. Now the batteries for the ruin of the flanks, can be conveniently made only within about 10 yards of the head of the glacis, and their length not greater than about 40 yards, containing at most about 6 or 7 cannon: But if the besieged can by any artifice preserve the cannon in the opposite flank, and have a greater number there than are in the battery opposed, consequently their fire will be superior, and ruin the battery intended to offend them; and this was expected to be accomplished by *double, or cazemated flanks*.

In some places two or three tier of cannon were placed one above the other, the upper ones being supported by vaulted work, as in the Tower of London; but such having been found very inconvenient, on account of the great smother contained in the vaults, they have been generally constructed open.

The cazemated flanks have been condemned by many writers, because they may be rendered useless by the enemy's bombs; was this objection just, it would hold against any small work; for the besiegers having once got the length to any place, they may be supposed to render that place also useless to the besieged; and it would thence follow, that it were best not to have any places of defence: But as the utility of such works have been commended by some of the most eminent engineers, it was judged proper here to mention how these flanks may be constructed.

SECTION

SECTION V.

117. *Of Works for the defence of the Foss.*

Such works are usually called *tenails*, or *ramshorns*, or *caponiers*.

A **TENAIL** is a work raised in the ditch before the curtain upon the lines of defence.

A **RAMSHORN** is a curved tenail raised in the foss before the flanks, and presenting its convexity to the covered way.

A **CAPONIERE** is a kind of covered way across a dry ditch before the middle of the curtain; and limited between the lines of defence and the counterscarp.

The *tenails* never rise above the common ground level, but are frequently two or three feet lower: They consist of a rampart about 36 yards thick, covered by a parapet of cannon proof, with one or two foot-banks. These works are placed at such a distance from the curtains and flanks, that the splinters of stone knocked off them by the besiegers cannon may not hurt the troops posted there to flank the enemy in their passage of the foss; and this is their principal use.

The *caponiere* is of like use, but is usually sunk about two or three feet lower than the bottom of the foss: It is about two or three yards broad, and is covered by a parapet about six feet high on each side, which slopes away like a glacis about 15 yards broad. It is frequently covered by a roof made with hurdles, to sustain a floor of earth about a foot thick, and the whole supported with thick planks of musket proof, leaving convenient loop-holes for the defendants to fire through.

118. *To construct Tenails. Plate XIII*

First sort with flanks, Fig. 11. Parallel to the curtains and flanks, draw the lines *eg* at 6 yards distance, and *ee*, *Gg*, at 10 yards distance: From *F*, and *D*, the middles of *CG*, *CE*, draw the flanks *Ff*, *Dd*, at right angles to the lines of defence *Ag*, *Be*; draw the curtain *df*, and put in the rampart, parapet, and foot-bank.

Second sort, with faces only, Fig. 12. Take the distances from the curtain and flanks, the same as before; and to the lines of defence annex the rampart and parapet.

Third sort, with faces and curtain, Fig. 3. When the lines of defence cross one another in *D*, so near the curtain that there is not room for the thickness of the rampart and its distance from the curtain, make the faces *ab*, *cd*, each of $\frac{2}{3}$ of *AD*, and draw the curtain *db*.

119. *To construct a Caponiere. Fig. 12.*

From the exterior flanking angle *D*, draw a line *Dp* to the angle of the counterscarp *p*; on each side of *Dp* draw parallel lines, at the distances of $1\frac{1}{2}$, $2\frac{1}{2}$, and 15 yards from *Dp*; these lines being limited by the lines of defence and counterscarp, form the caponiere.

This work, which can be used only in a dry ditch, serves, besides its defence of the foss, for a convenient passage between the town and its outworks; and in this case there is usually a circular excavation made in the

the angle of the counterscarp, out of which the troops may defile without being seen by the enemy.

120. *To construct a Ramshorn.* Plate XIII. Fig. 13.

A line $D a$ drawn from D the angle of the shoulder, at right angles to the line of defence, its intersection a with $B I$, the other line of defence continued, gives the centre a ; from which, with the radius $a D$, describe the outline AC of the ramshorn, CD being equal to 10 yards; and annex a parapet, foot-bank, and walk, as in the other tenailles.

This work seems preferable to either of the other tenailles, both on account of its simplicity, and the defence for which it is constructed.

SECTION VI.

Of Outworks.

121. OUTWORKS are all those works advanced beyond the foss, serving to augment the defence of the town, to cover the gates, bridges, and other weak parts; to inclose those eminences, not too far distant, which command the town; to surround and defend the suburbs, if any; and in fine, to throw so many difficulties in the way of the besiegers, that by the length of the siege, the enemy may waste both his time and troops; besides, such protraction frequently affords an opportunity of obliging him to raise the siege, either by the approach of a relieving army, or by the setting in of a season unfit for military business.

The outworks recorded by engineers are various, such as *ravelins*, *half-moons*, *lunets*, *counterguards*, *horn-works*, *crown-works*, *tenails*, and several others: But many of them having been found defective, are now disused in all new works and alterations; and therefore those only which are now in most esteem will be here treated of, referring the more inquisitive readers to the writings of *Mallet*, *Dedier*, *Le Blond*, *Muller*, and others.

122. The constructions of outworks depend on much the same principles as those of the place itself; to which may be added:

1st. Every part of an outwork ought to be flanked or defended either from some neighbouring work, or from the town itself; whereby the enemy may not find any place of real shelter among the outworks, without being at the labour of throwing up a covering himself.

2d. As the chief defence depends on the musket, therefore the parts mutually flanking one another, ought to be within the ordinary reach of that weapon; that is, not exceeding about 300 yards.

3d. Every outwork ought to be commanded from the town; and therefore the more advanced works are ever to be the lowest. The gradation usually observed from work to work is about three feet; that is, the ramparts of all works advanced before a place gradually diminish three feet in height from the rampart of that place: Thus if the town ramparts are 18 feet high, the rampart of the next outwork is 15 feet high, that of the work next advanced is 12 feet, and so on; whereby the more advanced outworks are commanded by those next within.

N n n n

4th. The

4th. The flanked angle of an outwork ought not to be less than 60 degrees.

5th. Each outwork should be surrounded with a ditch communicating with the foss of the town: When all the ditches can be filled with water, they ought to be equally deep; but when that cannot be obtained, the ditches of outworks should be shallower than that of the town; whereby the fronts of the outworks will be better defended from the parts which flank them. The breadths of these ditches are usually from about 20 to 24 yards, and rounded at that breadth before the salient angles, with the counterscarps parallel to the faces or outlines of the work they encompass.

6th. The parapets of outworks are to be equally strong and high with those of the town, as they are to be cannon proof, and to cover the troops posted on the ramparts of those works: So that it is usual to make the parapets of outworks of about 18 feet thick, 6 or $6\frac{1}{2}$ feet high, with a foot-bank $4\frac{1}{2}$ feet lower than the crown or crest of the parapet; and the walk of the rampart about seven or eight yards broad, with a slope within of about once and a half its height.

When the plan of a fortification is to be drawn, in which are outworks, the counterscarp of the great foss is to remain in black lead pencil, until the outworks are annexed in pencil also, with their fosses; then the covered way and glacis are to be drawn surrounding the fosses of all the works.

123. *Of the Ravelin.* Plate XIII. Figs. 14, 15, 16, 17.

A RAVELIN is a work made before a curtain, serving to cover it and the adjoining flanks from the direct fire of an enemy: It is composed of two faces, AB, AC (fig. 14), and the two demigorges pB , pC , taken on the counterscarp.

To construct a Ravelin. In a line drawn at right angles to the curtain thro' its middle, take the capital pA equal to about 100 yards, and draw the faces AB, AC, directed to points in the faces of the next bastions, about ten yards distance from M, the angle of the shoulder.

In this work put a rampart of about 16 or 20 yards, and a parapet of about six yards; and before the faces put a foss of about 24 yards wide, rounded before the salient angle A; also annex ramps in the slope of the rampart, and if proper, construct a barbet in the salient angle.

The gorge of the ravelin, or the part next the town is left open, without any rampart; because the enemy cannot attack it there; and also, that it may be entirely commanded from the ramparts of the town,

R E M A R K S.

124. It is proper to make a ravelin before every curtain, as the flanks are thereby covered from the enemy, and cannot well be ruined before he is master of the ravelin; without which, he would have it in his power to batter the flanks from the counterscarp, both from before the curtain and the face of the bastion; the latter of these being the only place he can do it from, when a ravelin is interposed, of which he is not master;

master: But the difficulty of erecting a battery on the head of the glacis against the face of a bastion, is rather too great to be undertaken, while this place is defended by the whole fire of the adjacent face of the ravelin; and therefore the besiegers can scarcely hurt the flanks before they have taken the ravelin; consequently such works greatly augment the defence of a town.

125. As the face and foss of a ravelin is defended by part of the face of the opposite bastion; therefore should the faces of the ravelin be directed somewhat within the angle of the shoulder, at least as much as is the breadth of the parapet and foot-bank of the flank; besides, by this construction they more effectually cover the flanks, which is one of the principal things wanted, as the flanks are of most importance to be preserved the longest.

126. In wet ditches, where the troops pass from the town to the ravelin in boats, it is proper to make in the gorge of the ravelin, a kind of harbour, where the boats may lie covered from the fire of the enemy; and this may be done various ways: One is, by making an excavation limited by an arc described from A (fig. 15), the angle of the counterscarp, with a radius of about 20 yards. In dry fosses, it is proper to have either ramps or stairs in the gorge of the ravelin to preserve a free communication, should the bridges be broken by the enemy's shot.

127. In some ravelins (fig. 16) there is constructed a kind of *redout*, or *KEEP*; which is a small work similar to the ravelin, with a ditch of about 10 or 12 yards wide: The capital may be about 30 or 40 yards; and the work covered in the faces by a wall of a foot or two thick, furnished with loop-holes for the musketry to fire through.

This *keep* serves to secure a retreat for the troops who defend the ravelin, and on this account the defence of the breach in the ravelin may be more obstinately maintained; and even when the defendants are obliged to give way to a superior force, they may, when retired into the *keep*, prevent the enemy from making a lodgment in the outward part of the ravelin, or at least greatly obstruct it; and cannot be drove from this place until the enemy has erected a battery, and brought cannon on the ravelin to batter the keep.

The construction may be otherwise done, by taking on the gorge of the ravelin the distance BA equal to about 20 yards; and out of the rest form the keep with its foss; the scarp and counterscarp being drawn parallel to the faces of the ravelin, whose two fronts form a work covering the keep, and is usually called a Countergard.

128. The ravelins, called by some authors *half-moons*, have sometimes their corners at B and C, fig. 14, cut off either parallel to the capital p A, or at right angles to the foss; thereby making a kind of flank of about 12 yards long, with its rampart and parapet, and serves to give a more direct fire along the cover'd way, which they are proposed more particularly to defend: But then this partly lays open the flanks, which the ravelins were originally intended to cover.

129. The celebrated General Coehorn, in the ravelins which he built at *Bergen op zoom*, contrived a very good defence for the covered way before the faces of the bastions, by making retired flanks in the breasts of the ravelin, where one or two cannon might be placed as secure from

the enemy's fire, as those behind the orillon of a bastion. These covered cannon in the ravelin seem to have been intended for the defence of the covered way on the last emergency, like as those in the bastion were for the defence of the breach when the besiegers brought on the storm: But the covered cannon in the ravelin seem to have this preference; that as towns are not usually given up until the enemy is master of the covered way, the secreted guns in the ravelin have an opportunity of being employed in the use for which they were placed there; whereas the guns covered way by the orillons are very rarely of any use, as there are few garisons which stand the storm of the breach in a bastion.

130. Coehorn's ravelins may be imitated by the following construction. In the gorge of the ravelin take ab , fig. 17, equal to about 20 yards, draw the flank bc at right angles to ab , make bc equal to about 12 or 14 yards, and draw the cover cd parallel to ab : To the flank bc annex a rampart, parapet, embrasures and ramp; and to prevent these flanks being raked from the enemy's batteries, particularly by the *ricochet*, or bounding shot, there may be made a traverse of about 10 feet thick across the top of the flank at c , with a small mine under it, to blow up the traverse when the besieged are obliged to quit the ravelin, lest it should serve to cover the assailants from the fire of the town.

Coehorn has also made flanks at the corners d of the ravelin, which may be constructed by taking an equal to 10 yards, and drawing ne parallel to bc , meeting cd in d ; then de is the flank required.

It seems proper, when such ravelins are used, that the faces should spread on those of the adjoining bastions 10 or 12 yards farther from the angle of the shoulder, than in ravelins which have no flanks; and this is done at *Bergen op zoom*, where the faces of the ravelins would fall between 20 and 30 yards from the angle of the shoulder on the faces of the bastions.

131.

Of the Countergard.

A COUNTERGARD is a work generally serving to cover the faces of a bastion: It is composed of two faces, forming a salient angle before the flanked angle of a bastion.

To construct it. Plate XIII. Fig. 18.

Having described the ravelins and drawn their fosses; then in the counterscarp of the ravelin take ab equal to about 24 yards, and draw the face bc parallel to the counterscarp of the foss before the bastion; and the outline of the countergard will be determined; the inner boundary being the counterscarp of the grand foss.

In this work put a rampart of about 16 or 18 yards broad, with a parapet of six yards, and annex the ramps and a barbet if necessary: Also make a foss of 24 yards broad, the counterscarp being parallel to the faces.

The countergard seems to be, next to the ravelin, one of the most useful outworks; for it occupies but little ground, is of no great expence, covers the faces of the bastion so effectually, that they cannot be battered in breach until the enemy has made himself master of this work; and

and when he has it, he will meet with difficulties enough in finding earth to cover himself and erect his batteries; and must therefore be at a considerable trouble in bringing it from beyond the foss, while he is exposed to the fire of the faces of the neighbouring ravelins; which, with the flanks of the bastions, are the defences of the countergard.

132.

Of the Tenaillon.

A TENAILLON is a work composed of a ravelin and two detached works called *lunettes*, which either partly or wholly cover the faces of the ravelin from an enemy's direct fire.

When the detached works cover only the lower part of the faces of the ravelin, they are called *small lunettes*: But when they cover the whole faces, they are called *great lunettes*, and this is the work which is here meant by the word *tenaillon*.

133.

To construct a Tenaillon. Plate XIII. Fig. 19.

Having described the ravelin and its foss, whose counterscarp is rt ; then in the face ef continued, take tv equal to about 60 yards, and on the counterscarp of the grand foss take rs equal to 25 or 30 yards; then drawing sv , the figure $rsvt$ is the *great lunette*; to which a rampart, parapet, and ramps are to be annexed against its faces tv , and branches vs , of the same kind with those in the ravelin.

134. A great lunette may be considerably strengthened by making across its middle, at A, a retrenchment, consisting of a rampart, parapet, and a foss of about six or eight yards broad, and seven or eight feet deep; the parapet being drawn at right angles to the counterscarp of the ravelin against the middle of its face; and the ditch of the retrenchment communicating with that of the ravelin.

A *tenaillon* is a work capable of affording a great defence to the besieged, as was evident at the siege of Lisle in 1708, where the besiegers were twice or thrice drove out of a *tenaillon* they had taken and retaken.

135.

Of a Horn-work.

A HORN-WORK is a fortification composed of two long sides, called branches, tending towards the town, and of a front formed by a curtain between two half-bastions.

This work, when used, is sometimes put before a bastion, but oftner against a curtain.

136. *To construct a Horn-work before a curtain. Plate XIII. Fig. 20.*

In a line drawn from o , the angle of the counterscarp, at right angles to the curtain, take oc equal to about 250 or 260 yards; thro' c draw AA at right angles to oc , and make CA , CA , each equal to half oc , so AA is equal oc .

Take

Take the *normal* CD equal to $\frac{1}{2}$ of AA, thro' D draw the lines of defence AI; make the faces AB equal to $\frac{1}{2}$ AD; transfer the distance BB from B to I in the line of defence, draw the flanks BI and curtain II, and draw the branches AQ towards some point in the face of the bastion, about 8 or 10 yards from the angle of the shoulder.

Against the faces, flanks, curtain and branches, put a rampart and parapet as in the ravelins; make a foss of 24 yards broad before the flanked angles A, the counterscarp of the foss in the front being directed towards B, the angle of the shoulder, and the rest drawn parallel to the branches.

A small ravelin is usually put before the curtain of a horn-work, for the same uses as that before the curtain of the town: Make the capital CP about 60 yards, draw the faces as in the large ravelins, and make a foss of about 15 or 16 yards wide, whose counterscarp is parallel to the faces of the ravelin, which are to be furnished with a rampart and parapet.

137. Retrenchments are sometimes made in the horn-work as in the great lunette, and for the same reason, namely, to increase the difficulties of an enemy in a siege.

Also a ravelin with its foss is made before the curtain of the town, independant of the horn-work which it commands.

138. There is another work, called a Crown-work, whose front consists of a whole bastion, two half-bastions, and two curtains, and is bounded on the sides by two long branches directed towards the town. This work is like two horn-works connected together, by their branches being put side to side.

Horn-works and *Crown-works* have been chiefly used to inclose some rising or hollow ground, or a suburb; but are now in general disesteem, on account that their defence is very small in proportion to their cost, a circumstance by which all works should be partly rated.

There are several other works called outworks, which may be found in the treatises wrote on Fortification, but are here omitted on account of the intended brevity of this work.

139.

Of the Covered way and Glacis.

When all the intended outworks are annexed to the plan, and the several fosses are drawn, communicating one with another, then the covered way and glacis are to be drawn parallel to the counterscarps of the fosses; and a foot-bank is to range quite round the covered way, at the foot of its parapet. The top of this foot-bank should be at least four feet broad, that there may be convenient room left for the troops to stand on after the palisades are planted, a row of them being always to run quite round the covered way. The manner of setting them, as recommended by the most celebrated engineers, and particularly by the great Marshal Vauban, the most experienced man in military works that ever lived, is to put the palisades on the upper foot-bank at about a foot distant from the foot of the parapet, sunk in the ground at least three feet, and their top on a level with the head or crest of the glacis.

Palisades

Palisades thus placed are not exposed to the cannon of the besiegers; the most they can do will be only to brush their points: Neither can they be easily jumped over by the troops commanded to the attack of the covered way, nor readily cut down; and their distance from the parapet leaves a space sufficient for men to go along to repair any disorder.

140.

Of Places of Arms.

PLACES OF ARMS in the covered way, are spaces made at the entring and salient angles, considerably larger than the street of the cover'd way, and formed by turning the head of the glacis into two faces projecting towards the country: Those at the salient angles are formed by the roundings of the counterscarp; but those at the entring angles are thus constructed. Plate XIII. fig. 3.

In the head of the glacis take the gorges co, co , from 20 to 30 yards, draw the faces ot, ot , so that the angles toa be between 90 and 100 degrees; and parallel to these faces draw a glacis of about 50 yards.

The faces ot of the *entring places of arms* should not make angles greater than 100 degrees with the *branches of the covered way* oa , because those branches could not then be defended so well by the musketry, as when their shot was delivered directly along the glacis nearly parallel to the branches, which can only be done when the defences are nearly at right angles to the places defended.

Some writers have advised to round all the salient angles of the places of arms, because those angles could then be defended by more men: But perhaps there might be some advantage in notching the faces somewhat like a redan, as mentioned at article 69.

141. The great *Coehorn* was so conscious of the advantage of places of arms at the entring angles for the defence of the covered way, that he was very particular in their description, and took such care in the construction of those at *Bergen op zoom*, that they very well answered his design, as was evident in the famous siege of this place in the year 1747.

Coehorn's places of arms may be thus imitated.

Plate XIII. fig. 21. where the work is on a larger scale to be plainer.

Take the gorges ab, ab , about 45 yards, and the faces bc, bc , about 50 or 60 yards: At 10 yards distant from the faces, draw the *first lunette* A with a ditch before its faces, both together being about ten or twelve yards broad, its ends being terminated by the line ab : Within this lunette, at about 10 yards distance, draw a second B of about five yards broad, its ends stretching to about half way the breadth of the covered way: Behind the ends of each lunette, at the distance of two or three yards, put a traverse of about five yards thick, reaching from the counterscarp to about two or three yards farther than the ends of the lunette: And about the angle of the counterscarp make a screen of about six yards in each face and gorge; which screen is of brickwork with loop-holes for the musketry, to favour the retreat of the defendants when drove from their works in this well-contrived place of arms.

142. *Of*

142. *Of Traverses in the Covered way.*

When the besiegers have pushed on their approaches so far, as to be within 30 or 40 yards of the head of the glacis, it will be in their power to rake the covered way from some of their batteries; to prevent which, and also to shelter the troops in some measure from the ricochet or rebounding shot, traverses are thrown across the covered way, particularly on each side of every place of arms, and in every branch of the covered way where the length of that branch exceeds 50 or 60 yards.

Each traverse is furnished with a foot-bank, and a row of palisades planted on the foot-bank; and the passage round its end should be furnished with klinkets or doors, to shut them up when necessary.

143. *Of Sally-ways.*

In the middle of some faces of the entering places of arms, is made a sally-way, which is a passage cut thro' the parapet, or head of the glacis, for to facilitate the sallies of the besieged. To make sally-ways in the places of arms salient, would be improper; because as those places are the most convenient to be first attacked, they should not by any means be weakened.

These passages are cut oblique to the faces of the places of arms, and wind towards the salient angle in such a manner, as not to be easily enfiladed or raked; they rise gradually from the foot of the parapet, until they meet the slope of the glacis at the distance of seven or eight yards from its head. The roads to the town commonly pass thro' these sally-ways; every one of which should be furnished with a good barrier, to shut up against surprizes and sudden attacks.

144. *Of Communications.*

Every detached work in a fortification should have a ready and free communication preserved, otherwise they might be disadvantageous to the place.

Communications are preserved various ways, otherwise than by bridges, which are liable to be destroyed by the enemy's cannon. In dry fosses it is thus:

From the town to the tenaille, by a subterraneous way under the rampart in the middle of the curtain into the foss, and rise thence by ramps or stairs into the tenaille: From the tenaille by a subterraneous way into the caponier, and thence to the ravelin either by ramps, stairs, or a timber stage: From the ravelin to the works beyond, the communication is kept in like manner; or sometimes a way is sunk across a ditch, and covered with a parapet next the enemy; and the way from the foss to the places of arms is also by stairs, or by a timber stage.

Over wet ditches, the troops are conveyed by boats or rafts, or by little bridges about three feet wide lying nearly level with the water's edge; but the latter are inconvenient in the night, and much more so when the defendants are obliged to quit the work precipitately.

When

When wet fosses are to be represented in plans, it is done by giving them a tint with distilled verdigrease along the borders of their scarps and counterscarps; but in printed plans, it is expressed as in the ditch of the ravelin at fig. 15; and dry ditches are shewn in coloured plans, by washing the said borders with a tint of brown bistre; but in prints, dry fosses are expressed as in the ditch of the ravelin at fig. 14: And a *cuvet* is represented by a double line drawn along the middle of the foss, as against the front BB of the polygon.

145.

Of Works beyond the glacis.

Sometimes, when earth is wanting to raise the several works to their proper height, or when the defence is to be increased, a foss of about 20 or 25 yards broad is sunk at the foot of the glacis, encompassing either the whole or some convenient part thereof: But then it is proper that the glacis should slope quite to the bottom of this foss, if it is a dry one, or a wet one which can be drained; for otherwise, when the enemy comes to be possessed of it, he would find a trench ready dug to screen him from the fire of the covered way, and other places.

To this advanced foss is usually annexed, when it can be, a covered way and glacis; and there are not wanting fortifications which have three covered ways and glacis's one before the other.

146. Before some of the salient angles of the glacis there are sometimes constructed a kind of work called a *lunet*, or by some an *arrow*. It consists only of a parapet and foot-bank forming two faces, making a salient angle, whose legs are parallel to the head of the glacis, thro' whose ridge runs a narrow passage to the lunet.

147. Sometimes works called *redouts* are constructed before the salient angles; some of these works have fosses, covered way and glacis; others are made to depend on their own defence: But to all these detached works a proper communication must be preserved with the covered way, and in such a manner as not easily to be cut off. When this passage is two or three times as long as the glacis is broad, the passage should have one or more traverses across it, to prevent its being raked; also, on either side, it should be covered by a parapet and small glacis.

148.

Of Profiles.

The **PROFILE** of a work is a figure expressing the several heights, depths and breadths of that work.

The profiles of works are regulated according to their use, and also according to the quality of the materials with which those works are to be made.

Thus the ramparts of the town are ever thicker than those of out-works; the breadth of the parapet only remains nearly the same in most works: Besides, a rampart made of good firm earth, need not be so thick as one made of loose and sandy materials: From 20 to 24 yards will do in the former case; but less than 25 or 30 yards will not suffice with poor earth.

O o o o

Profiles

Profiles are also regulated with regard to the expence intended to be bestowed on the works; that is, according to the outside coating or walling of the rampart and parapet, namely, whether it be done with earth or intire masonry, or half one and half the other.

149. When a work of earth is raised, and the surface is covered or plaistered with a tenacious substance made of well-tempered earth or loam; this coating or plaistering is called *placcage* or *crusting*.

And when the surface is covered with grafs turf, or, as it is most common, with clods of fat mould cut in the form of wedges, and called *gazons* or *fods*: This kind of coating is called *gazonage* or *sodding*.

150. When the whole front of a work, from the bottom of the fofs to the crown of the parapet, is covered or fronted with a wall of stone or brick: This kind of work is called *intire masonry*.

151. When the front of the rampart, from the bottom of the fofs to the horizontal line, or ground level, or a little higher, is fronted with a wall of brick or stone, and the remaining part of the front is coated with earth; then this work is called *half masonry*.

Construction of Profiles.

152. When the works are coated with earth. Plate XIII. Fig. 22.

Let $AHLQ$ represent the horizontal, or ground line, taken thro' the face of a bastion, and across the fofs, and thro' the cover'd way and glacis: In this line take the following measures; namely,

$AR = 2$ or 3 yards; $Rc = 10$ yards; $cd = 6$ feet; $de = 1\frac{1}{2}$ feet; $eg = 18$ or 20 feet; $ga = 12$ or 14 feet; $aH = 9$ feet; $Hl = 12$ feet; $lm =$ breadth of fofs; $mL = 12$ feet; $LM = 5$ yards; $Mp = 7\frac{1}{2}$ feet; $pQ = 50$ yards.

Thro' the points $R, c, d, e, g, l, m, M, p$, draw lines perpendicular to AQ .

In these lines take $RB = 2$ or 3 yards; $cC = RB + 2$ feet; $dE = cC + 1\frac{1}{2}$ feet; $eF = dE + 4\frac{1}{2}$ feet; $gG = cF - 2\frac{1}{2}$ feet; $lI = 18$ feet; $mK = 18$ feet; $Mo = 2$ feet; $pP = 8$ feet.

Draw the lines $AR, BC, EF, FG, Ga, HI, KL, Lo, PQ$. Then draw ED parallel to AH , and equal to 3 feet, and draw DC ; also dispose of the space between o and P , in like manner as between c and F .

Then will the several parts of the profile be compleated, and represented; namely, BA the inner, and Ga the outside, slopes of the rampart, whose walk is BC ; FE the inside, and FG the crown of the parapet; CD the slope, and DE the top of the footbank; aH the berm or foreland; HI the scarp, KL the counterscarp, and IK the bottom, of the ditch; LM the cover'd way; oN the slope, and NO the top, of the footbank; OP the inside of the parapet, and PQ the slope of the glacis.

The palisades k are put on the top NO of the footbank.

153. In ramparts coated with earth, it is usual to set a row of palisades in the front of the rampart, as at h , about 5 or 6 feet below the crown of the parapet. These are not planted upright, but nearly horizontal, the point dripping a little below the level; and this work, which is called *fraising* or *staccage*, is done to prevent surprizes by scaling, and also to hinder desertions.

154. In most ramparts coated with earth, whether of the town or outworks, it is usual to plant on the berm, either a row of upright palisades, as at *f*, or a quick-set hedge of thorn, which should be kept constantly cut, to make it grow thick: Either of these methods serve to cover the troops in going their rounds. Sometimes both palisades and hedge are used, the hedge standing foremost.

The salient angles of all earth works should be rounded off; otherwise the weather would soon destroy the angular edges, and make them look worse than the other parts.

155. *When the coating of the work is intire masonry.* Fig. 23.

Having determined all the parts thro' the rampart to the line *Gg*, as at art. 152; in *Gg* take $GI = 3$ or 4 feet, for the upright face of the parapet; on *I*, with the radius of one foot, describe a semicircle representing the *cordon*, which is a large semicircular moulding that ranges along the whole front; take *gr* equal to 18 feet, and draw *rcst* parallel to the ground line, and this represents the bottom of the foss: Take *rc* and its parallel *ia*, each equal to 5 feet; draw the scarp *ic*, and draw *ab* parallel to *Gg*, which continue to about 6 feet below *rc*, and this line represents the back of the wall; thro' *b* draw a line *bdmn* parallel to the ground line, and this represents the bottom line of the foundations, which is usually about 6 feet below the bottoms of the fosses, tho' in some cases it must be more; and let the foundation line *bd* project about 2 feet before the scarp line *ic*: Draw a line about 3 feet behind the front line of the parapet *GI*, parallel to it, and the front wall of the parapet will be thereby represented; to which must be annexed, at *G*, a little projection, of about 6 inches in sally and height, to express the coving stone, and the outlines of the profile of the rampart will be completed.

156. From the directions already given, it will be easy to conceive how to regulate the profiles of outworks, whether they are to be fronted with turf or masonry; and according to art. 155, were the other profiles of the masonry annexed in the figures 24 and 25; that of 24 being a tenail before the curtain, whereof no more than the parapet rises above the ground level; and consequently the walls to support this work need not be so thick as those are which sustain the lateral thrust of higher banks; and for the same reason, the counterscarp walls are of a less strength. In the ravelin, fig. 25, the front is half masonry, or the walling is carried up to the ground level, and the front of the rampart and parapet is supposed to be coated with earth; but the walling in this case should project about 2 or 3 feet before the foot of the rampart, in order to preserve a proper berm.

157. The walks of all ramparts, and also the cover'd way, should slope backwards, in order to carry off the rain; this sloping may be about half an inch, or an inch in a foot.

158. The scarping usually given to earth facings, is about 8 inches on 12, or the base of the slope is $\frac{2}{3}$ of the height; and the scarping generally allowed to fronts of masonry is about $\frac{1}{5}$ or $\frac{1}{6}$ of its height; and with regard to the thickness of the walls, Mr. Vauban usually allowed about 5 feet at top, and gradually thickned towards the bottom by their

scarping of 1 foot in 5 of height ; and this profile he has experimented in the works of 150 towns, wherein he has directed the building of more than 4000000 of cubic yards of walling : But in order to add a greater strength to his walls, he puts, at the distance of 18 feet on the inside, buttresses built of rough stone, whose length was about the thickness of the wall at bottom, and their breadth, on a mean, about half the length ; and these buttresses being carried up as high as the cordon, greatly contributed to take off the thrust of the bank against the wall.

159. At the flanked angles of bastions, and other works, the parapet should rise about a foot or two higher for about 12 or 14 yards on either side the angle, than along the rest of the faces ; and the same should be observed for about 4 or 5 yards on each side of the more exposed salient angles of the cover'd way : For by these means, the troops ranged along those faces will be somewhat better defended from the ricochet, or bounding shot.

160.

Of Irregular fortification.

When a town or other place is to be fortified, it generally happens that the polygon which best circumscribes its plan will be very irregular, and consequently the fortifying of that polygon will not fall under the rules already given for regular fortification : It is therefore proper to point out to a learner how he may proceed on such occasions.

161. Let the plan of the place be circumscribed, if possible, by a polygon, whose angles are all salient, none less than 60 degrees, nor greater than 100 ; and neither of the sides less than 200 yards, nor greater than 350 : This polygon, which is to be the interior one, should have its sides not less distant from the houses than 50 yards ; and the fortification is to be made from the interior side.

Thro' every angle of the polygon draw a line bisecting it ; and on these lines, continued without the polygon, lay from the angular points, the capitals (in the table, art. 86) of those polygons, whose angles are nearest to the angles in the given irregular polygon : Also on each side of the capital, from the said angles, lay on the sides of the polygon the demi-gorges corresponding to the capitals used.

Then the lines of defence being drawn, and the corresponding faces laid thereon, the *master line* of the fortification to the given irregular polygon may be readily compleated (as in art. 91) ; to which the rampart, foss, and proper outworks may be annexed, as before shewn.

162. In the circumscribing of the given place with a polygon proper to it, should it so happen that the sides, when drawn at the distance of 50 yards from the houses, make a polygon very different from that above directed ; then should the positions and lengths of all those lines, or of some of them, be changed, and tried various ways, when the situation will admit of those various trials, until one is fitted to the place that will come near the foresaid limits ; notwithstanding the sides should, in some places, run many more yards from the houses than 50, and a space of ground greater than the first proposed spot should be included within the polygon : For although by these means the fortification will be more extended, and thereby incur a greater expence both in building and defending ; yet as it will, by its nearer approach to regularity, be stronger
in

in itself, and thereby more effectually answer the end of fortifying, the latter considerations may on some accounts be preferred to the former.

163. It should be observed, that regularity is not strictly confined to such polygons only as can be circumscribed by a circle, or nearly so; for a polygon inscribed in an oval kind of figure will have a great regularity, and may be made pretty nearly equally strong, by putting the stronger kind of outworks at the narrow ends: For as these ends, on account of their small breadth, in proportion to the longer sides, cannot flank the approaches of an enemy so well as the longer sides, and the enemy thereby not necessitated to extend his works; consequently the smaller ends may be more advantageously attacked, and therefore should have their outworks of the stronger sort.

164. If the situation is such, that one or more of the sides of the circumscribing polygon cannot by any easy means be made less than 400 yards or more; then should such sides be divided into parts of some number of yards, between 200 and 350, as many as its length will admit of, which may then be considered as composed of so many polygonal sides: At every division, make such a bastion as the limits will admit, taking care that the faces of each are commanded by the adjacent flanks of the next bastions.

165. When any of the angles become too acute to make in that place a whole bastion, the angle may be diminished only on one side, making there a half bastion, and let the other side be either left as one line, or make in it two or three small flanks, in the nature of a redan (71); observing that the whole front is commanded by the flank of the next bastion: Or should the angle be too small even for a half bastion, then both sides may be fortified in the manner of redans, and commanded from the flanks of the next bastions: But in the case of these acute angles, the angular point should be made very solid, and the sides about such angle shorter than the others.

166. Should the polygon have one or more entring angles which cannot be avoided; when that angle is near 90 degrees, and the adjacent sides do not each exceed 250 yards; those sides, with the assistance of the flanks of the next bastions, will very well defend one another, and also the entring angle: But should the angle exceed 120 degrees, then put against that angle a work like a bastion, if it can be; or if there is not room for the projection of the flanked angle, make but one face, observing that it be raked from one of the adjacent flanks.

167. Where a town has been fortified in the ancient manner, and has already a good rampart and foss, such a town may be put into a good state of defence against the modern methods of attack, by adding outworks so disposed as to cover the town wall, be mutually defended by one another, and commanded from the ramparts of the town; such a place, with the outworks judiciously disposed, may hold out as many days as a place formed on a regular plan; which has been evinced from the good defence made by several towns fortified in this manner.

168. There are a multitude of other things that might be said on the business of irregular fortification, which, on account of the designed brevity of this tract, must be omitted: But after all, the best notions are acquired in these matters from studying the plans of the fortified towns in several parts of the world. Collections of these plans may be met with at most print shops.

PART II.

Of Marine Fortifications.

SECTION I.

Of the kinds and properties of Harbours.

169. A HARBOUR or PORT is a hollow cavity in the land, into which the sea runs with a depth of water sufficient to float ships or other vessels capable of performing voyages on the ocean, and where such vessels may lie secured from the tempests of the sea.

As harbours may be of very different forms, so there may be a multitude of ways proposed for their defence: But herein the business will be confined to a few of the more common situations.

Harbours in general may be distinguished into two kinds, namely, artificial and natural.

170. *Artificial harbours* are such inlets of the coast, as are rendered safe for shipping to ride there, by building about its entrance certain works called MOLES or PIERS, which defend the ships within from the surges of the sea without: And when the *moles* or *piers* can be brought so close, as to be shut up by *sluices* or *gates*, the harbour is then called a BASON: But the inner part of a harbour, where the ships ride in still water, is usually called the *basin*.

All artificial ports of these kinds, are easily fortified by building on the moles or piers either batteries, redouts or forts, which command the entrance into the harbour: And as these works are usually done at the time when the moles are made, there need no more be said of them in this work.

171. *Natural harbours* are those where the natural disposition of the land forming them, is such as contributes to the safety of the shipping therein, by sheltering the vessels from the fury of the sea; and this happens in various manners, such as a shoal lying off the harbour's entrance, which breaks the waves, and thereby keeps the water still within; or by the points of land forming the harbour, stretching themselves so far into the sea, and coming so near to one another, that the surges are broke at their entrance; or by a narrow inlet that runs into the land for a considerable way, and where the shores near the entrance receiving the shock of the waves, renders the upper parts smooth enough to anchor in, of which kind are navigable rivers; or by a large deep bay, wherein islands, or other obstructions, contribute to render a part secure for the riding of ships at anchor, &c. &c.

172. The properties of a good harbour, are, a sufficient depth of water for large ships to enter at any time, whether the tide be in or out; good offing, and an easy access out of all danger from the winds; free entrance, without rock or sand bank; the entrance not too wide to be easily barred and defended upon occasion; not subject to overflow, and where ships may lie close to the keys; where the vessels are sheltered from all winds by the high mountains surrounding the harbour, and from whence there is an extended view of the sea; lastly, that the shipping therein be out of all danger, either from being set on fire, bombarded, or cannonaded from sea by the ships of an enemy.

173. There are many places in which ships anchor, that lie open to the sea, and are only safe to ride in when the weather is calm, or when particular winds blow; such anchoring places are called *roads*.

SECTION II.

Of the fortifying of Roads and Harbours.

174. FIRST SORT. *When a town lies open to the sea upon a curved or strait bold shore, and has before it a sufficient depth of water and good anchorage:* As such places can afford shelter for shipping, only at certain times of the year, when they are generally sure to have calm weather, or gentle gales, in which vessels may safely ride at anchor, they cannot so well be secured from the attacks of an enemy, as in places more inclosed: However, the shipping lying there during the season of trade, may be well defended by forts built near the water's edge on each side of the anchoring place; such forts being so contrived as to have two or three batteries one higher than the other, and these furnish'd with many cannon, carrying shot from 24 to 48 pounds, will awe the ships of an enemy in general from undertaking any thing against the vessels that are riding in that road.

175. But in order to defend the town itself, there should be a rampart or wall, well flanked, built along the shore, beside the fortifications on the land side. The works along the shore should be carried so near to the water's edge, that were any troops to land under the cannon of a fleet, they might not find any ground to intrench themselves on.

176. Forts built on a bold shore, to which ships of war can come within musket shot, are in general liable to be easily taken, unless some precautions are used: For as the batteries on shore are seldom raised much above the water's level, in order to be more certain of their mark; the troops posted at the guns are liable to be commanded by the marines quartered in the round tops, and other elevated parts of the ships: However, this may be prevented, either by driving several rows of piles (which are long pieces of timber, or the bodies of trees, pointed, and shod with iron at one end when necessary) in the water before the fort, whereby the shipping cannot approach it; or by fixing along the battery a kind of *shed* made with planks of musket proof, and covered with raw hides or earth, that they cannot be set on fire by combustibles thrown from the ships: These sheds are not only convenient to preserve the troops posted at the guns from musket shot, during the time of an attack, but they would also preserve the cannon and carriages from the weather at all times; for the expence of a few rough planks, used now and then to repair such sheds, added to the first cost, might perhaps, in a length of time, be found of less expence than the gun carriages they would preserve; and should it turn out otherwise, the framing and planks might be prepared in time of peace, and set up only when there are apprehensions of an attack; but it should be observed to fix them so, as to be the least exposed to the cannon from the ships. In such sheds, as they would be open behind, the smoak would be soon dissipated, and might be sooner thrown off

off by making holes in the shed over every gun ; which holes may be covered by a short plank raised before them.

177. SECOND SORT. *When a harbour, being a bay, has a shoal or small island lying before its entrance :* Such a place is fortified by building a strong fort on the island, in a place where it can command the entrance on both sides, when the size of the island is not too large for this purpose : But when the mouth of the harbour, and the extent of the island, is too large for one fort on the island to guard the entrance, then two or more forts should be raised thereon, in the most commodious places that can command the avenues to the bay.

178. Beside the forts raised on such an island or shoal before a harbour's entrance, there should be others raised on the most convenient points of land forming the mouth of the bay ; whereby the defence of the passage is made more secure, should the ships of an enemy by any accident pass the advanced defences.

179. When forts are thus built, it is not enough to contrive them in such a manner that their batteries may annoy the shipping which attempt to pass without leave ; but they should be also carefully fortified, to resist attempts from an enemy ashore ; for should this be neglected, a body of marines, or seamen, landing on the island or place where the battery or fort is, would frustrate the intention of such fortifications or defences ; of which there are many instances to be met with in history.

180. As there may be several spots about the points judged proper to erect the defences on, that will command in the manner as above, it is of considerable importance to know which will have most advantages, and fewest disadvantages ; for both circumstances are to be brought into the account, and that only chosen which has the most favourable balance. Thus, the soil should be the firmest, and most capable to bear the cannon and building to be erected—Of a height sufficient to be out of the reach of tides and floods, and not much higher than is barely necessary to avoid those inconveniences ; for the cannon commands best when near the water's level—A place difficult for an enemy to land in its neighbourhood ; and if landed, where they may be incommoded in their march to the fort—A place where succours may easily approach—Where the troops on duty may without much trouble obtain the conveniences of life—And where there are the fewest noxious exhalations.

When the most convenient place for a fort or battery is chosen, the next thing to consider, is whether the work is to be temporary or perpetual ; that is, whether the work is only to serve for a present emergency, or to be kept in repair under a probability of its being frequently attacked ; and whether the work be temporary or perpetual, it is necessary that it be fitted to the materials at hand, or that can be procured with the least trouble and expence.

181. As it may frequently happen, that in such situations here referred to, the passage between the island and main is rather too wide for cannon to command across ; therefore, if the place is of sufficient importance, there may be many ways contrived to remedy this defect, when it is possible to be avoided. The best are those which oblige shipping to keep in one track or channel ; and when this can be accomplished, either by sinking of old shipping, running out of moles, or contriving to form

form sand banks or shoals, or by any other means that the situation of the place can furnish; then may forts or batteries be advanced so far into the water, as to command that channel: Or if the passage cannot be reduced to less than two channels, then a fort should be erected, if possible, between them.

182. In the building of these works, great care should be taken that the foundations are laid sufficiently firm to support the superstructure; and as these places are generally low, either in marshy or sandy situations, a great number of piles should be drove into the soil, to the depth of 10, 20, 30, or more feet, according as they drive with more or less difficulty: If a quantity of such piles be driven, at the distance of 12 or 18 inches from one another, over the place intended for the foundation, and the tops of them be cut level some feet below the surface, then may a floor of timber be laid over the tops of the piles, and the building raised thereon.

183. THIRD SORT. *When the harbour is in a bay, whose points forming the entrance stretch into the sea, and approach one another within cannon shot:* Such a harbour is fortified by building on both sides its entrance one or more forts; and if it is possible, let a fort be also built within the harbour's mouth, in such a manner, that its cannon can rake the shipping fore and aft as they come in; for this would be a good reserve, should an enemy's fleet force their way between the forts which command the entrance: But should there be no proper spot for such a fort, then let others be erected on the most convenient points that can command the turning to the right and left within the mouth of the bay; for by these means an enemy will be obliged to endure a kind of running fire, which, if the forts be well constructed, he will scarcely be able to sustain.

184. FOURTH SORT. *When a harbour is formed by a cluster of islands:* Such a harbour is not difficult to fortify, when the channel between the islands is not too wide for the command of cannon from one or both shores; the directions how to place the forts in the foregoing cases being equally applicable to this. And in places where the channel is more than cannon shot from either shore, the shipping that can ride there, must depend upon being defended from the batteries ashore, as in the first case, when the shore is bold enough for the ships to run under the cannon.

185. FIFTH SORT. *When the harbour lies in an inlet or river some miles above its mouth.* If the passage to the port lies straight, and can be commanded from side to side, then a fort built at each point of the entrance, and two others between them and the harbour, but not opposite to one another, unless the width of the channel requires it, will in most cases be a proper security for the shipping in such a harbour: And when the channel or river is winding, the forts should be built where they can command a reach at least; or be so placed at the bends, as to command two adjacent reaches; for as vessels must tack near the forts so placed, they will at those times be more under the command of the batteries, than they would be when sailing by them on a direct course.

In the directions for the placing of forts in this, and the other cases, there has been no mention of the figure which it would be most proper

to construct them of; for the engineer, who has the charge of such works, generally does, or should take care, to fit the spot with a figure properly adapted to it: But for the sake of the mariner, it was judged proper to point out the methods usually taken on such occasions.

186.

Of the figure and size of Forts.

Beside the forts described as redouts (art. 68, 70), there are many others, such as squares, pentagons, hexagons, or the halves of these figures, which are fortified with intire bastions, by much the same kinds of constructions, as are the polygons in the table art. 86; only the measures of the *exterior side, normal* and *face*, are much less than those for a town; and are usually about the following numbers.

Make the side or front about 180 yards; the normal one seventh, or one sixth of the side; and the face about twice the normal, or a little less; with a ditch, if necessary, of about 20 yards broad; a rampart broad enough for a platform, and a parapet of about 12 feet thick.

The exterior sides of forts may be considerably less or greater than 180 yards, even between the limits of 60 yards to 200; according to the size of the spot on which it is to be built, joined to the importance of the place it is to defend: For it would be a great absurdity, either to build a large expensive fort to secure a harbour or port of little more consequence than to give shelter to a few fishing boats; or, on the contrary, to build a small trifling work to guard a haven where many ships of the utmost importance are laid up, or riding at anchor.

It is not always necessary, or even convenient, to make these forts on figures perfectly regular; for the situation may require some of its sides to be shorter than others: Neither is it necessary that the figures should be fortified with bastions, or half bastions, at the angles; for there are many cases where it will answer the purpose equally well, or even better, to make a bastion on the side of a figure towards the middle; but then the faces of the bastion should be so drawn, that they may be raked from the extremities of that side, by a defence of at least 10 or 12 yards.

187.

Of proper Forts, and their disposition.

If the place on which the fort is to be built is a spot surrounded with water, forming a channel on both sides; then the figure is to be disposed, so as to have either one or two of its fronts turned towards each channel, whereby the batteries in the flanks will scour both up and down the stream, while those in the faces and curtains command the passage directly before them. That part to seaward, may either have a front extended along it, or have one of the points of the figure directed that way, according as it may be judged most convenient; and if before these fronts there is any place at which an enemy can land, then one or more batteries should be so formed, that their guns may be brought to bear on that landing place; and if this be not thought sufficient, a small outwork should be built near that place, to make the landing more dangerous; and on account of the convenience of landing, the fort should be furnished with a ditch, covered way, and glacis: The part next the harbour will

will be sufficiently secured by a line thrown up properly flanked. Should the situation be on a rock, where there is no danger of an enemy's landing; then a good line, capable of covering the troops and cannon, will suffice. *If there is only a channel on one side fit for shipping to pass through;* then that side should be fortified as in the former case; and least the enemy's boats should get through on the other side, it should be also put in a state of defence: But here a slight redan, furnished with a few pieces of small cannon, may be found a sufficient defence against the boats.

188. When a fort is built on a point of the main, the same kind of disposition used in the foregoing case, next to the channel, will also obtain in this; but it is necessary to take proper care of the land side, which in this case is more liable to be attacked than in the former. In the fortifying of the land side, the works should be proportioned to the strength of the enemy that may probably ever appear before it: For there are many places, where there is no likelihood of ever seeing an enemy stronger than the force which can be exerted by a fleet of three or four ships of war; and as these cannot be furnished sufficiently for undertaking a regular land siege, therefore a well flanked line, with a ditch, if properly defended, will secure the place against the attacks of sailors or marines unprovided with artillery: Or should a few pieces be brought against the place, their passage to it will be very dangerous, unless they can be brought near enough to act under cover; and to leave such a cover would be an unpardonable oversight in those who had the care of constructing that fort. But when the importance of a place may bring before it a large fleet, and a land army properly appointed, then should the fortifications on the land side be put in a state capable of sustaining a vigorous siege.

There are many low points on which batteries may be raised for the defence of a channel, over which a lofty ship may have a considerable command: In such forts proper traverses should be placed, that the line of the battery be not raked by the ships guns.

189. In the constructions of all forts, it should be remembered, that the figure of fewest sides and bastions, that can probably answer the expected defence, is ever to be preferred; as the works on such plans are sooner executed, and with less expence; besides, fewer troops will serve, and they are more readily brought together in cases of necessity: And with regard to the executive part of such works, enough has been already said, observing that where stone and brick are scarce, and there is plenty of wood, then timber works are to be most used; and, on the contrary, stone or brick earth being plenty, and wood scarce, masonry is to be preferred; taking care never to use sea water in the lime, nor to make the parapet intirely of stone; for the splinters drove about by an enemy's cannon, will make it dangerous for the defendants to continue their posts.

SECTION III.

190.

Of the fortifying harbours by Booms.

Notwithstanding the forts built for the security of a harbour, or passage, there are many instances that may be produced to shew that they alone are not sufficient. The great Sir Walter Rawleigh says, “ In this age, a valiant and judicious *man of war* will not fear to pass by the best appointed fort of Europe with the help of a good tide, and a leading gale of wind; no, tho’ 40 pieces of great artillery open their mouths against him, and threaten to tear him in pieces.

“ In the beginning of Queen Elizabeth’s reign, when Denmark and Sweden were at war, our Eastland fleet, bound for Liefland, was forbidden by the King of Denmark to trade with the subjects of his enemies, and he threatned to sink their ships if they came thro’ the streights of Elfseneur. Notwithstanding this, our merchants, having a ship of her majesty’s, called the *Minion*, to defend them, made the adventure; and sustaining some vollies of shot, kept on their course. The king made all the provision he could to stop them on their return: But the *Minion*, commanded (as I take it) by William Burrough, leading the way, did not only pass with little loss, but did beat down with artillery a great part of the fort of Elfseneur, which at that time was not so well rampar’d, as now perhaps it is; and the fleet of merchants that followed him went thro’ without any wound received. Neither was it long since, that the Duke of *Parma* besieging *Antwerp*, and finding no possibility to master it otherwise than by famine, laid his cannon on the bank of the river, so well to purpose, that he thought it impossible for the least boat to pass by: Yet the *Hollanders* and *Zealanders*, not blown up by any wind of glory, but coming to find a good market for their butter and cheese, even the poor men attending their profit, when all things were extream dear at *Antwerp*, passed in boats of ten or twelve tun, by the mouth of the duke’s cannon, in despite of them, when a strong westerly wind and tide of flood favoured them; as also with a contrary wind, and ebbing tide, they returned back again: So as he was forced in the end to build his *stockado* overthwart the river, to his marvelous trouble and charge. The fort of *St. Philip* terrified us not in the year 1596, when we entered the port at *Cadiz*; neither did the fort at *Puntal*, when we were entered, beat us from our anchoring by it, tho’ it played upon us with four demi-cannon, within point blank, from fix in the morning till twelve at noon. The siege of *Ostend*, and of many other places, may be given for proof, how hard a matter it is to stop the passage of a good ship, without another to encounter it. Yet it is true, that where a fort is so set as that of *Angra* in *Tercera*, that there is no passing along beside it, or that ships are driven to turn upon a bow-line towards it, wanting all help of wind and tide; there, and in such places, it is of great use and fearful: Otherwise not.”

But since Sir Walter’s time, the Sound has been better fortified, yet in 1700 Sir George Rook passed it, notwithstanding the Danish fleet lay to impede his passage. The actions at Londonderry, Vigo, Porto-bello, Cartha-

Carthage, and at many other places, are all sufficient proofs, that neither forts nor castles can stop the enterprizes of the brave; and therefore it has been judged proper on many occasions, besides forts and batteries, to lay booms and other obstructions in a stream, to secure the shipping there from the attacks of a powerful enemy.

191.

To lay a Boom.

Provide a great number of wooden battlings or spars of about 20, 30, or 40 feet long, more or less, and between 5 and 10 inches diameter: Then moor two boats, having a sheet anchor in each, in the place near one side of the river where it is intended the boom shall begin; bend two cables to these anchors, and round them place the spars or poles, frapping on each with ratling stuff, or better with four inch rope, until the boom is 7, 8, 9, 10, or more feet diameter (according to the hands employed), the cables being in the middle; next, with iron hoops rivetted together, worm the boom, and drive thro' the hoop a nail into almost ever spar: After having wrought a good birth from the anchors, drop them, and continue the work till it is brought near the other edge of the river, and there drop two anchors more with the cables bent to them: Then over all lash the spare yards and top-masts with the top chains, so far as the channel goes.

To that part of the cable in the channel that makes the boom, let other cables be fastened, and these bent to anchors laid down the stream; over the clinch of these cables let battlings or spars be wrought for near ten fathoms down the stream, that the enemy may not cut these cables: These will be serviceable when the enemy's ships come *stemlings* against it; for if he force it in one place, the whole will not be opened by that fracture.

In places where wood is scarce, and proper spars are not to be readily had; or in cases where there is not time sufficient to prepare them; it may be found sufficient to woold two cables together with old ropes, lashing to them the oars, top-masts, and yards, and worming the whole about with iron hoops; and let every part be well *payed* with pitch, and small gravel strewed on it while the pitch is warm. A boom so prepared cannot be easily cut.

If on any particular occasion it should be found convenient and safe to open the passage, it may be done by tripping the anchors that are on one shore and those down the stream, whereby the boats may swing down the stream upon the ebb, and upon the flood the boom may be relaid, if the enemy appear in sight, which he must do the first of the tide, for upon the ebb there is no danger of his coming; because, if the wind be right in, a prudent enemy will not adventure against the tide, a ship then making such wild steerage; and should she ground, she must lie there till flood, which may prove fatal to her from the batteries ashore; and against both wind and tide the enemy cannot come in. On the contrary, should the boom be carried up the flood, and the enemy appear at the beginning of the next flood, the boom cannot be relaid till the ebb, and before that time the enemy may have accomplished his design.

192.

192. If there be good store of timber at hand, then a *stockado* may be made by driving several rows of piles in the channel before the boom; or should the depth of water be too great for this work, the trees may be usefully applied, by making a raft to ride before the boom by good anchors, observing that the cables are so made fast, that the enemy can't come to cut them: These rafts will be of singular use in putting the enemy into disorder, by laying rubbish upon them to make fires of when the enemy appears, which will put him into some consternation, and perhaps may cause him to chop to anchor, and lose time or his tide; and this may be done each flood, observing not to have the smoke drive into the works, which might give the enemy too great an advantage.

193. *To lay a boom in a streight channel.*

Suppose the wind for the most part to blow obliquely across the river or channel on the starboard quarter going up the river: Now if the boom be laid directly thwart the channel, the ships coming with the tide of flood and a leading gale will run stemlings against it, and so hazard its breaking, by striking with a force that comes direct; whereas, could the boom be laid obliquely thwart the river, nearly in a line with the wind, so as the ship must take it with her bow, the blow would be diverted by the ship's casting; because in this position the enemy cannot run stemlings against it; for in sailing up the river, the ship must be near before the helm; and to bring her head to the boom, the helm must be put down, and then it is ten to one whether she come too so nicely as to take the boom: Not to mention the mistakes, in a confusion, committed by the man at the *helm*, and he upon the *cond*, nor the smok; for exclusive of all these, it may be taken for granted the ship would cast along side the boom: Then the batteries at the end of the boom on the larboard side would rake him fore and aft, while the opposite batteries on the starboard side playing on his broadside with *double-round* and *partridge*, must make a great slaughter among his men cutting at the *boom*. And should he not swing along side the boom, but lie stemling against it, then will the batteries on the starboard side the river, which are to be made above the weather end of the boom, rake him fore and aft, while those on the larboard side, against the leeward end, play on his broadside.

194. The ships within the boom, which it is to protect, should be moor'd in a kind of half-moon, with their broadsides flanking the boom. And several old ships, or those the least useful, may be sunk as soon as a signal, signifying that the boom will be cut, is made at one of the forts which defend it; for this purpose, those ships should have large scuttles ready cut: And for a farther security, it would be very proper to have a small boom to divert the enemy, that the ships may be sunk in the channel before he boards them.

As to the rest of the ships, they are left to their commanders judgment; and if there is no other remedy, the people may get ashore to windward if they can, and there make the best defence in their power against the enemy's boats and fireships. From duly weighing the whole of these circumstances, it will be found that the chief strength is in the boom;

boom ; therefore if a double, triple, or fourfold boom was laid, if the materials could be procured, and the value of the shipping and cargoes were of sufficient importance, it would make the place so much the stronger, and the enterprize of the enemy more hazardous. In stretching these booms, the trouble of many anchors may be spared, by making all the cables fast to the first, and so let them float in a bight, and by a small anchor ride upon the ebb, to keep clear of one another.

195. *To set up a boom in the bend of a river.*

From the point formed by a bend, stretch two booms across the channel, one towards the middle of the opposite bight, and the other so much higher, as to lie directly thwart the channel, leaving a kind of angular space between them.

Next the point from whence the two booms stretch, erect a proper fort or battery to command the channel both below and above the bend.

On the other side the river, erect another fort or battery against the bight a little above the end of the lower boom, and so disposed, that its cannon may rake the channel coming up to the boom, as well as command both the booms.

A boom being thus disposed, it is a great chance if a ship answer her helm so timely in *bearing* or *loofing* about the point as to take it stemlings, and if she smite it with her bow, she casts; and in either case she will be raked fore and aft by one fort, and have her broadside battered by the other.

Let some old vessels be fitted up for fireships, and placed between the two booms; from each ship let two hawsers be carried ashore, one on each side, and fixed to cramps set up; so that as soon as the enemy has passed the first boom, these ships being set on fire, and heaved in their way, nothing can hinder the enemy's destruction: As soon as fire is set to the train, the boat may pull ashore, under covert of the ship, without any apparent danger; for such will be the enemy's consternation, that they'll soon leave firing. The hawsers may be fastened to clamps below the water-line, that they be not burnt, nor in sight of the enemy to cut them. If the fireship is clapt aboard the weathermost ship, they may be both heaved together aboard another to leeward.

The ships to be defended, may be moored in a half moon, with their broadsides so laid, as to batter the enemy when he attempts the boom: And if other works than what are here directed be judged necessary, they must be adapted to the situation of the place.

196. When no strong attacks by land are to be fear'd, the mooring of ships behind a point is best, on account of laying the boom: Now should the boom be forced, which must be upon the flood, a fireship, instead of falling on board a ship thus moored, will by the tide be hurried beyond her; and if the place is any ways favourable to the ships moored there, it will be found impracticable to board a ship thus moor'd, with such a wind and tide as the enemy must have to break the boom, unless he expose his boats in carrying out an anchor to *warp* over, which will be a very dangerous attempt; or some unaccountable accident intervene.

197. It may be objected, that ships thus moor'd cannot bring their broadsides to bear upon the enemy when coming to force the boom; and that they may be battered by the enemy over the point. To this it may be answered, first: It is no matter whether the broadsides do or not, for perhaps it might be the best way to get the guns ashore, if there is time, to be there used; and to put the smaller fort only on the lower deck, and turned to flank the boom. Secondly, The enemy will have but an uncomfortable *birth* in lying to batter the ships over the point, as they will have to deal with the fire from the fortifications ashore. What is here said, is only to be understood when five or six ships are to be defended; for the wake of a point would not hold any considerable fleet.

198. *Maxims to be observed in fortifying a harbour and mooring the shipping therein.*

I. Let all the land marks be removed that may direct the enemy's steerage into, or up the river or harbour, when there is a probability of an attack.

II. Choose such a place if possible for erecting the works on, that cannot be laid under water, either by hasty rains, or by any art of the enemy.

III. Let the eminencies which command the works within cannon shot, be secured by erecting small forts on them, furnished with small cannon.

IV. All woods within cannon shot of the fortifications are to be cut down, and the timber employed about the works.

V. Secure a spring of fresh water by a fort and proper guard.

VI. Take all the buoys from the anchors that ride the booms, that the enemy may not trip them.

VII. Let the powder and other munitions be separated and kept in different places, that if an accident happens, the whole may not be destroyed.

VIII. The ships should not be posted near any town or village, unless they are intended to cover that place, or the thing be unavoidable; lest the shot and bombs which miss the ships destroy the town.

IX. The ships should never be moor'd before the fortifications.

X. Moor the ships so as to rake the enemy fore and aft when he batters the works.

XI. Let not the ships when moor'd touch one another, that the enemy may be obliged to burn each ship singly.

XII. Unrig the ships, and strip the shrouds from the mast-head, that the grapplings of fireships may have no hold.

XIII. Let the sails be carried ashore for tents, and most of the gunner's stores; that if the worst happen to the ships, there may be store of ammunition for the forts.

XIV. Unless the cargo be ashore, let not the chests nor cloths of any person, from the captain to the swabber, be carried out of the ship; whereby every man may have some inducement to exert himself in the defence.

XV. Let outguards and centinels be placed upon and near the banks of the river or harbour; and let some hands in nimble boats, armed, pass down the stream each night to watch the motions of the enemy.

SECTION

SECTION IV.

199. *Of the mooring of ships in fortified harbours.*

Before the boom is laid, it will be necessary to appoint the place where to moor the vessels, that they may be best secured from the enemy's fire-ships, and yet annoy their men of war; and as this can be only done either in the stream or out of it, the advantages and disadvantages resulting in both cases will be here considered.

200. *Considerations in mooring with regard to the stream.*

FIRST. *The advantages an enemy has of a ship moor'd in the stream.*

1st. A ship moor'd in the stream will find some difficulty in bringing her broadside to gaul an enemy in his approach, who always comes with the stream; and if the tide runs too swift for one to lie thwart, it certainly does so for the other; both having ground tackle equally good.

2d. In this situation there is no hindering a fireship, that has passed the boom, from being aboard you, as none but the chase guns can be brought to bear on him; and should you heave thwart, he will be the surer of grappling you, unless by bringing your whole broadside to bear, you sink him; for should the officer, thro' want of courage, forsake the fireship, yet the tide will bring him aboard: Moreover, in such a position the enemy's ships of war may board you, and then your batteries ashore are useless.

3d. In riding in the stream, the cables are exposed to the enemy; which if hit by a chance shot, the ship will swing upon the tide, and so the enemy have the opportunity of raking you fore and aft; and if the enemy has any conduct, he may lay his anchors so as his cables shall be secured from your shot.

4th. Should you moor thwart the stream, the enemy may drop a small anchor out astern, veer thwart your hawse, and so constrain you to alter your position, or rake you fore and aft.

5th. In riding thwart the stream, the cables by bearing a more than ordinary strain are apt to break, or start an anchor; whereby the ship swinging on the tide, it is a great chance but the other anchor starts, and she swing to leeward. But these advantages are only on the enemy's side while the tide sets in; for the defendants have them when the tide sets out: Therefore, in such places where the stream runs continually out, the best way is to moor in it, if the wind does not blow always in.

201. *SECONDLY. The advantages a ship moor'd in a stream has over an enemy.*

1st. A ship moor'd thwart the tide, lies convenient to rake an enemy fore and aft in his approach, and so may do him a considerable damage before he is thwart the stream to batter.

2d. In riding thus in the stream, according to the common situation of rivers, the enemy cannot batter you under covert.

202. *THIRDLY. A ship moor'd out of the stream has these disadvantages.*

1st. That she cannot rake the enemy fore and aft so well as while lying in the stream; unless by some particular situation of the place.

Q q q q

2d. The

2d. The enemy's men are not much exposed in mooring; for it is but dropping his anchor and he may veer along your side; or if you lie thwart the river, across your hawse: But this may be prevented by riding in the wake of a point; for, should the enemy drop anchor before he be about the point, he cannot veer along side; and if he let go his anchor after he is about the point, he may be astern before he is brought up.

203. FOURTHLY. *A ship moor'd out of the stream has these advantages.*

1st. There is no danger of the fireships driving aboard her, unless directed by a strong wind: And for the same reason, the enemy cannot be thwart your hawse, unless he tows there with his boats, and then they and their crews are exposed to your cannon.

2d. A ship thus moor'd is in no danger of driving should her cables be shot, if it is calm, as it commonly happens in engagements.

Considerations in mooring with regard to the wind.

204. FIRST. *With regard to windward.*

Beside the advantage of being free from the smoke, you may cause the enemy to drive to leeward, should you hit his moorings; and if he tries to heave off by carrying an anchor to windward, his boats will be exposed to your cannon; and if the boats *lay to pass*, his ship while he heaves will be raked fore and aft, which must do great execution among his men while they stand thick at the capstan: But it is highly probable, a ship so grounded could not be got off.

When to windward, the cables are cover'd from the enemy by the ship, and may be *seized to clamps and cleats* nailed to the side, and so the mooring secured from random shot: And this ought to be carefully done for preventing the only disadvantage that attends ships moor'd to windward, which is, that if their cables are shot they drive out upon the enemy, or swing head at wind; if the former, they may be boarded by the enemy's fireships, or ships of war, and then your precautions are rendered fruitless; if the latter, they may be raked fore and aft by the enemy; and if neither of these happen, the ship may drive to leeward and be grounded.

205. SECONDLY. *With regard to leeward.*

A ship so moor'd cannot prevent the enemy's being sent on board, unless a shoal or stockado lie between them.

In this position, should you shoot the enemy's cables through his carelessness in covering them with his hull; yet as he drives ashore, or upon you, his boats are cover'd with his hull while they carry an anchor to windward to heave off.

Besides your moorings lying to windward, are exposed to the enemy; which should they be cut, you will swing head at wind, and in that position be raked fore and aft; and if you attempt to carry out an anchor to heave your broadside against the enemy, your boats will be too much under his command; indeed, instead of carrying out an anchor, it may be best to clap a spring upon the cable, but this will fail if the ship is too near the shore.

Upon the whole, it appears best to moor to the windward in the wake of a low point, when such a situation can be obtained; but the circumstances being so various, it must be left to the judgment of the commander, who, if he rightly considers what has been said, will chuse for the best.

SECTION

SECTION V.

206. *Of some remarkable actions concerning the attack of ships in fortified harbours.*

Having shewn the methods that have been directed by experienced men, to be used in the defence of harbours, we shall now think of closing this work with the accounts of some of the most remarkable actions recorded within the last 100 years, wherein the English have had any share; and this was thought useful on two accounts: First, as examples to the preceding precepts; and secondly, to point out to the young mariner the means by which many have acquired great reputation and honour.

207. *The battle of Santa Cruz in the year 1657.*

Santa Cruz is a town and bay on the west side of the island of Teneriff, one of the Canary islands: The bay, which lies rather open than inclosed, has deep water near in shore; but the best anchoring is about half a mile from shore, in 30, 40, or 50 fathoms water, black slimy ground: If there be many ships, they must ride close one by another. The shore is generally high land, and in most places is steep to the water.

Admiral Blake lying with some ships near Cadiz, to watch for the return of the Spanish plate fleet, had intelligence they were in the bay of Santa Cruz, in the island of Teneriff; hereupon he broke ground the 13th of April, and on the 20th arrived at Santa Cruz, where he found the Spanish fleet, to the number of sixteen, moor'd in the bay in a half moon: Near the mouth of the bay was a castle well furnished with heavy cannon; and besides that, there were seven forts round about the bay, with 6, 4, or 3 great guns in each, all united by a line of communication from fort to fort, and well lined with musketeers. *Don Diego Diagues*, the Spanish general of the fleet, upon sight of the English fleet, caused the ten smaller ships to be moored close to the shore, and set six great galleons, well mann'd, further out at anchor, with their broad sides to sea; in this posture the Spanish admiral vainly thought himself so secure, that a Dutch merchant ship going out of the harbour, he sent a message by him to *Blake*, that he might now come if he durst.

The English admiral having well viewed their posture, saw it would be impossible to bring off the galleons; however, he resolved to burn them, and to that end sent in Captain *Stayner*, commander of the *Speaker* frigate, with a squadron to attack them: He soon forced his passage into the bay, whilst other frigates entertained the forts and lines with continual broadsides: These were presently supported by *Blake* himself with the whole fleet, who placing some of his ships to batter the castles and other forts, he with *Stayner* continued to engage the galleons; over whom, in six hours, he gained (notwithstanding they had reinforcements of men several times from the shore) a complete victory, set them on fire, and every one of them was burnt to the water's edge. They had no sooner done this, than the wind luckily turn'd, and carried the fleet, without the loss of one ship, out of the bay, and put them safe to sea again.

Londonderry is a small but pretty strong town in the north of Ireland ; it stands on the west side of the river *Loughfoyle*, about three miles from a lake of the same name ; which is about ten miles broad and fifteen miles long, communicating with the sea by a streight of about a mile broad, so that the town stands about 18 miles from the sea : The river next the town is about $\frac{1}{4}$ of a mile over, and keeps this breadth nearly, between the town and lake, where its mouth is defended by Culmore fort, which is a large square with four bastions.

This town was besieged by the Irish and French in the year 1689, and the garison brought into great straits for want of provisions, the besiegers having strongly fortified the river. There was a boom framed of a chain and cables, covered with timbers, stretched across the river about two miles below the town, from a point on the east side : Each end of the boom was defended by a fort. Between the boom and Culmore fort were two others, one on each side the river, and another fort above the boom opposite the town. The sides of that narrow river were intrenched and lined with musketeers, and several boats were sunk, and stockadoes drove, armed with large iron spikes.

Notwithstanding all these preparations, the boom was forced, the town relieved, and the siege raised, by the assistance brought by three ships only, in the following manner : The *Montjoy of Derry*, commanded by Captain *Browning*, and the *Phoenix of Colrain*, by Captain *Douglas*, being both laden with provision, were sent towards the town, under the convoy of the *Dartmouth* frigate, commanded by Captain *John Leake*. They were forced to stand a furious fire of the enemy from Culmore fort, and from the other forts and intrenchments on both sides of the river, which they received and returned with the greatest bravery imaginable. The *Montjoy* made a little stop at the boom, occasioned by her rebound, after striking and breaking it, so that she was run aground. Upon this the enemy gave a loud and joyful shout (tho' a dreadful one to the besieged), fired all their guns upon her, and were preparing their boats to board her. The trouble and concern of the besieged, to see their last hopes disappointed, is not to be expressed ; but by great providence, firing a broadside, the shock loosened the ship so, that she got clear and passed the boom. Captain *Douglas* was engaged all this while, and the *Dartmouth* gave the enemy very warm entertainment, till at length the three ships, having overcome the great difficulty of breaking and passing the boom, got up to the city, and brought unspeakable joy and transport to a garison, which reckoned only upon two days life, having nothing left but nine lean horses, and a pint of meal to each man. This brave and successful undertaking so discouraged the enemy, that they raised the siege the next day, and retreated by night in the utmost confusion, but made a most miserable havock in the country, by robbing and burning all before them for several miles.

The boom appears to have been laid directly thwart the stream, and so was easier to force, than if had been laid obliquely across.

The English fleet under the command of *Sir George Rook* and *Sir Thomas Hobson* were convoying of a very large fleet of merchant ships towards the Mediterranean; but falling in with the French fleet under the command of Admiral *Tourville*, the English admiral gave the signal for the merchantmen to shift for themselves; and among others, four heavy laden Turkymen; namely, the *Shandois*, the *Italian merchant*, the *Asia*, and the *Loyalty*, stood away for the Spanish ports, and put into the bay of Gibraltar; where the commanders of the Turkey ships, being hindered from prosecuting their voyage by some dissention on board, and the fear of the French fleet, thought proper to fortify themselves.

For this purpose, the commanders posted their four ships in a line within the new mole, which stretches to the NW., with their broadsides flanking towards the north, or entrance into the mole; the ships were unrigg'd, and consequently having no sails to manage, they turn'd all hands to the cannon, bringing as many as they conveniently could to bear on one side.

On the SE. end of the mole, or land side, was a castle, in which was mounted ten small cannon, four pointing into the bay towards the west, three commanding the mole, and the other flanking the NE. shore: Opposite to the mole-head was a kind of redent, very advantageously built to flank the entrance into the mole; but neither on this, nor the mole-head, had the Spaniards any cannon. From the castle there ran northward a line of communication, or a strong stone wall, to the south end of the town, where was a battery of cannon, but too far from the mole to be of any service in defending it.

The English stretch'd a cable from the mole-head to the redent, the ends being bent to two sheet anchors, and to this cable they lashed their *spare yards* and *top-masts*. Upon the mole-head was planted six of the *Shandois's* guns to flank the boom, and this battery was committed to the care of some English seamen, who behaved themselves very well. The commanders intended to plant some guns at the other end of the boom, upon the redent, but being obliged to go thro' the castle, the draw-bridge broke, and down went the first gun that came upon it, into a ditch near forty feet deep, with some of the *Shandois's* men; by this it may be judged in what condition the Spanish works and garison were.

This was the posture things were in, when the French came into the bay with fourteen or sixteen men of war and two bomb-ketches; and standing within shot of the castle, the Spaniards shewed their dexterity in managing their ordnance, by firing about one shot in half an hour. The commanders perceiving their awkwardness, sent a quartermaster from on board the *Shandois*, and the gunner's mate of the *Loyalty*, to their assistance, and some dozens of cartouches; but thro' want of judgment in the Spaniards, they were both blown up. All this time not a ship could bring a gun to bear, and so without any damage the enemy passed by into the bay where they anchored.

In the afternoon they sent down two men of war to view in what posture the ships lay; as soon as they came within shot, the English fired briskly at them, but before many broadsides were returned, they stood away to their squadron, and

and continued quiet that day: Next morning a ship of upwards of fifty guns came towards the boom, and it proving calm when she came within shot, there was pretty brisk firing on both sides, on which she endeavoured to tow off, but her boats were sunk from the battery on the mole-head, and had not boats come from the squadron to her assistance, she had most certainly been sunk, having above 100 cannon playing upon her from the English. The enemy finding such entertainment provided for them before the boom, sent off four *men of war* and the two *bomb-ketches*, who lying to the southward of the mole, and firing over it, raked the English fore and aft; who on account of their former disposition could not bring one gun to bear upon the enemy, neither from the four ships nor from the mole; under these circumstances, the commanders judged it best to sink the ships. In the evening the French stood out of the bay, and next day stood in again; and having posted some *men of war* and their two bomb-ketches against the castle and mole, they sent in a storeship they had turned into a fireship, and burnt the upper works of three sinking ships; for the *Italian merchant's* fore-castle being under water, saved the Shandois.

210.

Disposition of the English fleet at Cadiz 1695.

The 16th of October Sir George Rooke arrived in the bay of Cadiz with the fleet under his command, and on the 21st it was resolved in a council of war, that as there were but 30 ships of the line, which was not a force sufficient to oppose the enemy, who were expected to be above double that number, all that could be proposed till the fleet should be reinforced from England, was to protect the trade: But lest the French should appear at *Cadiz* before this reinforcement arrived, it was agreed that the ships should be removed within *Puntal* castle, and formed in three lines; which was all that could be done, the ships being in general but half manned, and the Dutch ships very foul.

These three lines were to be as follows: The first, to consist of the largest English ships, to lie from *Puntal* thwart the channel, to the creek's mouth called *Truccadero*, next within the north castle: The second, to be composed of the smallest English and Dutch ships, along the shoal on the south side of the harbour: And the third, to be of the largest Dutch ships, to begin from the upper end of the second line, and to trench away thwart the channel, to the mouth of the upper creek, which goes to *Port Real*; and the small frigates, bomb-vessels, and fireships, were to be posted to the best advantage, as the wind and other circumstances might permit. However, the fleet was not attacked, and about the middle of March Sir George put to sea on his return to England, and arrived in the Channel the 22d of April.

211.

Fortifications in the harbour of St. John in Newfoundland 1697.

Commodore Norris being sent with a fleet to recover Hudson's bay, put into the harbour of St. John in Newfoundland, upon hearing a large French fleet were upon the coast of North America; and not knowing the strength of the enemy, he judged it prudent to secure his force, that
it

it might be employed in the service he was sent on. He therefore put his squadron into a line of battle, which was composed of four fourth rates, two fifth, and two sixth rates, with two fireships, two bomb-vessels, and a hag boat: Two booms were laid across the harbour, and Colonel Gibson's regiment, that was carried by the squadron from England, were embarked, and several guns mounted on the batteries ashore.

When the French squadron came in sight, the foregoing disposition was altered: All the *men of war* lay in a half moon to the harbour's mouth; and each ship's broadside commanded the two booms: Colonel Gibson's regiment was posted at the two batteries, and about the harbour's mouth, and Captain Richards (who was the engineer for fortifying the harbour) was making such works as he judged necessary on this occasion; and that the squadron might be better able to do service, all the men were taken from the merchant ships and put on board them.

The French fleet made no attempt on the harbour; but, on the contrary, were glad to find that the English had been misinformed of their strength, and so shaped their course for Europe.

212.

The battle of Vigo, in the year 1702.

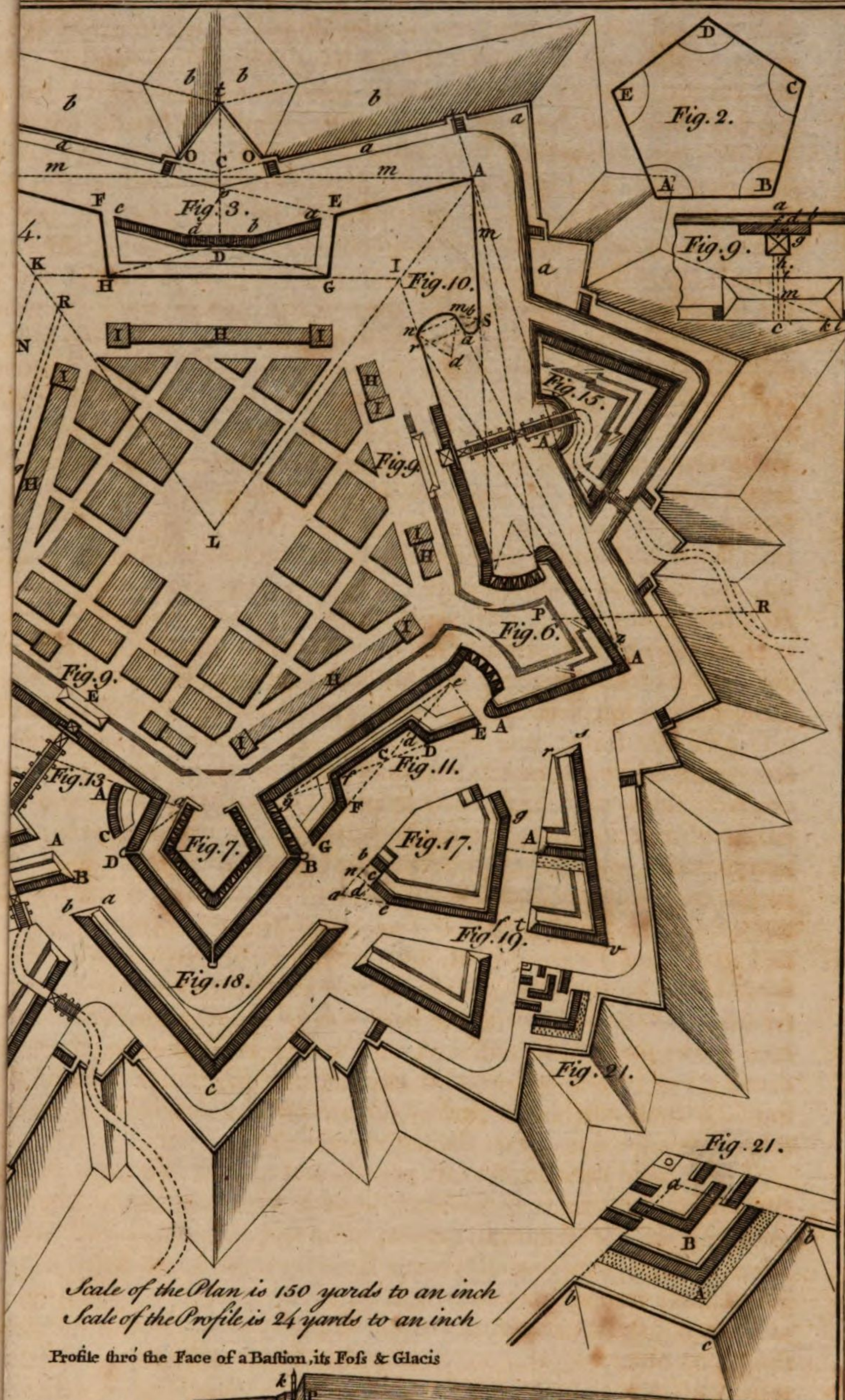
Monsieur *Chateau-Renault* arriving early in September at Vigo in Galicia, with the Spanish galleons under his convoy, received information of the English and Dutch fleets being before Cadiz; of which timely intelligence he made all the advantage he could in preparing for their reception.

In the upper end of *Vigo bay* is a river running into the bay; up this river is *Redondella* harbour, which is surrounded with hills in such a manner, as to be capable of being made very strong, the harbour's mouth not being above a musket shot over. On the north side the entrance was a battery of eight brass, and twelve iron guns, and on the south side a platform with twenty stately brass guns, and twenty good iron guns, besides a stone fort, with a trench about it, mounted with ten guns, and defended by 500 men. A little above the battery on the right, from two points in a narrow gut, was stretch'd a cable, to which was lashed masts, yards, casks, &c. with their top-chains, and frap'd about with four inch rope very thick: The bight of the boom was rid by three anchors down the stream, that should the English ships come stemlings against it, they might not bring home the anchors upon the shelf at the end of the boom; or if they cut it in one place, the whole might not be open: To flank the boom, there was moor'd the *L'Esperance*, a ship of seventy guns, on the south side, and the *Le Bourbon*, a ship of seventy-six guns, on the north side; also five men of war, of sixty and seventy guns each, lay moor'd with their broadsides flanking the gut. The French admiral, to give him his due commendation, had taken all human precautions to secure his fleet.

When the confederate fleet came before the place, Sir George Rooke called a council of war, and it was resolved to attempt the forcing of the harbour the next morning. As soon as the land forces were got on shore, the 12th of October in the morning, the admiral gave the signal to weigh, the line was formed, and the squadron was briskly bearing up

to the boom; but when the van was got within shot of the batteries, it fell calm, so that they were necessitated to come to anchor: However, a fresh gale springing up not long after, Vice-admiral *Hopson*, in the *Torbay*, being next the enemy, immediately cut his cables, clapt on all his sails, and bearing up directly against the boom, amidst all the enemy's fire, broke thro' it at once, cast anchor between the *L'Esperance* and *Bourbon*, and with unparallel'd resolution received several broadsides from them. The rest of Vice-admiral *Hopson*'s division, and Vice-admiral *Vandergoe*'s with his detachment, having weighed at the same time, sailed abreast towards the boom, to add the greater weight and force to the shock, but being becalmed they all stuck, and were obliged to hack and cut their way thro'; but a fresh gale blowing again, the Dutch Vice-admiral made such good use of it, that having hit the passage which the valiant *Hopson* had made, he went boldly in, and made himself master of the *Bourbon*. All this while Admiral *Hopson* was in great danger, for being clapt on board by a French fireship, whereby his rigging was presently set on fire, he expected every moment to be burnt; but it very fortunately fell out, that the French ship, which indeed was a merchantman laden with snuff, and fitted up in haste for a fireship, being blown up, the snuff in some measure extinguished the fire, and preserved *Hopson*'s ship from being consumed, which however received very considerable damage in this action: At the same time Captain *Bokenham*, in the *Association* of ninety guns, laid his broadside against a battery of seventeen guns on the left side the harbour, while Captain *Wynell* in the *Barfleur*, a ship of like force, was sent to batter the fort on the other side. Thus for a long while there was a considerable firing of great and small shot on both sides, till the French admiral seeing the platform and fort in the hands of the English, his fireship spent in vain, the *Bourbon* taken, the boom broke, and the confederate fleet pouring in upon him, he set fire to his own ship, and ordered the rest of the captains under his command to follow his example, which was done in great confusion; yet could he not be so punctually obeyed, but that several men of war and galleons were taken by the *English* and *Dutch*.—In this action it should be observed, that the success of the fleet was in a great measure owing to the land forces under the command of the Duke of *Ormond*, who not only drove away several bodies of troops destined for the defence of the shore, but also took the castle and platform on the south side, which might otherwise have done great damage to the fleet.

There have been in the last war several gallant attempts made on fortified harbours, such as those of Admiral *Vernon* at *Porto-bello*, and *Cartagena*; of the Admirals *Anson*, *Warren*, *Knowles*, and other brave officers; and also several booms have been set up in different harbours; but as these transactions are fresh in every one's mind, and the accounts of them common enough to be met with, they are here omitted: But it should be observed, that all the booms which have been lately set up, were placed oblique to the stream; and is now become a general maxim among the marine officers.



Robertson, John

The Elements Of Navigation; Containing The Theory and Practice With all
the necessary Tables ; To which is added, A Treatise of Marine
Fortification ; For the Use of the Royal Mathematical School at Christ's
Hospital, and the Gentlemen of the Navy ; In Two Volumes
Bd.: 2

London (1754)

Math.u. 170-2

urn:nbn:de:bvb:12-bsb10594239-5